Engineering 203
Dynamics Lab Manual
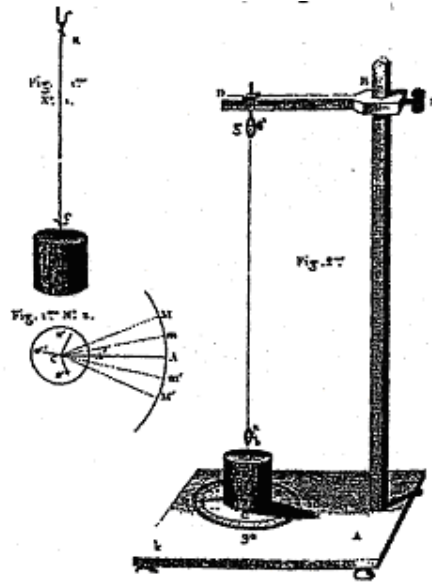
TAM 203 Lab Introduction
Last Updated: March 10, 2007

PURPOSE

The laboratories in dynamics are designed to complement the lectures, text, and homework. They should help you gain a physical feel for some of the basic and derived concepts in dynamics: *force*, *velocity*, *acceleration*, *natural frequency*, *resonance*, *normal modes*, and *angular momentum*. You will also get exposure to equipment and computers which you may use in the future. Some mathematics from courses you have taken recently or are now taking will be used. We hope this will help you make the connection between mathematics and physical reality that is essential to much of engineering. The labs may come either before or after you cover the relevant material in lecture. Thus, they can be either a motivation for the lecture material or an application of what you have learned depending on the timing.

Lab groups are small enough (usually 2 or 3 people) that you can get direct experience with the instruments and equipment. Both you and your lab partners should learn how to do all aspects of the lab. The laboratory teaching assistants will have scheduled office hours so that you can return to use the facilities independently or ask questions. These hours will be posted on the course website. You may also ask the lab TA about other course material if time allows.

CONTENT
There are four labs:

1. Forced Mass-Spring-Dashpot System

2. Multiple Degrees-of-Freedom Oscillator

3. Slider-Crank Mechanism

4. Gyroscope

These labs will be done with physical equipment and some will also involve computer simulations. It is essential that you read through the lab (especially the procedure section) and answer the pre-lab questions before coming to lab. It is not necessary that you understand all of the material perfectly before the lab period.

COURSE INFORMATION
The dynamics laboratories are in Thurston 101.

Each of the four labs is taught for two or three weeks (depending on enrollment). You will be scheduled to attend lab during one of the three weeks. The dates for your laboratory section will be posted outside room 101 Thurston and on the course website. In general, you will have a lab once every three weeks, but be aware that this may vary due to exam and break schedules.

See the Secretary in Kimball 212 if you have any problems with your lab schedule. You'll need to get his or her approval for any changes so that the lab sections do not become overly full. **Turning in a course change form to the registrar is not enough.**

LAB REPORTS

Lab reports are due one week from the day you performed the lab, at 10:00 in the morning unless your TA specifies another time.

Turn in reports in the boxes in the Don Conway room, 102 Thurston Hall. Put your report in the correct box corresponding to the TA in charge of your **lab** section. Reports placed in incorrect boxes might not be found.

Missed Lab and Late Report Policy

All make-up labs must be arranged with your TA. If you know in advance that you'll be gone, you should make advance arrangements. Labs may also be made up at the end of the semester. Sign up with your TA.

If you show up for lab after it is under way, your lab instructor may ask you to leave and to perform the lab another time. The ENGRD 203 lab equipment should not be used without proper training.

Pre-Lab Questions

Each lab has pre-lab questions which should be answered before you come to lab. These questions encourage you to read through the laboratory procedure before attending the lab. Answers to pre-lab questions are due at the beginning of lab and will not be accepted for credit later.

Academic Integrity

Your pre-lab answers and lab reports should be in your own words, based on your own understanding and your own calculations. You are encouraged to discuss the material with other students, friends, TAs, or even faculty. **Any help you receive from such discussion must be acknowledged on the cover of your lab report, including the name of the person or persons and the exact nature of the help.** Violations of this policy will be reported to the academic integrity board.

You may, however, do a joint report with your lab partners (turn in one report for your lab group). All partners get the same grade on the report but separate grades on pre-lab questions.

When you are done in the lab you must have your TA sign one of your data sheets. This sheet must include the name of your lab partners and the time and date the lab was performed. The TA will not sign this sheet until your work station is clean and all equipment is accounted for. **No lab reports will be accepted without this signed sheet.**

Credit and Grading

Each lab is graded out of 15 points. The grade breakdown for each lab report will be determined by your lab TA. This grade will be given to your recitation TA.

Problems and Complaints

1. Your teaching assistant. The lab TA's job is to help you. See your TA if you have problems with the pre-lab questions, lab, or lab report. In-lab office hours will be available if you need to redo some part of the lab or want to collect additional data (For this you can see any of the lab TAs.)

2. Dan Mittler, 218 Kimball, (5-9172), dm68@cornell.edu. See Dan if you have problems with equipment operation. You may also arrange with him to redo part of a lab or perform additional lab work.

3. Alan Zehnder, 317 Kimball, (5-9181). See Professor Zehnder about problems with laboratory content or policy.

4. Secretary, 212 Kimball (5-5062), pmm4@cornell.edu. See her if you have problems with your lab schedule or need to make up a missed lab (read the policy above).

Laboratory Notes

A rule of laboratory work is to keep a neat, complete record of what has been done, why it was done, how it was done, and what the result was.

The success or failure of an experiment in a research laboratory often depends critically upon the record made of the experiment. The outcome of a poorly documented experiment becomes a matter of personal recollection, which is not reliable enough to serve as a basis for further work, especially by someone else. You should take copious notes. If in doubt, **write it down.** One can ignore what is written, but one can not resurrect that which was never recorded. Similarly, **never** erase in your lab notes. If an erroneous reading was made, strike it out with a single line and record the new data. You may later decide that it was not in error.

All lab notes, in their original form, must be submitted with your report.

THE LAB REPORT

Your laboratory report should be typed. Do not crowd your writing. Make sure there is room for comments by your TA. This report should communicate clearly and convincingly what was demonstrated or suggested by the lab work. Your TA is looking for evidence of thought and understanding on your part. Your logic and methods are as important as results or "correct" answers. It is essential that you provide information and calculations which indicate how you arrived at your conclusions. It is permissible (and a good idea if you want a very good grade) to discuss observations and material relevant to the lab which is not specifically asked about in the questions.

The lab reports should contain the following material in the order specified.

1. Cover page: A plain sheet, firmly attached to the rest of the report with staples or another binder. The cover should contain the following (with appropriate substitutions for the words in quotes):

"NAME OF THE LAB"
TAM 203

By: "Your name and your signature (both partners if a joint report)"
Performed: "Date"
Performed with: "Name of person(s) with whom you performed the lab"
Discussed lab with: "Names of people with whom you discussed the lab, and nature of the discussions"
TA: "Lab TA's name"
TA signed the data on page: "Page #"

2. Procedure: (1/2 page maximum) This section should be included only when you deviate from the procedure specified in the lab manual. This section will be needed when there are problems with malfunctioning equipment or if you develop your own procedure.

3. Answers to questions: Concisely answer the questions that are asked and number them as they are numbered in the lab manual. Include any necessary plots, data, or calculations. Your answers should be self-contained and presented in an orderly fashion (i.e. the reader of the report should not have to refer back to the questions that are asked, nor should he or she have to hunt through the report to find your answers). While many questions require that you perform calculations, written explanations of what you are doing and sketches can be very helpful. Show all calculations that you perform in arriving at your answers. If you are performing repetitive calculations you need show only one sample calculation.

4. Observations and conclusions: If you did anything or observed anything in the lab which was not covered in your answers to questions this is the place to discuss it. This is optional.

5. Supplemental procedures and questions: If you choose, or are required, to do additional lab work, repeat sections II, III, and IV for each topic.

6. Mistakes and suggestions: This is an optional section. Point out errors in any of the documentation or oral information you were given. Make suggestions for changes in the lab procedure, instructions, content, etc. Please put this section on a separate page so that it may be kept by the TA for future reference.

7. Appendix: Append ALL notes and records taken in the laboratory (including data sheets signed by your TA). If you have used an x-y plot or data table in your previous answers to questions you need not include it here again.

DATA ANALYSIS AND PRESENTATION

1. Significant figures: When reporting numerical data, an appropriate number of significant figures should always be used. Large numbers should be written in scientific notation, so that the number of significant figures is not ambiguous.

   The numbers 3.840, 0.003840, and 3.840×10^5 each have four significant figures.

   When multiplying two numbers together, the general rule of thumb is to write the answer using the same number of digits as the multipliers. When the multipliers have different numbers of

significant digits the smallest is used. Thus $0.3526 \times 1.2 = 0.42$ (not 0.42312). This same method should be used for division.

Addition is different. Consider the example: $0.2056 + 14.25 + 576.1 = 593.1$. An answer of 593.1276 is not appropriate because the last three digits (.0276) add nothing to the accuracy of the results, since one of the numbers being added (576.1) is accurate only to tenths. Subtraction should be done in a similar manner.

2. Percentage difference calculation: Percentage difference calculations can be used to quantify how well experimental results agree with theoretical or expected values. Rather than writing "the experimental results agree very well with the theoretical calculations," this phrase can be changed to make a quantifiable statement; "the experimental results are within 5 percent of the theoretical calculations." Percentage difference is calculated as:

$100\% \times$ (Value being compared - Reference value) / (Reference value)

While formal error analysis can be used if it is necessary to make a point, your answers should include some discussion of the types and relative sizes of errors in your data.

3. Units: The dimensions of all physical quantities should be clearly presented in all calculations, tables and figures.

4. Graphs: Figure 4 is an example of how your graphs should appear in lab reports. The following is a checklist of the items your graph should include.

   • Use a computer program such as Matlab or Microsoft Excel.

   • Curves should not be drawn between discrete data points unless the type of fitting used is explained and the equation of the curve given.

   • All graphs should be titled and all axes labeled, with the appropriate units listed in parentheses.

   • Numerical values on the axes should be set at reasonable intervals and scales chosen so that all of the data points can be displayed on the graphs.

   • On graphs with more than one curve a legend should be used to identify the curve. Data points can be enclosed by some symbol (i.e. circle, rectangle, etc.) to distinguish different data sets.

   • The independent variable should be placed on the horizontal axis.

5. Linear, semi-log and log-log plot interpretation:

   • Linear: a straight-line plot on linear graph paper indicates a relationship of the general form $y = mx + b$, where m is the slope and b is the y-axis intercept. Choose two points along the line - (x_1, y_1) and (x_2, y_2), preferably well separated. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{1}$$
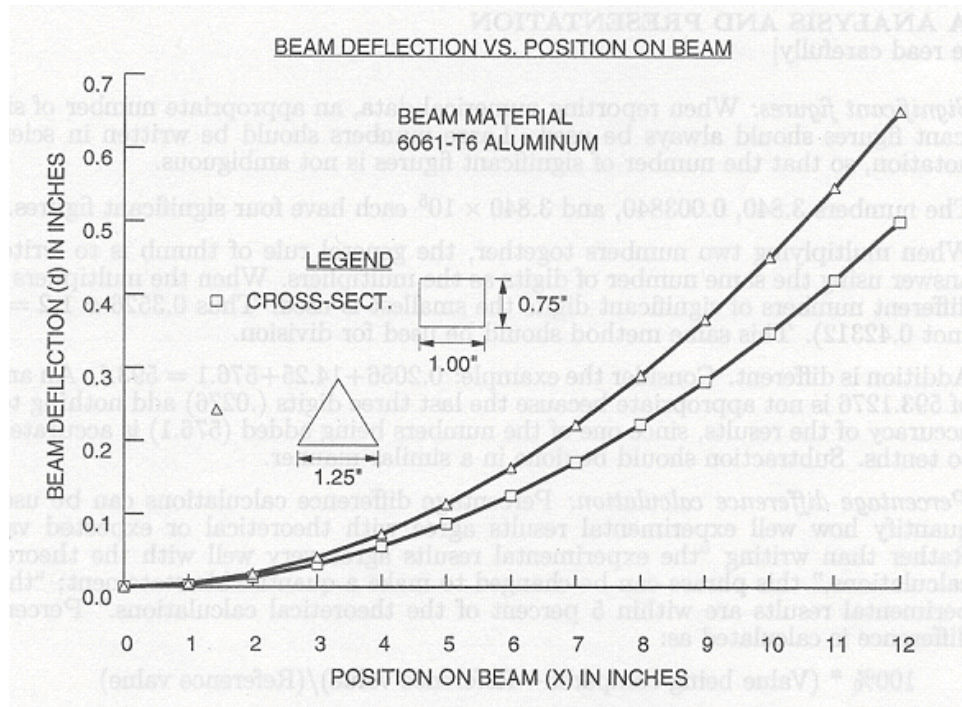
Figure 1: An example graph.

Since $y_1 = mx_1 + b$ we have

$$b = y_1 - mx_1 \tag{2}$$

Sometimes it is useful to plot a function of x and/or y, instead of plotting x and y directly. For example, if y^2 is proportional to x, you could plot y^2 vs. x, and obtain a straight line. The slope of the line then gives you the constant of proportionality. If you don't have (or don't want to use) semi-log or log-log graph paper, you can plot $\ln y$ vs. x or $\ln y$ vs. $\ln x$ respectively. (Logs to other bases will also work.) This is often very useful with computer-generated graphs. The procedure to obtain an equation from the graph is similar to the one described above. For a graph of $\ln y$ vs. $\ln x$ which forms a straight line

$$\ln y = m \ln x + b \tag{3}$$

Find two well-separated points on the line, and write

$$m = \frac{(\ln y)_2 - (\ln y)_1}{(\ln x)_2 - (\ln x)_1} \tag{4}$$

Since $(\ln y)_1 = m(\ln x)_1 + b$ we get

$$b = (\ln y)_1 - m(\ln x)_1 \tag{5}$$

Then

$$e^{\ln y} = e^{m(\ln x)+b} = e^b e^{\ln x^m} \tag{6}$$

and therefore we get a power-law relationship.

$$y = e^b x^m \tag{7}$$

- Semi-log: a straight-line plot on semi-log paper indicates an exponential relationship of the general form $y = ae^{cx}$. Choose two points along the line, as above, reading the values from the graph paper scales. Then

$$c = \frac{\ln y_2 - \ln y_1}{x_2 - x_1} \tag{8}$$

Since $y_1 = ae^{cx_1}$ we get

$$a = \frac{y_1}{e^{cx_1}} \tag{9}$$

If you want to plot your data on a computer, use a log scale for the y-axis, or plot $\ln y$ vs. x with linear scales, as described earlier.

- Log-log: a straight-line plot on log-log paper indicates a power-law relationship of the general form $y = ax^n$. Choose two points along the line, as above, reading the values from the graph paper scales. Then

$$n = \frac{\ln y_2 - \ln y_1}{\ln x_2 - \ln x_1} \tag{10}$$

Since $y_1 = ax_1^n$ we have

$$a = \frac{y_1}{x_1^n} \tag{11}$$

If you want to plot your data on a computer, use log scales for both axes, or plot $\ln y$ vs. $\ln x$ with linear scales, as described earlier.

Lab #1 - One Degree-of-Freedom Oscillator

Last Updated: March 10, 2007

INTRODUCTION

The mass-spring-dashpot is the prototype of all vibrating or oscillating systems. With varying degrees of approximation, car suspensions, violin strings, buildings responding to earthquakes, earthquake faults themselves, and vibrating machines are modeled as mass-spring-dashpot systems. This laboratory is aimed at demonstrating some of the basic concepts of the mass-spring-dashpot system. Additionally, the computer solution of the governing differential equations will be demonstrated with a computer simulation program. Phrases connected with some of the key ideas are: *natural frequency*, *resonance*, *forcing function*, and *frequency response*.

PRELAB QUESTIONS

Read through the laboratory instructions and then answer the following questions:

1. Derive the equation of motion for a mass-spring-dashpot system with forcing term $f(t)$. Assume a constant linear spring constant k and linear damping constant c.

2. Solve the equation of motion you derived in #1 if the forcing term is given by $x_s(t) = A = $ *constant*.

3. Repeat #2, this time numerically integrating the equation using *Matlab*. Choose $m = 1$, $k = 5$, $c = 0$ (undamped), and $A = 3$ and integrate over the time period $0 \leq t \leq 10$. Assume the mass starts from rest with an initial displacement of $x(0) = 1$ m. What is the period of the oscillation? Turn in a plot and an m-file of your code.

4. Define in your own words: *natural frequency*, *damping coefficient*, *critical damping coefficient*, *underdamped*, *overdamped*, *resonance*, *phase-shift*, and *amplitude ratio*.
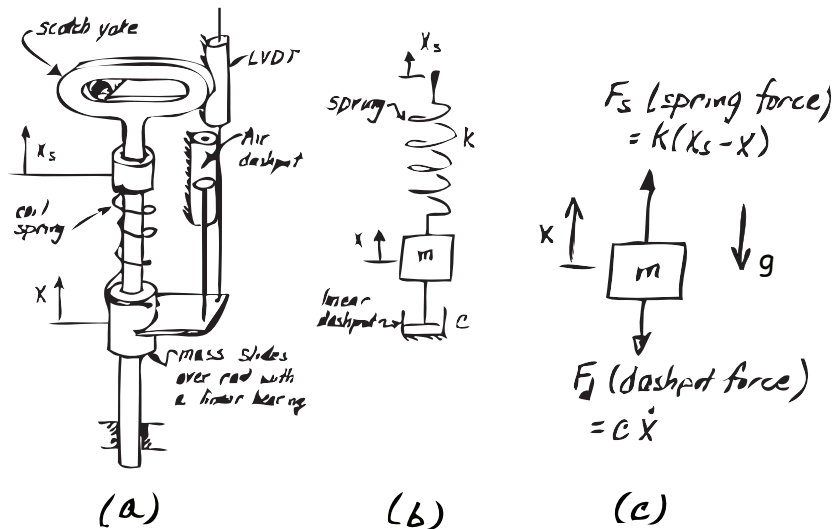

THE MASS-SPRING-DASHPOT SYSTEM

The picture in Figure 2a shows (crudely) the laboratory mass-spring-dashpot, or one degree-of-freedom oscillator. A mass is supported by a spring and is constrained to slide on a rod. In this lab you will record the vertical motion of the mass both with a fixed support and with the support oscillating vertically. Figure 2b shows an idealization of the laboratory apparatus. The spring is modeled as linear (the force it applies is proportional to its increase in length) with proportionality constant k. The damping is produced by a linear air dashpot. The force transmitted by a linear dashpot is proportional to the rate at which it is being stretched with proportionality constant c. The vertical displacement of the mass is $x(t)$ and the vertical displacement of the support is $x_s(t)$.

Pictured in Figure 2c is a free body diagram of the mass. Neglecting gravity (*Why can we neglect it?*), the mass has two forces acting on it in the $\hat{e}_x$-direction:

$$F_{sp}(t) = k\,(x_s - x) \tag{12a}$$

$$F_d(t) = c\dot{x} \tag{12b}$$

where $F_{sp}(t)$ is the linear spring force and $F_d(t)$ is the linear damping force. The system is a one degree-of-freedom system since a single coordinate is sufficient to describe the complete motion of the

Figure 2: Three models of the mass-spring-dashpot system.

system. (The support displacement $x_s(t)$ does not count as a degree of freedom since it is specified by the motor position, over which we assume we have complete control.)

From Newton's second law the equation of motion for this system is

$$\left\{\sum \underline{\mathbf{F}}\right\} \cdot \hat{\mathbf{e}}_{\mathbf{x}} \Rightarrow -F_d + F_{sp} = m\ddot{x} \tag{13}$$

Assuming a linear spring (12a) and a linear dashpot (12b) this becomes

$$m\ddot{x} + c\dot{x} + kx = F_s(t) \tag{14}$$

where $F_s(t) = kx_s(t)$ is the (presumably specified) excitation "forcing function". In this case the forcing function is the position of the end of the spring as a function of time multiplied by the spring stiffness.

The air dashpot provides resistance to motion by drawing air in and out of the cylinder through a small opening at the top of the cylinder. Due to the small, but nonzero viscosity of air, a pressure drop is created across the opening that is linearly proportional to the speed of the air flowing through. This produces linear damping. Nonlinearities are introduced due to the friction between the piston and the cylinder. Note also that the compressibility of the air in the dashpot introduces some springiness to the system in addition to the coil spring. The compressibility of the air may be thought of as a spring in series with the dashpot.

In the first part of this experiment you will attempt to determine the value of the viscous damping constant c by measuring the rate at which oscillations decay towards zero. In addition, the system response to both free vibration and "forced" motion will be observed experimentally and through computer simulation.

A REAL-WORLD EXAMPLE: THE LOUDSPEAKER

A speaker, similar to the ones used in many home and auto speaker systems, is one of many devices which may be conveniently modeled as a one degree-of-freedom mass-spring-dashpot system. The one you will observe in this lab is typical (see Figure 3). It has a plastic cone supported at the edges by a roll of plastic foam (the surround), and guided at the center by a cloth bellows (the spider). It has a large magnet structure and (not visible from outside) a coil of wire attached to the point of the cone which can slide up and down inside the magnet. (The device described above is, strictly speaking, the speaker driver. A complete speaker system includes an enclosure, one or more drivers, and various electronic components.) When you turn on your stereo, it forces a current through the coil in time with the music, causing the coil to alternately attract and repel the magnet. This results in the vibration of the cone which you hear as sound.

In the speaker, the primary mass is comprised of the coil, cone, and (in this case) LVDT core. The "spring" and "dashpot" effects in the system are due to the foam and cloth supporting the cone and perhaps to various magnetic effects. Speaker system design is greatly complicated by the fact that the air surrounding the speaker must also be taken into account. Changing the shape of the speaker enclosure can change the effective values of all three mass-spring-dashpot parameters. (You may be able to observe this by cupping your hands over the speaker (gently, without touching the moving parts) and observing amplitude or phase changes.) Nevertheless, knowledge of the basic characteristics of a speaker (e.g., resonant frequency) is essential in speaker system design.
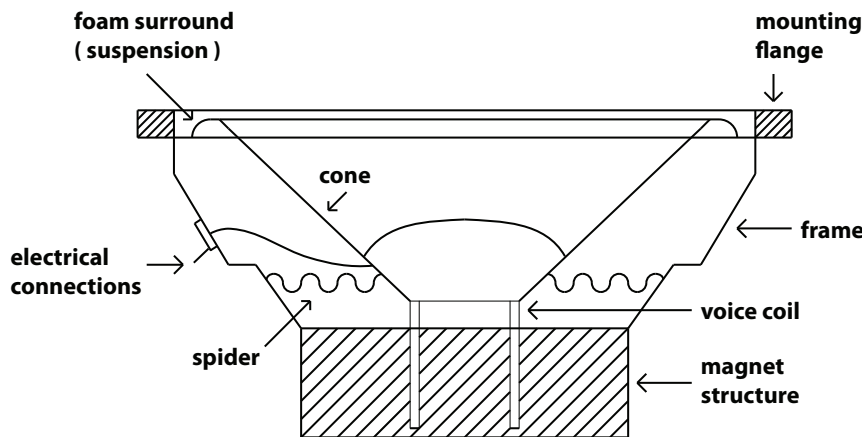
Figure 3: Cross-sectional view of a speaker.

The equation of motion for the speaker is similar to that of the laboratory mass-spring-dashpot above, except the forcing function is electrical, rather than mechanical:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t) \tag{15}$$

where the forcing function $F(t) = ai(t)$, $i(t)$ is the electrical current flow through the coil in amps, and a is the electromechanical coupling coefficient, in Newtons per amp. In the second part of this experiment, the current flow through the speaker will be generated, controlled, and measured using a waveform generation and a data acquisition program. Using this data, the effective mass, damping coefficient, and spring constant for the speaker will be calculated.

SOLVING THE EQUATIONS OF MOTION

Our goal is to know the motion of the mass, $x(t)$, for a given forcing $F(t)$. The two most important cases are unforced or "free" motion, where $F(t) = 0$, and sinusoidal forcing, given by $F(t) = kx_s(t) = kF\cos\omega t$.

Recall that the differential equation governing the motion is given by

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t) \tag{16}$$

Before solving for the motion, we define new variables that will help streamline our analysis. First, we define the *natural frequency*, ω_n, as

$$\omega_n = \sqrt{\frac{k}{m}} \tag{17}$$

Secondly, we define a quantity known as the *critical damping constant*, c_{crit} as

$$c_{crit} = 2\sqrt{km} = 2m\omega_n \tag{18}$$

Finally, we define the *damping factor*, ζ, as

$$\zeta = \frac{c}{c_{crit}} = \frac{c}{2\sqrt{km}} \tag{19}$$

We see that the damping factor incorporates all 3 of the physical properties that define the system - the mass, the spring constant, and the damping constant. Thus we can think of the damping factor as an indicator of the overall damping of the system's response.

We can now rewrite the governing differential equation (15) in terms of these new variables, giving

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n^2 x = \frac{F(t)}{m} \tag{20}$$

We will now solve (20) for the unforced case $F(t) = 0$. We assume an exponential solution of the form $x(t) = Ae^{\lambda t}$. Plugging this into (20), the amplitudes and exponential functions can be divided through, yielding the *characteristic equation*

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0 \tag{21}$$

We then use the quadratic formula to solve for the λ's, giving

$$\lambda_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\zeta\omega_n \pm \imath\omega_d \tag{22}$$

where ω_d is the *damped natural frequency* and is defined as

$$\omega_d = \omega_n\sqrt{1 - \zeta^2} \tag{23}$$

From the definition of the damped natural frequency we see that our analysis will depend on the magnitude of ζ. We will concentrate in this lab on underdamped responses, where $\zeta < 1$. The two other cases are overdamped ($\zeta > 1$) and critically damped ($\zeta = 1$) responses.

Having solved for λ_1 and λ_2, we can now write the solution to (20) as

$$x(t) = Ae^{-\zeta\omega_n t}\cos\left(\omega_d t - \phi\right) \tag{24}$$

where the amplitude, A, and phase, ϕ, are the unknowns. If this form of the solution appears unfamiliar to you, plug (24) into (20) and verify that it does indeed satisfy the governing differential equation. The two unknowns can be found from the given initial conditions, $x(0) = x_0$ and $\dot{x}(0) = v_0$.

THE LOGARITHMIC DECREMENT METHOD

The viscous damping constant, c, may be determined experimentally by measuring the rate of decay of unforced oscillations. The logarithmic decrement, which is the natural logarithm of the ratio of any two successive amplitudes, is used. The larger the damping, the greater will be the rate of decay of oscillations and the bigger the logarithmic decrement, D.

$$D = \ln\left(\frac{x_n}{x_{n+1}}\right) \tag{25}$$

where x_n and x_{n+1} are the heights of two successive peaks in the decaying oscillation pictured in Figure 4.
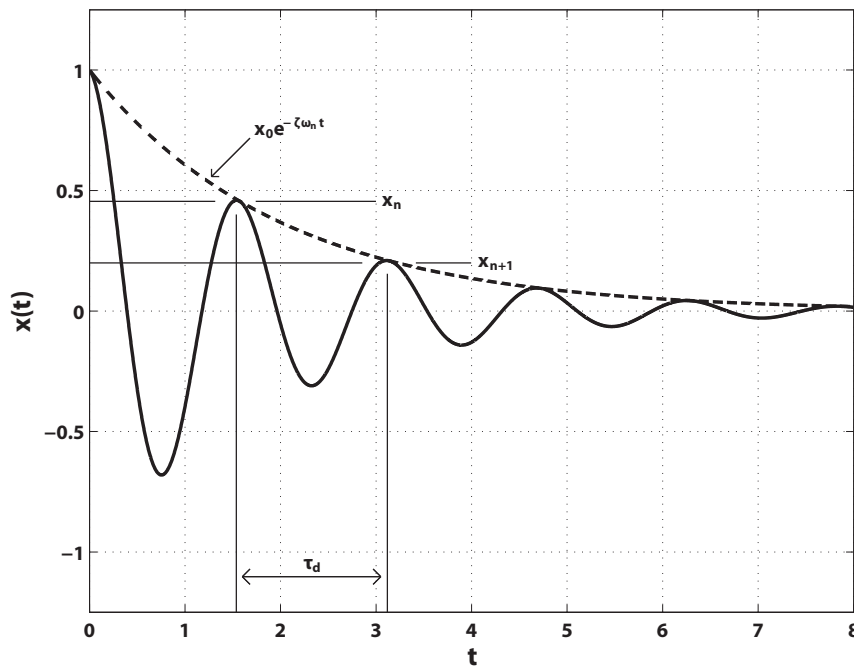
Figure 4: The logarithmic decrement method.

To find a theoretical representation for the logarithmic decrement D, we look at the exponentially decaying envelope for the damped oscillation, which is given by

$$x_{envelope}(t) = x_0 e^{-\zeta\omega_n t} \tag{26}$$

Using this equation we now write the logarithmic decrement as

$$D = \ln\left(\frac{x_{envelope}(t)}{x_{envelope}(t+\tau_d)}\right) = \ln\left(\frac{x_0 e^{-\zeta\omega_n t}}{x_0 e^{-\zeta\omega_n(t+\tau_d)}}\right) = \ln\left(e^{\zeta\omega_n\tau_d}\right) = \zeta\omega_n\tau_d \tag{27}$$

where τ_d is the period of the damped oscillation, i.e. $\tau_d = \frac{2\pi}{\omega_d}$. We simplify this expression by substituting in (19) for ζ and then solve for the damping constant c, yielding

$$c = \frac{2mD}{\tau_d} \tag{28}$$

We can also obtain an equation for k from (27) , yielding

$$k = \frac{c^2 \left(1 + \frac{4\pi^2}{D^2}\right)}{4m} = \frac{4\pi^2 m}{\tau_d^2 \left(1 - \zeta^2\right)} \tag{29}$$

Thus, using equations (28) and (29), we can find the damping constant c and spring constant k for the mass-spring-dashpot system from the experimentally obtained values for D and τ_d.

FORCED VIBRATIONS AND FREQUENCY RESPONSE
Next, we will solve (20) for the forced case. We assume the support is driven harmonically with its displacemnt given by $x_s(t) = F \cos \omega t$, where F is the displacement amplitude of the support and ω the natural frequency of its motion. Since the mass is coupled to the support via the spring, the force exerted by the support's motion on the mass is given by $F(t) = kx_s(t) = kF \cos \omega t$. The equation of motion (20) now becomes

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n^2 x = \frac{kF}{m} \cos \omega t \tag{30}$$

From ordinary differential equation theory we can write the general solution to (30) as the sum of a complimentary (also referred to as the transient or homogeneous) solution $x_c(t)$ and a particular (or steady-state) solution $x_p(t)$.

$$x(t) = x_c(t) + x_p(t) \tag{31}$$

In this lab we will only be interested in the steady-state solution after the transient response dies out. Thus we take the general solution to be of the form

$$x_p(t) = A \cos (\omega t - \phi) \tag{32}$$

where the amplitude of oscillation of the mass position, A, and the phase of the displacement with respect to the exciting force, ϕ, are two unknowns. To solve for the unknowns we substitute (32) into (30) to get (after some trigonometric reductions)

$$\left[2A\omega\omega_n\zeta \sin \phi + A\omega_n^2 \cos \phi - A\omega^2 \cos \phi - \frac{Fk}{m}\right] \cos \omega t$$
$$+ \left[-2A\omega\omega_n\zeta \cos \phi + A\omega_n^2 \sin \phi - A\omega^2 \sin \phi\right] \sin \omega t = 0 \tag{33}$$

While (33) looks intimidating, note that the coefficients of $\cos \omega t$ and $\sin \omega t$ are independent of t, i.e. they are constants. Therefore we can use the linear independence of $\cos \omega t$ and $\sin \omega t$ to claim that their respective coefficients must be identically equal to zero for (33) to hold. This gives us two equations and two unknowns, A and ϕ, to solve for. Solving for the unknowns yields

$$\tan \phi = \frac{2\omega\omega_n\zeta}{\omega_n^2 - \omega^2} \tag{34a}$$

$$A = \frac{\frac{Fk}{m}}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + 4\omega^2\omega_n^2\zeta^2}} \tag{34b}$$

with the restriction $0 \le \phi \le \pi$.

RESONANCE

Resonance as defined by *Merriam-Webster* is *a vibration of large amplitude in a mechanical or electrical system caused by a relatively small periodic stimulus of the same or nearly the same period as the natural vibration period of the system*. From basic ODE theory we know that resonance occurs when we force the system with a frequency $\omega = \omega_n$. If the system has zero damping $(c = 0)$ the response is unbounded, else we will see that the system's response amplitude is simply maximized.

Following the above definition, if we force the damped system (30) at a frequency $\omega = \omega_n$, the system's response's phase-lag (34a) and amplitude ratio become

$$\phi = \tan^{-1} \infty = \frac{\pi}{2} \tag{35a}$$

$$\frac{A}{F} = \frac{\frac{k}{m}}{2\omega_n^2\zeta} = \frac{1}{2\zeta} \tag{35b}$$

Thus when $\omega = \omega_n$ we should expect the response of the system to lag the forcing function by approximately 90 degrees and to have a finite amplitude dependent on the value of the damping factor. However, resonance in a damped mass-spring-dashpot system does not occur when the forcing frequency is exactly the undamped natural frequency ω_n. To find the resonance frequency, ω_r, we maximize the response's amplitude (34b) by differentiating with respect to the forcing frequency ω and setting it equal to zero.

$$\left.\frac{dA}{d\omega}\right|_{\omega=\omega_r} = 0 \Rightarrow \omega_r = \omega_n\sqrt{1 - 2\zeta^2} \tag{36}$$

PHASE DIAGRAMS

While performing the lab we will need to graphically determine if we are forcing the mass-spring-dashpot system near its resonance frequency. In Figures 7 and 9 we see that the *Labview* software provides a graph of $x_s(t)$ vs. $x(t)$, i.e. it plots the position of the mass as a function of the position of the support. For different values of the forcing frequency ω this graph will have different qualitative behavior.

For a support whose position is given by $x_s(t) = F\cos\omega t$, the amplitude and phase of the resulting mass oscillation were found to be (34b) and (34a) respectively. Now lets assume that $\omega \ll \omega_n$. The amplitude and phase of the steady-state oscillation then become

$$\tan\phi = \frac{2\omega\omega_n\zeta}{\omega_n^2 - \omega^2} \approx 0 \tag{37a}$$

$$A = \frac{\frac{Fk}{m}}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + 4\omega^2\omega_n^2\zeta^2}} \approx F \tag{37b}$$

From (32) we find that the position of the mass will be

$$x(t) \approx F \cos \omega t \tag{38}$$

We see that $x(t) = x_s(t)$. Therefore if we plotted $x_s(t)$ vs. $x(t)$ (parameterized by t) we would simply see a line through the origin with a slope equal to 1 (in other words the graph looks like $y = x$).

Now lets assume that we are forcing the system at resonance, i.e. $\omega \approx \omega_n$. In the previous section we found that the system's response would have amplitude and phase given by (35b) and (35a) respectively. Therefore the position of the mass is given by

$$x(t) \approx \frac{F}{2\zeta} \cos \left(\omega t - \frac{\pi}{2} \right) = \frac{F}{2\zeta} \sin \omega t \tag{39}$$

To see what the plot of $x_s(t)$ vs. $x(t)$ looks like, we note that from basic trigonometry we have

$$\frac{x^2(t)}{\left(\frac{F}{2\zeta} \right)^2} + \frac{x_s^2(t)}{F^2} = 1 \tag{40}$$

This equation represents (in general) an ellipse in the $x_s(t)$-$x(t)$ plane. For the special case of resonance the above equation reduces to

$$\cos^2 \omega t + \sin^2 \omega t = 1 \tag{41}$$

Thus the plot of $x_s(t)$ vs. $x(t)$ will appear to be nearly circular.

LABORATORY SET-UP

- **Mass-Spring-Dashpot System**
  The apparatus consists of a laboratory-model mass-spring-dashpot system with displacement transducers (Linear Variable Differential Transformers or LVDTs) for measuring $x(t)$ and $x_s(t)$. The output from the LVDTs is communicated to the computer via the data acquisition board. An electric motor and controller, acting through a scotch yoke, enable a sinusoidal forcing function to be applied to the system. Note that the controller dial readings are arbitrary; frequency and period data must be obtained from your computer plots.

- **Loudspeaker**
  The apparatus consists of a speaker on a stand with one LVDT to measure cone displacement. Waveforms are generated by the computer, amplified, and sent through a resistor to the speaker (approximating a current source). The computer is also used to measure current flow through the speaker and displacement of its cone (using the attached LVDT).

Please follow all safety precautions. Keep long hair and loose clothing well away from the electric motor, pulleys, and other moving parts.

- **Using the LabView Software**
  The four programs you will be using for part A of the lab are: *FreeAcq* (Figure 5) for making measurements of the unforced system; *FreeSim* (Figure 6) for simulation of the same; *ForcedAcq* (Figure 7) for measurements of the system with a sinusoidal forcing function; and *ForcedSim* (Figure 8) which may be used for simulation of the forced system. Although somewhat different in appearance and function, the programs share many key features.
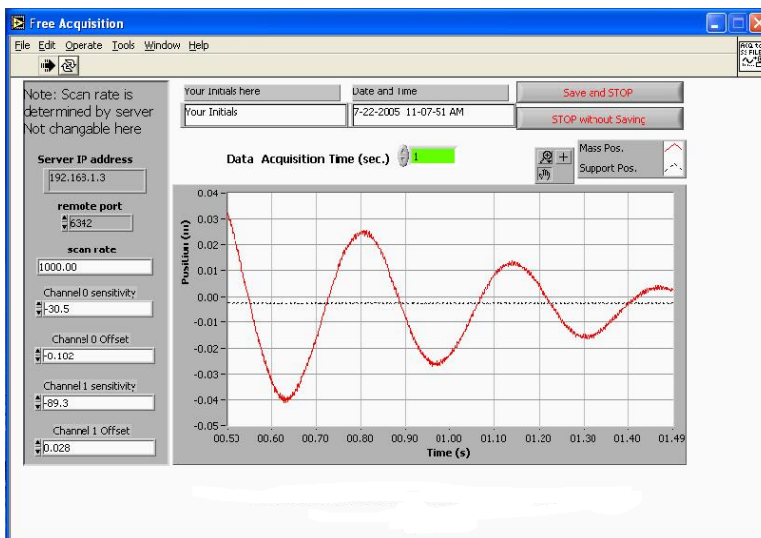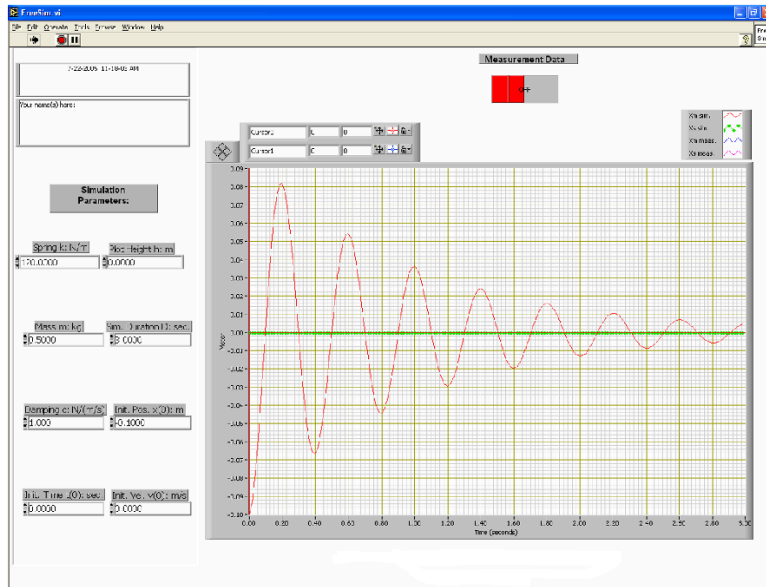

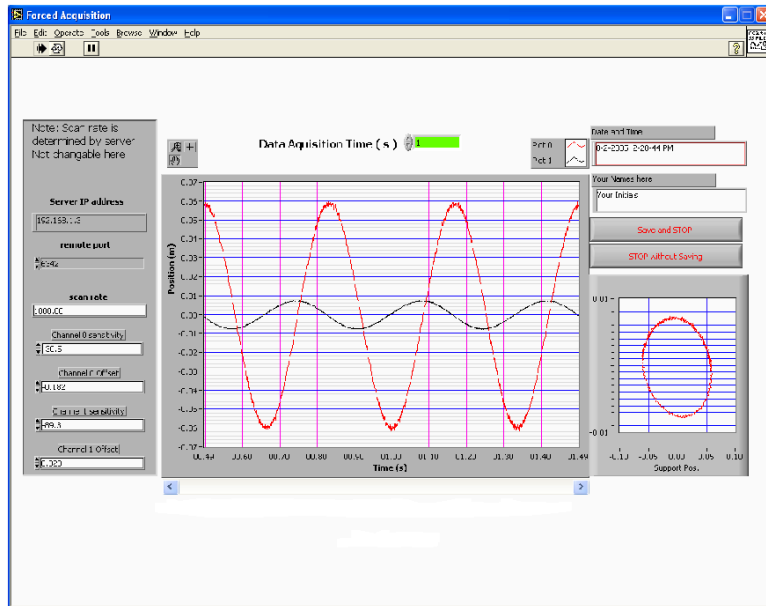**Figure 5:** The *FreeAcq* program.

Figure 6: The *FreeSim* program.
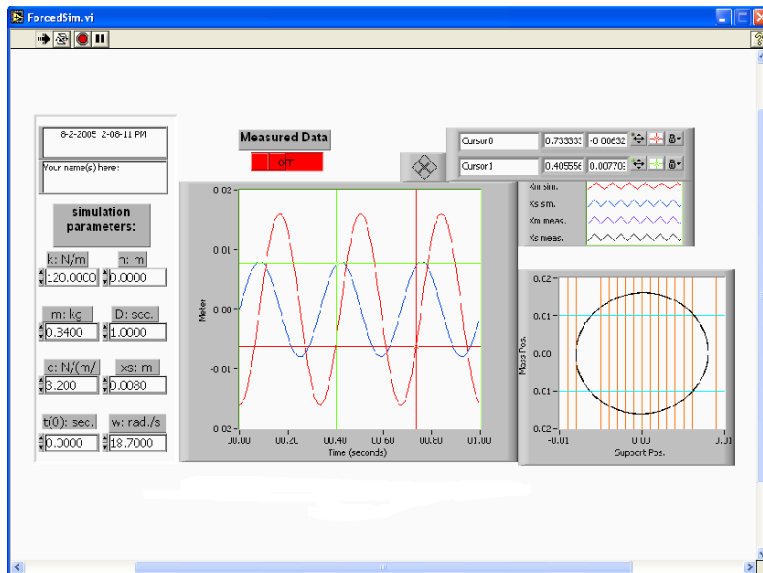

Figure 7: The *ForcedAcq* program.

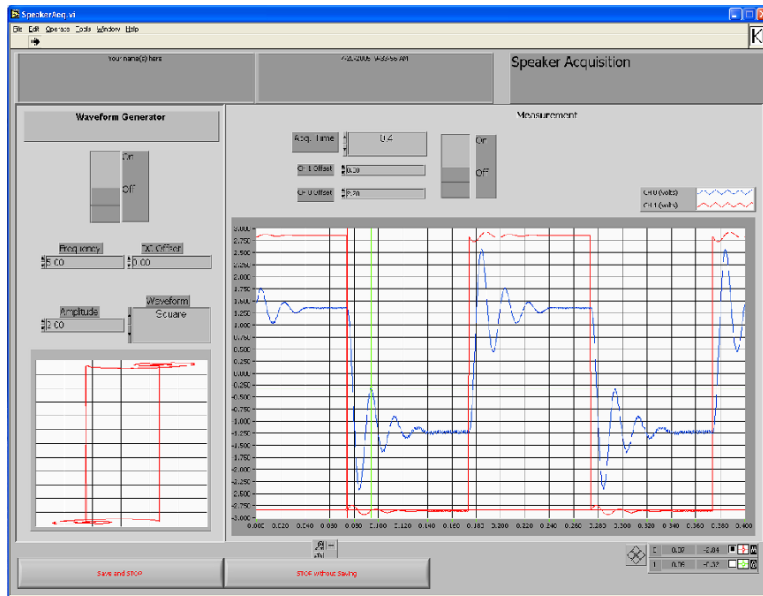**Figure 8:** The *ForcedSim* program.

Figure 9: The *SpeakerAcq* program.

The data acquisition programs automatically convert the voltage output of the LVDTs to meters. To do this, they need a set of conversion factors, which are on a label on the mass-spring-dashpot base board. After starting a measurement program, make sure that the sensitivity and offset values on the left hand side of the window match the values listed on a small sheet of paper in front of the apparatus. Also please enter your name in the box provided.

To run the program, you must hit the white arrow in the top left of the screen. If this arrow is black, that means that the program is already running. For the data acquisition programs, a green box on top will define the amount of time the program will record the motion of the mass-spring after hitting the arrow. To reset the data acquisition, press on STOP without Saving and then press the white arrow to begin again.

After getting data pressing the Save and STOP button stores your current data on disk, for comparison later to the simulation. Any previous data is erased. The data file is only used by the simulation programs *FreeSim* and *ForcedSim*; it is not available to the data acquisition programs.

You may find it convenient to obtain numerical data from your plots using the cursors, rather than using a ruler. Two cursors are available, one indicated by a circle and one by a square. To use a cursor, use the mouse to drag it to the point you want to measure. The x and y values of the point you have chosen will appear above the graph, in the row corresponding to that cursor. For best accuracy, you should utilize every full cycle on the screen. For example, if three cycles are displayed, you should measure the time elapsed from the start of the first cycle to the end of the third, and then divide Δx by three to obtain the period. If your cursor has vanished off the screen, you can enter an on-screen position for it into the x and y display boxes, and it will reappear in the desired location. You can also move the cursors around using the little arrow "buttons" on the screen in the cursor control box. If a cursor turns a darker color, it is locked on to a data point, and will trace the curve point by point if the left or right arrow is pressed. Zoom and other features are available for the cursors and graphs; see the *LabView* manual for details.

PROCEDURE

- **Free Vibration, Mass-Spring-Dashpot**

    1. Here you will be recording the motion of the mass after it has been pulled down from equilibrium and then released. First start up the *FreeAcq* program. Start with a data acquisition time of 6 seconds. Give the system an initial position with zero initial velocity (i.e. pull down the mass and hold it still). Then press the white run arrow in the top left of the toolbar and immediately release the mass. Repeat this procedure until you have a nice oscillation over the three seconds. Please note that the zero position is somewhat arbitrary and will depend on the position of the scotch yoke when the mass is released. Also, the zero level for the scotch yoke and for the mass may not agree exactly. Finally, you will need to take data long enough for the mass to stop oscillating in order to have a

good zero reference. Save your best oscillation on disk by pressing the `Save and STOP` button.

2. Next we will simulate the free vibration of the mass-spring-dashpot system. Start the *FreeSim* program. To compare the simulation data to your saved experimental data, turn the `Measured Data` switch on. To change a simulation parameter type in the value you want and press `Enter`. The following parameters for the simulation can be changed:

   - k is the spring constant in Newtons/meter.
   - m is the mass in kilograms. **You need to include both the mass of the spring and the mass of the weight since springs in the real world are not massless.**
   - c is the damping coefficient in Newtons/meter/second.
   - $t(0)$ (in seconds) lets you adjust the relative starting point of the simulation plot. It allows you to move the plot horizontally, as necessary, making comparison with the measured data easier.
   - h (in meters) lets you adjust the simulation plot vertically.
   - D is the duration in seconds for the simulation to be run. Set it equal to the duration of the measured data set if you are comparing them.
   - $x(0)$ is the initial position of the mass in meters. To start the oscillations you need to initially pull down on the mass.
   - $v(0)$ is the initial velocity of the mass in meters/second. This should be zero.

   Add the measured data to the graph by pressing the `Measurement Data` switch above the graph. Using the cursors, measure the logarithmic decrement D of the measured data and the period of the damped oscillation τ_d. Check if c is constant by measuring the logarithmic decrement for several separate cycles. Determine k, the spring stiffness. Make a print-out of one of your curves.

3. Simulate unforced motion by inputting the values of m, k, and c that you just determined into the *FreeSim* program. Obtain $x(0)$ from your measured data. Compare your simulation with your measured data. If agreement is not good, adjust k and c until you have good agreement. Make a print-out.

- **Forced Vibration, Mass-Spring-Dashpot**

  1. We will now be recording the motion of the mass as it undergoes sinusoidal forcing. Start up the *ForcedAcq* program. Set the acquisition time to 30 seconds, start the data acquisition, and turn on the motor. Two graphs will be displayed. The left one contains two plots. One is a plot of the mass position $x(t)$ vs. time and the second one is a plot of the spring support position (forcing) $x_s(t)$ vs. time. The right graph plots $x(t)$ vs. $x_s(t)$ and helps show phase relationships.

  2. For at least five different forcing speeds get nice plots of several cycles of motion. The forcing speeds should include:

     - The slowest speed for which the motor runs smoothly.
     - A very fast speed.
     - Resonance.

- A speed just slower than resonance.
- A speed just faster than resonance.

Make sure to save each data set to disk in order to analyze them in the *ForcedSim* program (print-outs are not necessary). Set the data acquisition time to 10 seconds and run the program in order to find the desired speed. Then hit `STOP without SAVING`. Reduce the data acquisition time to 1 second and then run the program again. Then hit `SAVE and STOP`.

3. Next we will simulate the forced vibration of the mass-spring-dashpot system. Open the *ForcedSim* program. Turn on the measured data switch to view your saved data. To change the current measured data set you must close and then re-open the *ForcedSim* program. Once experimental data is loaded, make your necessary measurements using the computer cursors. **You will need to make sufficient measurements in order to make plots of amplitude ratio, $\frac{x(t)}{x_s(t)}$, and phase shift, ϕ, vs. forcing frequency ω. In particular, you should measure the period, forcing function amplitude, mass motion amplitude, and phase-lag between the forcing function and the resulting mass motion. You need to do this for each of your five speeds.** You can then enter your calculated experimental values into the *ForcedSim* program and see how well the experimental data follows theory. The parameters in the *ForcedSim* program include most of the ones described for the *FreeSim* program, plus the following:

   - $x_s(t)$ is the amplitude (in meters) of the motion of the spring support, which is moved up and down by the motor and scotch yoke. This motion supplies the forcing of the system.
   - ω is the angular velocity of the spring support motion in radians per second.

You may also want to save the data to a USB storage device or write it to a CD for later analysis. To do this just copy the text files of the desired data onto your storage device.

- **Vibration of a Speaker**

   1. You will now experimentally measure the "free"-response of the loudspeaker. Open the folder named *speaker* on the desktop and then open the *Speaker* program. Set the waveform generator controls as follows in order to obtain a maximum-amplitude square wave at about 5 Hz. Set the `Waveform` control to `Square`, the `Frequency` control to 5, and the `Amplitude` control to 2. Leave the `DC Offset` control set to 0. Note that when changing waveforms or frequencies, you must wait a few seconds for the computer to equilibrate and display correct data. Set the data acquisition time control to about 0.4 seconds. The displacement of the cone (channel 0) is displayed as a blue line and the current flow through the speaker coil (channel 1) is displayed as a red line. You want to see a square wave with "ringing" wiggles after each shift in level that gradually damp out. Use a low enough frequency for the square wave so that the "ringing" damps out completely before the square wave changes levels again. **Be careful not to shake the table during the experiment as small vibrations can cause errors.** This is the step-response of the speaker, which is approximately equivalent (in this lab) to the "free"-response you obtained earlier for the mass-spring-dashpot system. When you have a good display of the "ringing" turn off the data acquisition.

2. Measure the logarithmic decrement D. Try to measure the coordinates of at least three successive peaks of the blue curve yielding at the minimum two values for the logarithmic decrement. **Remember to measure amplitudes relative to the equilibrium level (the level your exponential decay curve ends up at), not to the zero level of the plot.** Measure the period of the damped oscillation.

3. Next you will force the loudspeaker at its resonance frequency in order to experimentally determine the mass m and spring constant k of the loudspeaker. Set the Waveform control to Sine and the Amplitude control to 2. Leave the DC Offset control set to 0. Set the data acquisition time to 0.1 seconds. The CH 0 Offset and CH 1 Offset controls may be used to adjust the plots vertically if necessary. Turn on the waveform generator and data acquisition switches and adjust the Frequency control value until you observe resonance of the speaker cone. **To change the frequency you must press** STOP without Saving**, enter the desired frequency and then start the program again in order to observe the new frequency.** Neither the spring constant k nor the mass m of the speaker is easily measured at resonance. However, you can derive the approximate mass and spring constant by observing what happens when the mass is changed a known amount. Measure the mass of the rubber weight and then carefully press it onto the LVDT shaft. The best way is to spread the weight open, position it, and release it. Find the new resonant frequency.

LAB REPORT QUESTIONS

Please answer the following questions concerning the mass-spring-dashpot part of the lab within your lab report:

1. What is the spring constant k and damping constant c for your mass-spring-dashpot setup? Indicate the measured data and formulas you used to calculate these values. Is the damping constant c really constant? What does this say about the air dashpot acting linearly?

2. Compare your experimental data to the simulated data for unforced motions. Comment on any similarities or differences of interest. How did adjusting k and c to better fit your data change the simulation graph? Please attach print-outs from before and after you adjust k and c to better fit your data.

3. Make a plot of the amplitude ratio (peak mass displacement divided by peak forcing displacement) versus forcing frequency ω.

4. Make a plot of the phase-angle ϕ between $x(t)$ and $x_s(t)$ versus the forcing frequency ω.

5. For a typical value of damping constant c that you measured, what is the percent difference between the natural frequency ω_n and the damped natural frequency ω_d? Does the addition of a dashpot to a mass-spring system increase or decrease its oscillation frequency?

6. Discuss the plots from questions #3 and #4. Do they look like what you expect based on textbook solutions to the damped one degree-of-freedom oscillator? Relate the phase-angle plot to the $x(t)$ vs. $x_s(t)$ plots. Why do the ellipses change shape and rotate as you go through resonance? State your observations about the behavior of the mass as the forcing frequency is varied in words without using numbers, angles, graphs, or equations.

Please answer the following questions concerning the loudspeaker part of the lab within your lab report:

1. Calculate k and m for the speaker, using the resonant frequencies and mass you measured in lab.

2. Calculate c, the damping coefficient, for the speaker. Is the speaker overdamped or under-damped? How linear was the speaker damping?

3. Find another real-world vibrating system which could be reasonably modeled as a mass-spring-dashpot. Give the system a "push" and observe its response. Try applying a forcing function of various frequencies, and look for resonance.

   (a) Describe how you modeled your vibrating system as a mass-spring-dashpot. That is, what does the mass represent, what is the spring, and what is the dashpot? Be as specific as possible.

   (b) Is this system typically overdamped? Underdamped? If applicable, what was the resonant frequency (approximately)?

   (c) In what ways does the system you found most significantly differ from an ideal linear mass-spring-dashpot system?

CALCULATIONS & NOTES