"Solutions"

Your Name: Andy Ruina

Section day and time: ____________

T&AM 203 Prelim II

Tuesday Oct 26, 2004

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3 problems, 25 points each, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problems.

b) Full credit if

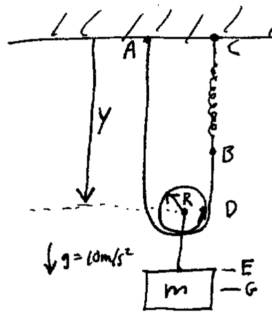
- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance is used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- →| reasonable justification, enough to distinguish an informed answer from a guess, is given; you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem 1:	/25
Problem 2:	/25
Problem 3:	/25

1) The system below is released from rest at $Y = 1.25$ m. In terms of the quantities given, what is Y at $t = 5$ s?

FBD



$L_{AB} = 2\text{ m}$ = length of string

$R = \cancel{0.25}\text{ m}$ $(0.5/\pi)\text{ m}$

L_s = rest length of spring $= 1\text{ m}$

$m = 3\text{ kg}$

$k = 70\text{ N/m}$

$Y(0) = 1.25\text{ m}$

$Y(5s) = ?$

Spring Law: $T = K\underbrace{\Delta}_{\text{spring stretch.}}$

Kinematics: $2Y + \pi R = L_{AB} + \underbrace{(L_s + \Delta)}_{\text{total spring length}}$

$$\Delta = 2Y + \pi R - L_{AB} - L_s$$

LMB $\quad \sum F_y = m\ddot{Y} \Rightarrow -2\underbrace{T}_{K\Delta = K(2Y + \pi R - L_{AB} - L_s)} + mg = \ddot{Y}m$

$$\Rightarrow \quad \ddot{Y} + \frac{4K}{m}Y = \frac{2K}{m}(L_{AB} + L_s - \pi R) + g$$

$$Y_h = A\cos\sqrt{4K/m}\,t + \underbrace{B}_{0 \text{ because } \dot{Y}(0)=0}\sin\sqrt{\tfrac{4K}{m}}\,t$$

$$Y_p = \frac{L_{AB} + L_s - \pi R}{2} + \frac{mg}{4K}$$

$$Y(t) = A\cos\sqrt{\tfrac{4K}{m}}\,t + \frac{L_{AB} + L_s - \pi R}{2} + \frac{mg}{4K}$$

$$Y(0) = Y_0 \Rightarrow \quad Y(t) = \left[Y_0 - \left(\frac{L_{AB} + L_s - \pi R}{2} + \frac{mg}{4K}\right)\right]\cos\sqrt{4K/m}\,t$$

$$+ \frac{L_{AB} + L_s - \pi R}{2} + \frac{mg}{4K}$$

1) cont'd

$$y(t) = \left[\left[1.25 - \left(\frac{2+1-1/2}{2}\right) - \frac{30}{280}\right] \cos\sqrt{\frac{280}{30}}\, t/s + \left(1.25 + \frac{30}{280}\right)\right] m$$

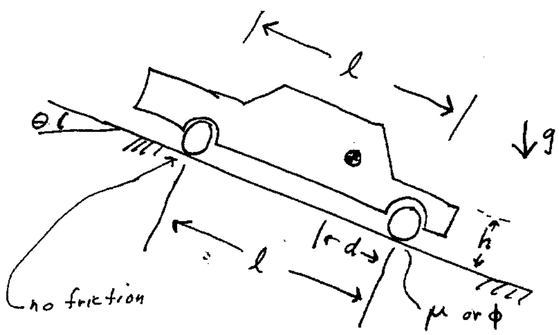
$$= \left[-\frac{3}{28} \cos\left(\sqrt{\frac{280}{3}}\, 5\right) + \frac{5}{4} + \frac{3}{28}\right] m$$

$t = 5\,s$

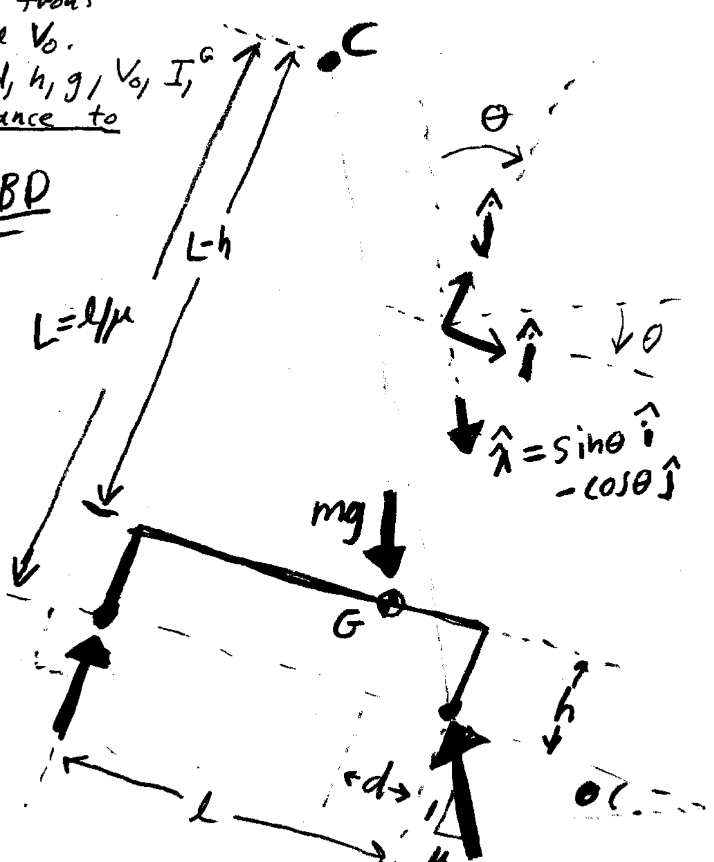
$$\boxed{y(5s) = \left[\frac{3}{28}\left(1 - \cos\left(5\sqrt{\frac{280}{3}}\right)\right) + \frac{5}{4}\right] m}$$

Prob 1

2) The car shown slams on its front brakes when it is going a speed V_0. In terms of some or all of ℓ, d, h, g, V_0, I^G, μ (or ϕ) and θ what is <u>the distance to stop?</u>



FBD



AMB$_{/C}$: $\sum \underline{M}_{/C} = \dot{\underline{H}}_{/C}$

$$\underline{r}_{G/C} \times (mg\hat{\lambda}) = \underline{r}_{G/C} \times (m\underline{a}_G)$$

$$\underline{a}_G = a\hat{\imath}\ ;\ \underline{r}_{G/C} = (\ell-d)\hat{\imath} - (L-h)\hat{\jmath}$$

$$\Rightarrow \left[(\ell-d)\hat{\imath} - (L-h)\hat{\jmath}\right] \times \left[mg(\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath})\right] = \left[(\ell-d)\hat{\imath} - (L-h)\hat{\jmath}\right] \times (ma\hat{\imath})$$

$$\left\{ \cancel{m}g\hat{k}\left[-(\ell-d)\cos\theta + (L-h)\sin\theta\right] = \cancel{m}a(L-h)\hat{k} \right\}$$

$$\{\}\cdot\hat{k} \Rightarrow \quad a = \left[\frac{-(\ell-d)}{(L-h)}\cos\theta + \sin\theta\right] g < 0 \quad \text{(slowing down)}$$

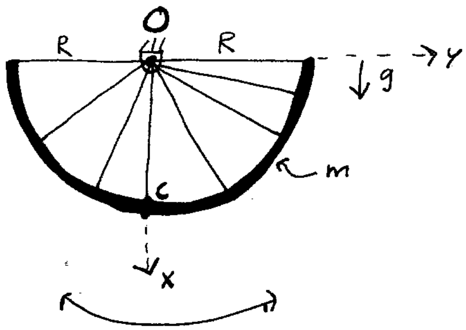
$$a = \frac{dv}{dt} \Rightarrow \quad av = v\frac{dv}{dt} \Rightarrow \quad a\,v\,dt = v\,dv \Rightarrow a\,dx = d(v^2/2) \Rightarrow a\Delta x = \Delta v^2/2$$

$$\left[\Delta(v^2/2) = -V_0^2/2\right]$$

$$\Rightarrow \Delta x = x = \text{stopping distance} = -2V_0^2/2a$$

$$x = \frac{V_0^2}{2\left[\frac{\ell-d}{(L-h)}\cos\theta - \sin\theta\right]g} = \boxed{\frac{V_0^2\left[\ell/\mu - h\right]}{2g\left[(\ell-d)\cos\theta - \left(\frac{\ell}{\mu} - h\right)\sin\theta\right]}}$$

3) A bicycle wheel is cut in half. <u>What is the period of oscillation?</u> Answer in terms of some or all of R, m, g & the release angle θ_0. Neglect the mass & weight of the spokes & hub. Assume small oscillations.



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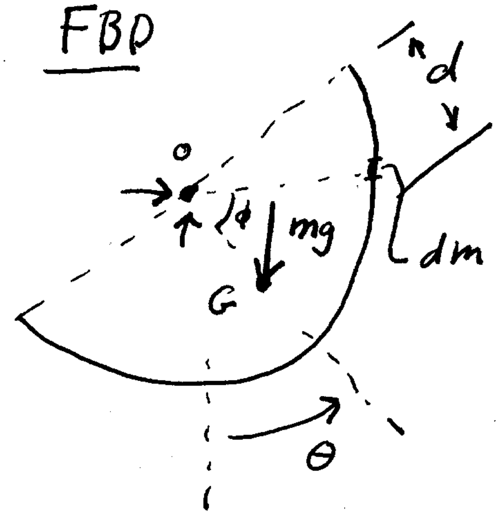


AMB/O:
$$\sum \underline{M}_{/O} = \dot{\underline{H}}_{/O}$$

$$-mdg\sin\theta\,\hat{k} = \int \underbrace{\underline{r}}_{\text{radial from } O} \times \underbrace{\underline{a}}_{(\text{radial part}) + \overbrace{(\text{circumferential part})}^{R\ddot{\theta}}} dm$$

$$\Big\{ \qquad = \ddot{\theta} R^2 m \hat{k} \Big\}$$

$$\{\ \}\cdot\hat{k} \Rightarrow \ddot{\theta} + \frac{dg}{R^2}\sin\theta = 0$$

small angles $\Rightarrow \ddot{\theta} + \frac{dg}{R^2}\theta = 0 \Rightarrow \theta = A\cos\left(\sqrt{\frac{dg}{R^2}}\,t\right) + B\sin\left(\sqrt{\frac{dg}{R^2}}\,t\right)$

t^* = period: $\sqrt{\frac{dg}{R^2}}\,t^* = 2\pi$

$\boxed{d = 2R/\pi \text{ from next page}}$

$$t^* = 2\pi\sqrt{R^2/dg}$$
$$= 2\pi\sqrt{R\pi/2g}$$

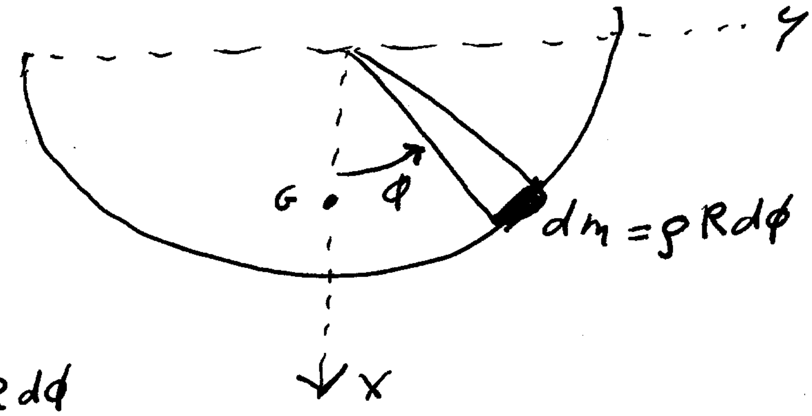
$$\Rightarrow \boxed{t^* = \sqrt{2}\,\pi^{3/2}\sqrt{\frac{R}{g}}}$$

prob 3

3 cont'd

Find d

$$\rho = \frac{m}{\pi R}$$



$$m X_G = \int x \, dm$$

$$= \int_{-\pi/2}^{\pi/2} (R\cos\phi)\, \rho R \, d\phi$$

$$= R^2 \left(\frac{m}{\pi R}\right) \int_{-\pi/2}^{\pi/2} \cos\phi \, d\phi$$

$$= \frac{Rm}{\pi} \sin\phi \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{2Rm}{\pi}$$

$$\Rightarrow \boxed{d = X_G = \frac{2R}{\pi}}$$ ← used on previous page