Your Name: MANISH AGARWAL

Section time: ____________________

T&AM 203 Prelim 2

Tuesday October 24, 2006

Draft October 24, 2006

3 problems, 25^+ points each, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problems.

b) Full credit if

- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance are used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- →| reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	4:	/25
Problem	5:	/25
Problem	6:	/25

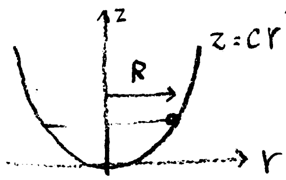
4) (25 pt) **Particle sliding in circles in a parabolic bowl.** As if in a James Bond adventure (in a big slippery Cornell-managed radio-telescope bowl in Puerto Rico), a particle-like human with mass m is sliding with negligible friction around in level circles at speed v. The equation describing the bowl is $z = CR^2 = C(x^2 + y^2)$

a) (20 points) Find v in terms of any or all of R, g, and C.

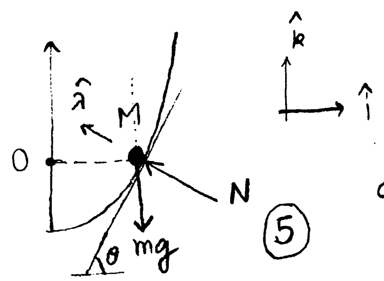
b) (5 points) Now say you are given ω, C and g. Find v and R if you can. Explain any oddities.

Sol

(a)



FBD



5

mass M executes circular motion about center O.

3

note $\quad \tan\theta = \left.\dfrac{dz}{dr}\right|_{r=R} = 2CR$ — (1)

also $\quad \hat{\lambda} = \cos\theta\,\hat{k} - \sin\theta\,\hat{i}$ — (2)

applying N2L

$$\vec{F}_{ext} = m\vec{a}$$

$$-mg\hat{k} + N\hat{\lambda} = m(\underbrace{a_z}_{0}\hat{k} + \overbrace{a_r}^{-\frac{v^2}{R}}\hat{i})$$

$\because$ motion in Level circle

$$\Rightarrow \quad -mg\hat{k} + N(\cos\theta\hat{k} - \sin\theta\hat{i}) = -\frac{mv^2}{R}\hat{i} \qquad [\text{using } (2)]$$

$\{\ \}\cdot\hat{k} \Rightarrow \quad -mg + N\cos\theta = 0 \Rightarrow N\cos\theta = mg$ — (3) 5

$\{\ \}\cdot\hat{i} \Rightarrow \quad -N\sin\theta = -\dfrac{mv^2}{R} \Rightarrow N\sin\theta = \dfrac{mv^2}{R}$ — (4) 5

dividing (4) by (3) and using (1)

$$\tan\theta = 2CR = \frac{v^2}{Rg}$$

$$\Rightarrow \boxed{v = R\sqrt{2Cg}} \quad 2$$

(b) ω, C, g given, v and R to be found

(2) $\omega = \frac{v}{R} = \sqrt{2Cg}$ from answer of part (a)

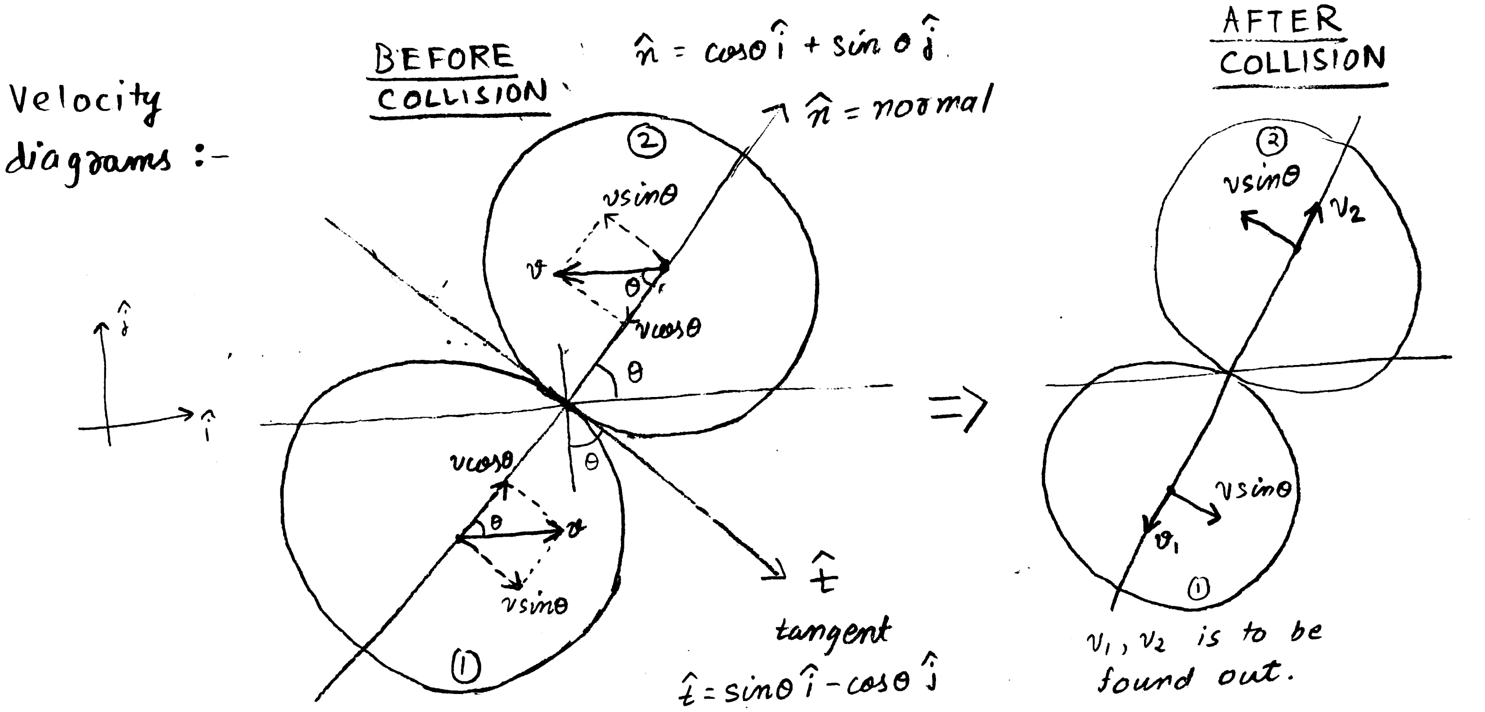
=> v and R can have any values untill

(3) $\frac{v}{R} = \sqrt{2Cg} = \omega$

this has to match in the given data.

5) **(25 pt) Collision.** Two equal mass m spherical particles have a frictionless collision with coefficient of restitution e. Before the collision their two velocities are $\underline{\mathbf{v}}_1^- = v\,\hat{\mathbf{i}}$ and $\underline{\mathbf{v}}_2^- = -v\,\hat{\mathbf{i}}$. The normal to their common tangent plane at contact is $\hat{\mathbf{n}} = \cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}$. In terms of some or all of $v, m, e, \theta, \hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, find the velocity of particle 2 after the collision.

Velocity diagrams :-



Since impact is only in $\hat{n}$ direction, only normal component of velocities change.

by conservation of momentum for system in $\hat{n}$ direction

$$m v\cos\theta - m v\cos\theta = m V_2 - m V_1$$

(10)

$$\Rightarrow \quad \boxed{V_1 = V_2} \quad — \; (1)$$

by definition of coefficient of restitution

$$e = \frac{\text{velocity of seperation}}{\text{velocity of approach}}$$

$$e = \frac{v_1 + v_2}{V\cos\theta + V\cos\theta} = \frac{v_2}{V\cos\theta} \qquad [\, v_1 = v_2 \text{ from (1)} \,]$$

(10)

$$\Rightarrow \quad v_2 = eV\cos\theta$$

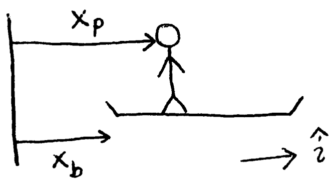
$\Rightarrow$ velocity of particle ② after impact $= -v\sin\theta\,\hat{t} + eV\cos\theta\,\hat{n}$

$= -v\sin\theta\,(\sin\theta\,\hat{i} - \cos\theta\,\hat{j}) + ev\cos\theta\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

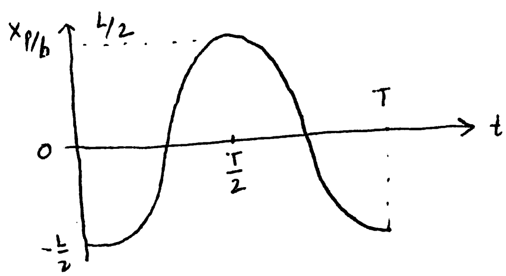
(5)

$$\boxed{v\left\{(-\sin^2\theta + e\cos^2\theta)\,\hat{i} + \sin\theta\cos\theta\,(e+1)\,\hat{j}\right\}}$$

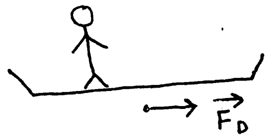
6) **(25 pt)** A person mass m_p walks up and back, all the way to the bow and to the stern, in a boat mass m_b. The person walks continuously and repeatedly, over and over and over again, moving relative to the boat sinusoidally in time, with period T. The length of the boat is L and the drag force on the boat from the water is $F = cv_b$. After a while the boat just moves back and forth also. How far does the boat go back and forth? (That is, the bow of the boat goes back and forth between two points, what is the distance between those two points?) Answer in terms of some or all of m_p, m_b, L and T.



motion of person wrt boat



FBD



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$$x_{P/b} = -\frac{L}{2}\cos\left(\frac{2\pi}{T}t\right)$$

$$\Rightarrow v_{P/b} = \frac{2\pi}{T}\frac{L}{2}\sin\left(\frac{2\pi}{T}t\right) \qquad \text{let } \frac{2\pi}{T} = \omega$$

Material property

$$\vec{F}_D = -c\vec{V}_b \qquad 3$$

$$\Rightarrow a_{P/b} = \frac{L}{2}\omega^2\cos(\omega t) \quad — \circledast$$

5

LMB $\quad \sum\vec{F} = \dot{\vec{L}}$

$$\vec{F}_D = m_p\vec{a}_p + m_b\vec{a}_b$$

$$-c\vec{V}_b = m_p\vec{a}_p + m_b\vec{a}_b \qquad 3$$

by, <u>KINEMATICS</u>

$$\vec{a}_p = \vec{a}_{P/b} + \vec{a}_b$$

given by $\circledast$

$$\Rightarrow \left\{ -c\vec{V}_b = (m_p + m_b)\vec{a}_b + m_p\vec{a}_{P/b} \right\}$$

$$\{\ \}\cdot\hat{i} \Rightarrow \quad -cV_b = (m_p + m_b)\dot{v}_b + m_p\frac{L}{2}\omega^2\cos\omega t \qquad [a_b = \dot{v}_b]$$

$$\dot{v}_b + \underbrace{\frac{c}{m_p + m_b}}_{C_1} V_b = \underbrace{-\frac{m_p\frac{L}{2}\omega^2}{m_p + m_b}}_{A}\cos\omega t$$

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$$\Rightarrow \boxed{\dot{v}_b + C_1 V_b = A\cos\omega t} \quad — (1)$$

$$\omega = 2\pi/T$$

$$C_1 = c/(m_p + m_b)$$

$$A = -\frac{m_p\frac{L}{2}\omega^2}{m_p + m_b}$$

we need to solve for V_b and then x_b

The exponential part of solution $\to 0$ as $t \to \infty$. For the oscillating part guess the solution of the form

$$V_b = B\cos\omega t + D\sin\omega t$$

Plug it in (1) to get

$$-B\omega\sin\omega t + D\omega\cos\omega t + BC_1\cos\omega t + DC_1\sin\omega t = A\cos\omega t$$

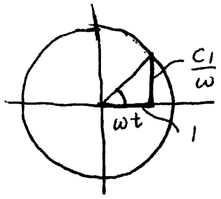
equating $\sin\omega t$ and $\cos\omega t$ terms

$$-B\omega + DC_1 = 0$$
$$D\omega + BC_1 = A$$

Solving it we get $\quad D = \dfrac{A\omega}{\omega^2 + C_1^2} \quad \& \quad B = \dfrac{AC_1}{\omega^2 + C_1^2}$

$$\Rightarrow \quad V_b = \frac{A}{\omega^2 + C_1^2}\left(C_1\cos\omega t + \omega\sin\omega t\right)$$

$$\Rightarrow \quad X_b = \frac{A}{\omega^2 + C_1^2}\left(\frac{C_1}{\omega}\sin\omega t - \cos\omega t\right) + \text{constant of integration}$$



$$\Rightarrow \quad \text{amplitude} = \frac{A}{\omega^2 + C_1^2}\sqrt{\left(\frac{C_1}{\omega}\right)^2 + 1} = \frac{A}{\omega\sqrt{C_1^2 + \omega^2}}$$

so, distance boat goes back and forth is $2 \times |\text{Amplitude}|$

$$\left|\frac{2A}{\omega\sqrt{C_1^2 + \omega^2}}\right|$$

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which is

$$\boxed{\dfrac{\left(\dfrac{m_p}{m_p + m_b}\right)\left(\dfrac{2\pi}{T}\right)L}{\sqrt{\left(\dfrac{c}{m_p + m_b}\right)^2 + \left(\dfrac{2\pi}{T}\right)^2}}}$$ Ans.

in dimentionless form dividing by $\frac{2\pi}{T}$ througout

$$\boxed{\dfrac{\left(\dfrac{m_p}{m_p + m_b}\right)L}{\sqrt{\left[\dfrac{cT}{2\pi(m_p + m_b)}\right]^2 + 1}}}$$ Ans.