Lab #2 - Two Degrees-of-Freedom Oscillator

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INTRODUCTION

The system illustrated in Figure 1 has two degrees-of-freedom. This means that two is the minimum number of coordinates necessary to uniquely specify the position of the system. The purpose of this laboratory is to introduce you to some of the properties of linear vibrating systems with two or more degrees-of-freedom. You have already seen a one degree-of-freedom vibrating system (the mass-spring-dashpot system) and should have some familiarity with the ideas of *natural frequency* and *resonance*. These ideas still apply to an undamped linear system with two or more degrees-of-freedom.

The new idea for many degrees-of-freedom systems is the concept of *modes* (also called *normal modes*). Each *mode shape* has its own natural frequency and will resonate if forced at that frequency. The number of modes a system has is equal to the number of degrees-of-freedom. Thus the system above has two modes and two natural frequencies.

The primary goals of this laboratory are for you to learn the concept of normal modes in a two degrees-of-freedom system – the simplest system which exhibits such modes. You will learn this by experimentation and calculation

PRE-LAB QUESTIONS

Read through the laboratory instructions and then answer the following questions:

1. Are the number of degrees of freedom of a system and the number of its normal modes related? Explain.
2. How can a normal mode be recognized physically?
3. What do you expect to happen when you drive a system at one of its natural frequencies?
4. Draw a free body diagram and derive the equations of motion for a three degrees-of-freedom system, with three different masses and four equal springs. Put them in matrix form. (See the derivation for a two degrees-of-freedom system in the lab manual. Your result should resemble equation (5).) Substitute in the normal mode solution (7) to get an eigenvalue problem similar to (9).
5. Using Matlab, find the eigenvalues and eigenvectors of the following matrix and print the results (HINT: Type `help eig` for assistance).

$$[A] = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \tag{1}$$

NORMAL MODES

The concept of normal modes can be expressed mathematically in the following way. Say the position of an n-degree of freedom system can be described by the n numbers x_1, x_2, x_3 $\ldots x_n$ (this is, in fact, the definition of an n degrees-of-freedom system). Since the system is dynamic, each of these variables is a function of time $x_1(t)$, $x_2(t)$ etc. A motion of the system corresponds to a specified list of these functions. In general these functions of time can be quite complicated. However, for linear undamped systems there turns out to be many solutions that are, in some sense, simple. In fact, there are n such simple solutions called *normal mode vibrations*. A fortunate and often used fact is that every possible solution of the system can be written as a sum of these solutions. (In the language of linear algebra one can say that the normal mode solutions *span* the space of all solutions.) A normal mode solution for a five degrees-of-freedom system looks like

$$\underline{\mathbf{x}}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} = \underline{\mathbf{v}}\,(A\cos\omega t + B\sin\omega t) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} (A\cos\omega t + B\sin\omega t) \tag{2}$$

A normal mode vibration is characterized by a mode shape $\underline{\mathbf{v}}$ and an angular frequency ω (the "natural frequency" for the given mode shape). The mode shape $\underline{\mathbf{v}}$ is a list of constants (v_1, v_2, ...) that determine the relative amplitude of motion for each degree-of-freedom of the system. The constants A and B determine the amplitude and phase of the vibration. Note that in a normal mode vibration each point moves exactly as in simple harmonic motion. All points are moving with the same angular frequency ω and are exactly in-phase or exactly out-of-phase, depending on the signs of the appropriate elements of $\underline{\mathbf{v}}$.

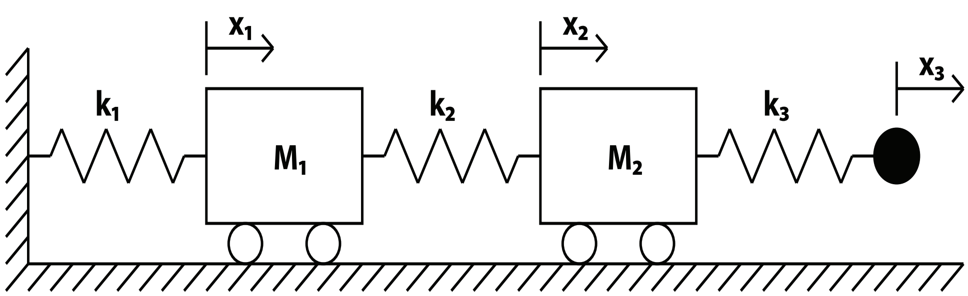

Figure 1: Illustration of a coupled mass-spring system.

The general motion of an n degrees-of-freedom undamped linear vibrating system can be written as the sum of normal mode solutions.

$$\underline{\mathbf{x}}(t) = \sum_{i=1}^{n} (A_i\cos\omega_i t + B_i\sin\omega_i t)\,\underline{\mathbf{v_i}} \tag{3}$$

The system is characterized by its natural frequencies ω_i and mode shapes $\underline{\mathbf{v_i}}$. The constants A_i and B_i are determined by the initial conditions and specify the amplitude and phase of the i-th normal mode. The mathematics involved in the discussion above is very similar to the mathematics for a set of first-order differential equations. (The governing equations for an n degrees-of-freedom vibrating system can, in fact, be written as a set of $2n$ first order equations.)

DERIVING AND SOLVING THE EQUATIONS OF MOTION

We will now derive the equations of motion for the two degrees-of-freedom air track experiment. The variables and physical setup are shown in Figures 1 and 3. We will draw the free-body diagram for each mass and work out its equation of motion.

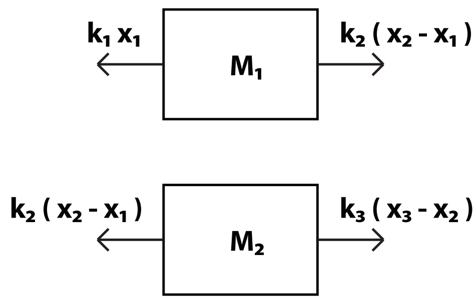

Figure 2: The free-body diagrams for masses m_1 and m_2.

To help get the signs right, assume that the displacements, velocities, and accelerations of the masses are all positive (i.e. to the right) with $x_1 < x_2 < x_3$. This puts all of the springs into tension relative to their equilibrium condition. The equations of motion (assuming equal spring constants) are

$$k\,(x_2 - x_1) - kx_1 = m_1\ddot{x}_1 \tag{4a}$$

$$k\,(x_3 - x_2) - k\,(x_2 - x_1) = m_2\ddot{x}_2 \tag{4b}$$

We can rewrite this in matrix form as

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{2k}{m_1} & \frac{k}{m_1} \\ \frac{k}{m_2} & -\frac{2k}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{kx_3(t)}{m_2} \end{bmatrix} \tag{5}$$

or as

$$\underline{\ddot{\mathbf{x}}} = [A]\,\underline{\mathbf{x}} + \underline{\mathbf{f}}(t) \tag{6}$$

We now take a lead from ODE theory and propose a solution to (6) (assuming $\underline{\mathbf{f}}(t) = \underline{\mathbf{0}}$, i.e. no external forcing) of the form

$$\underline{\mathbf{x}}(t) = \underline{\mathbf{v}}e^{\alpha t} \tag{7}$$

Substituting (7) into (6) and canceling out the exponentials yields

$$\alpha^2\underline{\mathbf{v}} = [A]\,\underline{\mathbf{v}} \tag{8}$$

Equation (8) is in the form of an eigenvalue problem from linear algebra. The values of α we need to find to complete our solution (7) are really the square roots of the eigenvalues of the matrix $[A]$. Furthermore the vector $\underline{\mathbf{v_i}}$ is the eigenvector associated with eigenvalue λ_i. Rearranging this equation we get

$$\big([A] - \alpha^2\,[I]\big)\,\underline{\mathbf{v}} = \underline{\mathbf{0}} \tag{9}$$

We would like to find a solution to this equation that doesn't involving setting $\underline{\mathbf{v}} = \underline{\mathbf{0}}$ since this is the trivial solution where neither mass is moving and the entire system is at rest. From basic linear algebra theory this requires the matrix $([A] - \alpha^2\,[I])$ be singular, i.e. non-invertible. Stated mathematically, we require the determinant of this matrix to be equal to 0.

$$|[A] - \lambda\,[I]| = \lambda^2 + \left(\frac{2k}{m_2} + \frac{2k}{m_1}\right)\lambda + \frac{3k^2}{m_1 m_2} = 0 \tag{10}$$

where we have substituted $\lambda = \alpha^2$. To simplify our calculations we now make the assumption that $m_1 = m_2 = 1$. This reduces the equation to

$$\lambda^2 + 4k\lambda + 3k^2 = 0 \tag{11}$$

with solutions $\lambda_1 = -k$ and $\lambda_2 = -3k$. The two eigenvectors associated with λ_1 and λ_2 are found by substituting each eigenvalue back into equation (8) and solving for $\underline{\mathbf{v}}$, giving us

$$\underline{\mathbf{v_1}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \underline{\mathbf{v_2}} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{12}$$

Therefore we can write the solution to (6) as

$$\underline{\mathbf{x}}(t) = c_1 \underline{\mathbf{v_1}} e^{\sqrt{-k}t} + c_2 \underline{\mathbf{v_2}} e^{\sqrt{-3k}t} \tag{13}$$

Using Euler's identity we can rewrite this in terms of trigonometric functions as

$$\underline{\mathbf{x}}(t) = \underline{\mathbf{v_1}} \left(A_1 \cos(\omega_1 t) + B_1 \sin(\omega_1 t) \right) + \underline{\mathbf{v_2}} \left(A_2 \cos(\omega_2 t) + B_2 \sin(\omega_2 t) \right) \tag{14}$$

where $\omega_1 = \sqrt{k}$ and $\omega_2 = \sqrt{3k}$. The coefficients A_i, B_i (for $i = 1, 2$) are the four unknowns to be determined by initial conditions (recall that our system is comprised of 2 second-order ODEs, thus the 4 required initial conditions).

Physically, equation (14) tells us that the motion of each mass can be written as a linear combination of a high-frequency and a low-frequency harmonic oscillation. These are the *normal mode oscillations*. To get a better idea of the physical significance of the normal modes, let us perform a simple initial value problem (IVP).

First we will assume that we initially displace both m_1 and m_2 by a positive distance x_0 (placing them in-phase with one another) and release them from rest. Plugging $t = 0$ into (14) the first initial condition yields

$$\underline{\mathbf{x}}(0) = A_1 \underline{\mathbf{v_1}} + A_2 \underline{\mathbf{v_2}} = \underline{\mathbf{x_0}} \tag{15}$$

Differentiating (14) with respect to t, the second initial condition (initially at rest) yields

$$\underline{\dot{\mathbf{x}}}(0) = B_1 \omega_1 \underline{\mathbf{v_1}} + B_2 \omega_2 \underline{\mathbf{v_2}} = \underline{\mathbf{0}} \tag{16}$$

Equations (15) and (16) give us four equations (since $\mathbf{v_1}$ and $\mathbf{v_2}$ are both 2 by 1 vectors) involving four unknowns. Solving for the unknowns gives us $A_1 = x_0$ and $A_2 = B_1 = B_2 = 0$. The solution to the IVP is then

$$\underline{\mathbf{x}}(t) = x_0 \underline{\mathbf{v_1}} \cos \omega_1 t \tag{17}$$

We see that by setting the system to be initially in-phase, the resulting motion consists only of the first normal mode. Since x_0 was chosen arbitrarily, we could easily just assume that $x_0 = 1$. Thus at $t = 0$ we have

$$\underline{\mathbf{x}}(0) = \underline{\mathbf{v_1}} \tag{18}$$

We see from (18) that it is the eigenvector associated with the normal mode that tells us the necessary initial displacements in order to excite that normal mode when starting from rest. Furthermore, since any scalar multiple of an eigenvector still satisfies the eigenvalue equation

(9), we do not need to worry about what units we take the eigenvector to be in (i.e. if the eigenvector tells us to move each mass by 1, we can move them 1 cm or 1 inch).

If we were to perform another IVP with initial displacements $x_1(0) = x_0$ and $x_2(0) = -x_0$ we would see that the solution would consist of only the second normal mode. Thus we can conclude that for the two degrees-of-freedom system the first normal mode represents in-phase motion while the second normal mode represents out-of-phase motion. One interesting result of our analysis is that the normal mode corresponding to in-phase motion has a lower natural frequency than the out-of-phase normal mode. *Why do you think that is?*

Fortunately for us we won't need to perform all this linear algebra during the lab. We can easily compute the eigenvalues and eigenvectors of the matrix $[A]$ using computer software. Each lab station computer has a numerical analysis program called *SciLab* installed on it and by following the directions given in the lab set-up you will be able to calculate the necessary values easily.

LABORATORY SET-UP

- **Air Track**
  The lab set-up consist of an air-track hooked up to the lab's air system, four or more air track gliders, four plug-in springs, a mechanical oscillator (for external forcing), a photogate timer, and a digital stopwatch. Please note that there are two somewhat incompatible styles of glider which should only be used on the appropriate air tracks. Each glider has a label listing its mass (including spring) and the air tracks on which it will work. **You should make sure to remeasure the masses of the gliders and springs at the start of your lab.**

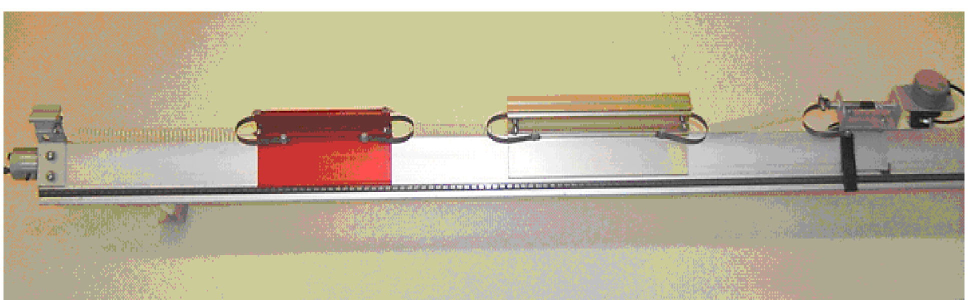

Figure 3: The laboratory set-up you will be working with.

- **Using the SciLab Software**

  1. Open *SciLab* by clicking on its icon located on the desktop of your computer. This program is a freeware program similar to *MatLab* and should look quite similar.
  2. To find the eigenvalues and eigenvectors of a matrix you must use the function `spec()` as shown below. Send the matrix $[A]$ as the function parameter and the program will return the eigenvalues along the diagonal of a square matrix and the eigenvectors as the columns of the second returned matrix.

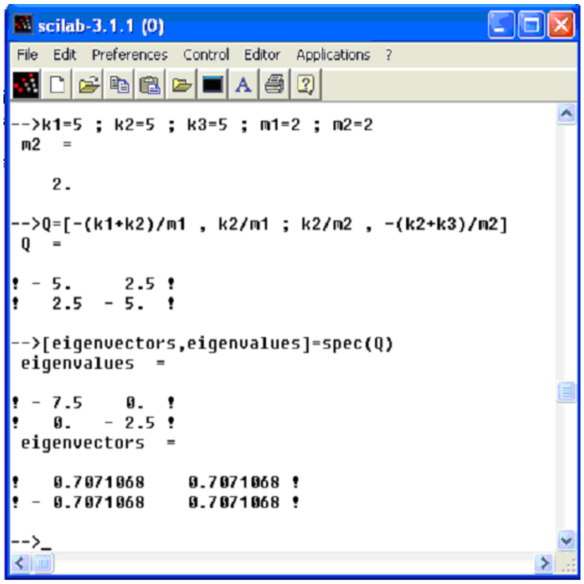

Figure 4: Screenshot of *Scilab* in use.

3. For the lab, however, you must find $[A]$ for a three degrees-of-freedom system. This example is for the two degrees-of-freedom simulation. If you wish to try this function on *MatLab*, everything is the same except for the function name. In *MatLab* you must enter `eig(A)` to find the eigenvalues and eigenvectors.

PROCEDURE

1. Play with the air track, gliders, and timer. Adjust the mechanical oscillator left or right so that each spring, at equilibrium, has a total length of about 20 cm (the oscillator is attached with a Velcro strap). Have the TA turn on the main air supply, if it is not already on, and turn on the valve at the end of the air track.

2. Find the spring constant for your springs. Attach a small weight (40 to 50 grams) to one end of the spring and hold the other end solidly against the tabletop. Pull the weight down a few centimeters and release it, and then measure the period of oscillation. Use $\omega = \sqrt{\frac{k}{m}}$ to find k. Remember to include part of the spring as well as the plug mass in m – half is a good approximation in this case. Check several springs to determine the variability in k.

3. Choose two gliders of different sizes and calculate the eigenvalues and eigenvectors for the two normal modes. The eigenvalues are the squares of the natural frequencies of the normal modes (in radians/sec.), and the eigenvectors describe the relative amplitudes of the mass motions. You may do the calculations by hand or use *SciLab* on the computer. Weigh the gliders if necessary; remember to include the mass of the plug-in springs.

4. The system is set into a normal mode oscillation by applying the appropriate initial conditions. First, place the system in equilibrium. One simple method is to turn the air track on and off repeatedly until the gliders stop moving. With the air off, displace m_1 an arbitrary distance d

(normally 1 or 2 centimeters) and displace m_2 a distance $d(\frac{v_2}{v_1})$. For example, if $\frac{v_2}{v_1} = -2$ and you move m_1 2 cm to the right, you should move m_2 4 cm to the left. (NOTE: The variables v_1 and v_2 represent the first and second elements of an eigenvector $\underline{\mathbf{v}}$, not the eigenvectors themselves.) Turn on the air track valve abruptly. The system should oscillate in a normal mode.

Find the angular frequency of oscillation (radians per second) corresponding to each normal mode and verify that they are approximately equal to the natural frequencies calculated. Note the phase difference between the two masses at each normal mode. The angular frequency of the masses is found by timing a number of oscillations (i.e. 10) and then converting the resulting period to ω. Digital stopwatches are available at the air track.

5. Use some arbitrary initial conditions and set the system into a non-normal mode oscillation. Observe the motion. (It should be difficult to see that it is the sum of normal mode vibrations.)

6. Attempt to obtain normal mode vibrations by driving the system at each natural frequency. The frequency of oscillation is obtained by timing the motion of the driving rod connected to the motor, using either a stopwatch or a photogate timer. With the air off, set the driving frequency to one of the natural frequencies you have calculated. Does the system resonate when you turn the air on? Be patient. Start the system from rest every time you change the motor speed. Time the frequency at resonance and compare it to the natural frequencies. Observe, as best you can, the relative phase between the scotch yoke and the masses at resonance.

7. Set up the air track with three (approximately) equal masses and four (approximately) equal springs. Adjust the mechanical oscillator to give an equilibrium spring length of about 20 cm. Verify by observation that $[1 \quad -1.414 \quad 1]^T$ is approximately a normal mode for this system.

8. Find another normal mode for this system by observation. Find another still. Are there any more? Use *SciLab* to find the normal modes and natural frequencies.

9. Using *SciLab*, find the normal modes and natural frequencies for a system with three unequal masses and four equal springs, and test them on the air track (free vibration only).

LAB REPORT QUESTIONS

1. List your values of k for the springs and a sample calculation. What is the average value of k, and what was the largest variation from the average (in percent)?
2. Did you obtain normal mode oscillations using initial conditions based on your eigenvectors? How could you tell?
3. How close were your experimental frequencies to those calculated? How does this experiment deviate from theory?
4. In what way did the block motions look like normal mode vibrations when you forced the system? In what ways did they not look like normal mode vibrations? Consider three cases:
   (a) forcing frequency = a natural frequency
   (b) forcing frequency close to a natural frequency (Was the amplitude of the oscillations constant in this case? If not, how did it vary?)
   (c) forcing frequency far from a natural frequency.
5. Write down the equations of motion for the system with two equal masses and three equal, massless, linear springs, as derived previously in the pre-lab. Assume x_3 (see Figure 1) is a given function of time: i.e. $x_3 = \sin t$.
6. Derive the equations again, this time with x_3 fixed at zero but with a known force F acting on m_2 in addition to the two spring forces.
7. Suppose that $F = k \sin t$. Show that the systems in #5 and in #6 are mathematically equivalent.
8. If the spring forces were given by the equation $F_{sp} = kx^2$ and the force F in #6 was given by $F = k \sin^2 t$, would the two systems still be equivalent?
9. How many normal modes are there in the three equal mass system? What are they and how did you recognize them as normal modes? How many were you able to find experimentally? How do they compare with those you calculated?
10. What were your calculated normal modes and natural frequencies for the system with three unequal masses? Did normal mode oscillations occur with these ratios and frequencies on the air track?

CALCULATIONS & NOTES

SOLVING THE EQUATIONS OF MOTION VIA A CHANGE OF BASIS

So far we have discussed how normal modes are the simplest oscillatory functions from which *all* motions of the two degrees-of-freedom system can be thought to be comprised of. Mathematically, the normal modes y_1 and y_2 satisfy the equations of motion for simple harmonic oscillators with natural frequencies ω_1 and ω_2 respectively.

$$\ddot{y}_1 + \omega_1^2 y_1 = 0 \tag{19a}$$

$$\ddot{y}_2 + \omega_2^2 y_2 = 0 \tag{19b}$$

Since the equations of motion for the normal modes are simple in terms of the y_1, y_2 coordinates, it would be nice if we could find some transformation between the physical coordinates x_1, x_2 and these new variables, i.e. $\underline{\mathbf{x}} = f(\underline{\mathbf{y}})$, so that we can solve the problem in terms of the easier coordinates and then transform back into the original ones. We can accomplish this mathematically by performing a change-of-basis from the original basis into the *eigenbasis* of $[A]$. We define our new normal mode coordinates by

$$\underline{\mathbf{x}} = [S]\,\underline{\mathbf{y}} \tag{20}$$

where the change-of-basis matrix $[S]$ is defined as

$$[S] = \begin{bmatrix} \underline{\mathbf{v_1}} & \underline{\mathbf{v_2}} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{21}$$

Plugging this change of variables into (6) we get the new equation

$$[S]\,\underline{\ddot{\mathbf{y}}} = [A]\,[S]\,\underline{\mathbf{y}} + \underline{\mathbf{f}}\,(t) \tag{22}$$

Left-multiplying both sides by $[S^{-1}]$ gives us

$$\underline{\ddot{\mathbf{y}}} = \left[S^{-1}\right][A]\,[S]\,\underline{\mathbf{y}} + \left[S^{-1}\right]\underline{\mathbf{f}}\,(t) = [\Lambda]\,\underline{\mathbf{y}} + \underline{\tilde{\mathbf{f}}}(t) \tag{23}$$

where

$$[\Lambda] = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} -k & 0 \\ 0 & -3k \end{bmatrix} \tag{24}$$

Looking at the unforced case, $\underline{\tilde{\mathbf{f}}}(t) = \underline{\mathbf{0}}$, we see from (23) that in the new normal mode coordinates we now have two uncoupled second-order ODEs,

$$\ddot{y}_1 + k y_1 = 0 \tag{25a}$$

$$\ddot{y}_2 + 3k y_2 = 0 \tag{25b}$$

the solutions of which are

$$y_1 = A_1 \cos\sqrt{k}t + B_1 \sin\sqrt{k}t \tag{26a}$$

$$y_2 = A_2 \cos\sqrt{3k}t + B_2 \sin\sqrt{3k}t \tag{26b}$$

Using (20) we can now transform back into the original x_1, x_2 coordinates giving

$$\begin{aligned} \underline{\mathbf{x}} &= [S]\,\underline{\mathbf{y}} = \begin{bmatrix} y_1 + y_2 \\ y_1 - y_2 \end{bmatrix} = \\ &\underline{\mathbf{v_1}}\left(A_1 \cos(\omega_1 t) + B_1 \sin(\omega_1 t)\right) + \underline{\mathbf{v_2}}\left(A_2 \cos(\omega_2 t) + B_2 \sin(\omega_2 t)\right) \end{aligned} \tag{27}$$

where we have substituted $\omega_1 = \sqrt{k}$ and $\omega_2 = \sqrt{3k}$. This is the same result we found before in (14), so you might not think much was gained by performing this change-of-basis. However, the real advantage of this method appears when we consider the forced case.

FORCED TWO-DEGREE-OF-FREEDOM SYSTEM

We now reconsider equation (23) when $\underline{\tilde{\mathbf{f}}}(t) \neq \underline{\mathbf{0}}$.

$$\underline{\ddot{\mathbf{y}}} = [\Lambda]\,\underline{\mathbf{y}} + \underline{\tilde{\mathbf{f}}}(t) \tag{28}$$

The two resulting equations are

$$\ddot{y}_1 + \omega_1^2 y_1 = \frac{kx_3}{2m_1} \tag{29a}$$

$$\ddot{y}_2 + \omega_2^2 y_2 = -\frac{kx_3}{2m_1} \tag{29b}$$

where $x_3(t) = F\cos\omega t$ and ω is the forcing frequency. Solving both of these non-homogeneous second-order ODEs yields

$$y_1(t) = A_1\cos\omega_1 t + B_1\sin\omega_1 t - \frac{Fk}{2m_1}\left(\frac{1}{\omega^2-\omega_1^2}\right)\cos\omega t \tag{30a}$$

$$y_2(t) = A_2\cos\omega_2 t + B_2\sin\omega_2 t + \frac{Fk}{2m_1}\left(\frac{1}{\omega^2-\omega_2^2}\right)\cos\omega t \tag{30b}$$

Once again we use (20) to transform back into the original coordinates to get

$$\underline{\mathbf{x}}(t) = \underline{\mathbf{x}_\mathbf{c}}(t) + \frac{Fk}{2m_1}\begin{bmatrix} \frac{1}{\omega^2-\omega_2^2} - \frac{1}{\omega^2-\omega_1^2} \\ -\frac{1}{\omega^2-\omega_2^2} - \frac{1}{\omega^2-\omega_1^2} \end{bmatrix}\cos\omega t \tag{31}$$

where we have suppressed the homogeneous (or complementary) part of the solution. We note that the particular solution becomes unbounded as the forcing frequency approaches either $\omega = \omega_1$ or $\omega = \omega_2$. In other words, *resonance* occurs when we force the two degrees-of-freedom system at one of the normal modes' natural frequencies. (Obviously the oscillations you will observe in the lab will not be unbounded as the lab set-up is not entirely frictionless.)

We now rewrite the particular solution as

$$\underline{\mathbf{x}_\mathbf{p}}(t) = \frac{F}{2}\begin{bmatrix} \frac{1}{\left(\frac{\omega}{\omega_1}\right)^2-3} - \frac{1}{\left(\frac{\omega}{\omega_1}\right)^2-1} \\ -\frac{1}{\left(\frac{\omega}{\omega_1}\right)^2-3} - \frac{1}{\left(\frac{\omega}{\omega_1}\right)^2-1} \end{bmatrix}\cos\omega t \tag{32}$$

where we have written it in terms of the ratio of the forcing frequency to the smaller normal mode frequency ω_1. Figure 5 graphically shows how the amplitudes of the particular (or steady-state) solutions change as the forcing frequency ω is varied.

The plot graphically illustrates what we found earlier – that when the forcing frequency is near the natural frequency of a normal mode, that mode resonates. As $\omega \to \omega_1$ the two masses move in-phase and when $\omega \to \omega_2$ the masses move out-of-phase.

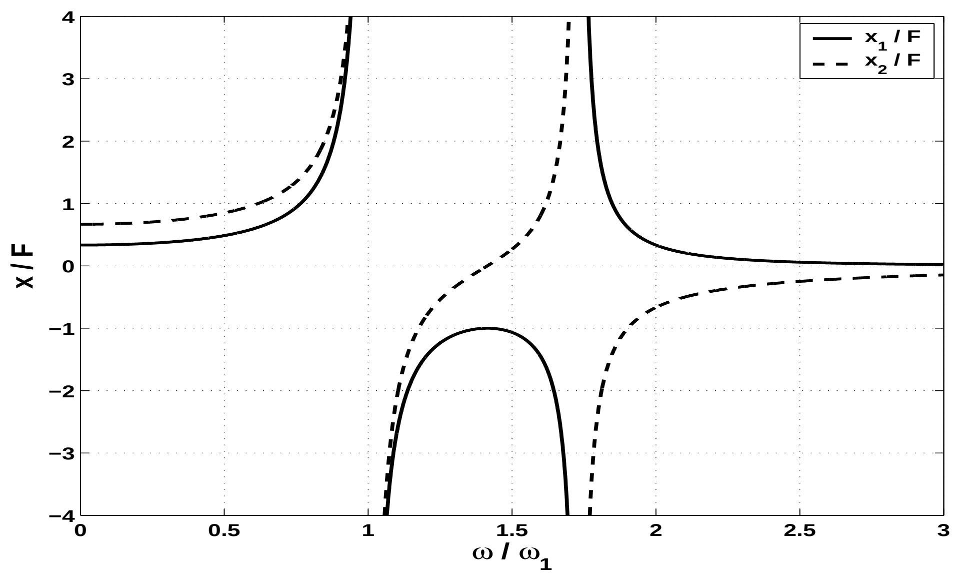

Figure 5: Plot of the response amplitude to forcing amplitude ratio for the forced two degrees-of-freedom system.