"SOLUTIONS"

Your Name: Andy Ruina

Section day and time: ____________

T&AM 203 Prelim 1

Tuesday Sept 28, 2004

Draft September 27, 2004

3 problems, 25 points each, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problems.

b) Full credit if:

- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance is used;
- correct vector notation is used, when appropriate;

↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;

± all signs and directions are well defined with sketches and/or words;

→| reasonable justification, enough to distinguish an informed answer from a guess, is given; you clearly state any reasonable assumptions if a problem seems *poorly defined*;

- work is I.) neat,
  II.) clear, and
  III.) well organized;

- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);

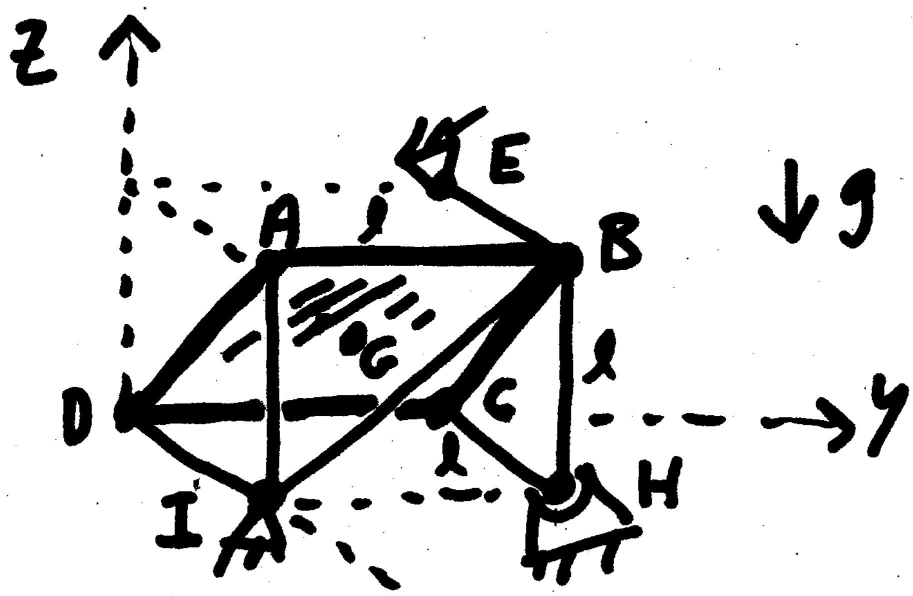
□ your answers are $\boxed{\text{boxed}}$ in; and

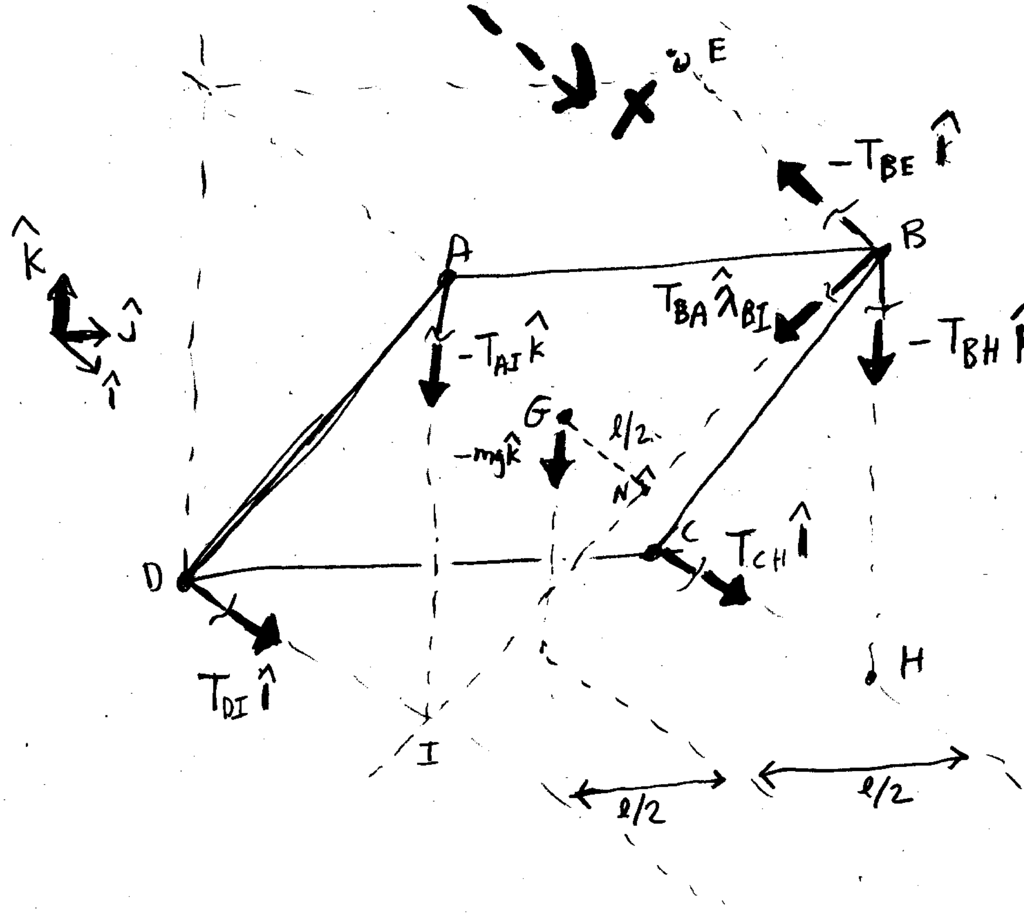
≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "theta7dot = 18". You will be penalized, but not heavily, for minor syntax errors.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	1:	/25
Problem	2:	/25
Problem	3:	/25

1) (25 pt) Statics. The uniform plate ABCD with mass m is held up by 6 bars (EB, HB, HC, IB, IA & ID). Find the tension in *any three* of these bars. Answer in terms of some or all of m, g, and ℓ.





$$\left\{\sum \underline{F}_i = \underline{0}\right\}\cdot\hat{j} \Rightarrow \quad T_{BI}\,\hat{\lambda}_{BI}\cdot\hat{j} = 0 \Rightarrow \boxed{T_{BI}=0}$$

(no other forces have $\hat{j}$ components)

$$\sum M_{/DI} = 0 \quad \Rightarrow \quad -mg\,\ell/2 + (-T_{BH})\,\ell = 0 \Rightarrow \boxed{T_{BH} = -mg/2}$$

↑axis

(all tensions besides T_{BH} either intersect axis DI or are parallel to it)

$$\sum M_{/EB} = 0 \quad \Rightarrow \quad -mg\,\ell/2 + (-T_{AI})\,\ell = 0 \Rightarrow \boxed{T_{AI} = -mg/2}$$

(all other tensions don't contribute)

$$\sum M_{/AB} = 0 \Rightarrow \quad \left(\frac{2}{\sqrt{2}}\,mg\right)\frac{\ell}{2} + T_{CH}\,\overbrace{\frac{\sqrt{2}\,\ell}{2}}^{d_{NH}} = 0 \Rightarrow \boxed{T_{CH} = -mg/2}$$

lever arm

force ⊥ to axis & lever arm

(No other tensions contribute)

$$\sum M_{/IC} = 0 \Rightarrow \quad \underbrace{\left(T_{BE}\frac{\sqrt{2}}{2}\right)\ell}_{} - \underbrace{\overset{-mg/2}{T_{BH}}\left(\frac{\sqrt{2}}{2}\,\ell\right)}_{} = 0 \Rightarrow \overset{*}{\boxed{T_{BE} = -mg/2}}$$

slide T_{BE} to E to see this

slide T_{BH} to H to see this.

(gravity & other tensions drop out)

$$\sum M_{/BH} = 0 \quad \Rightarrow \quad T_{DI}\,\ell = 0 \quad \Rightarrow \boxed{T_{DI} = 0}$$

* $\sum M_{/IH} = 0$ gives T_{BE} in one shot.

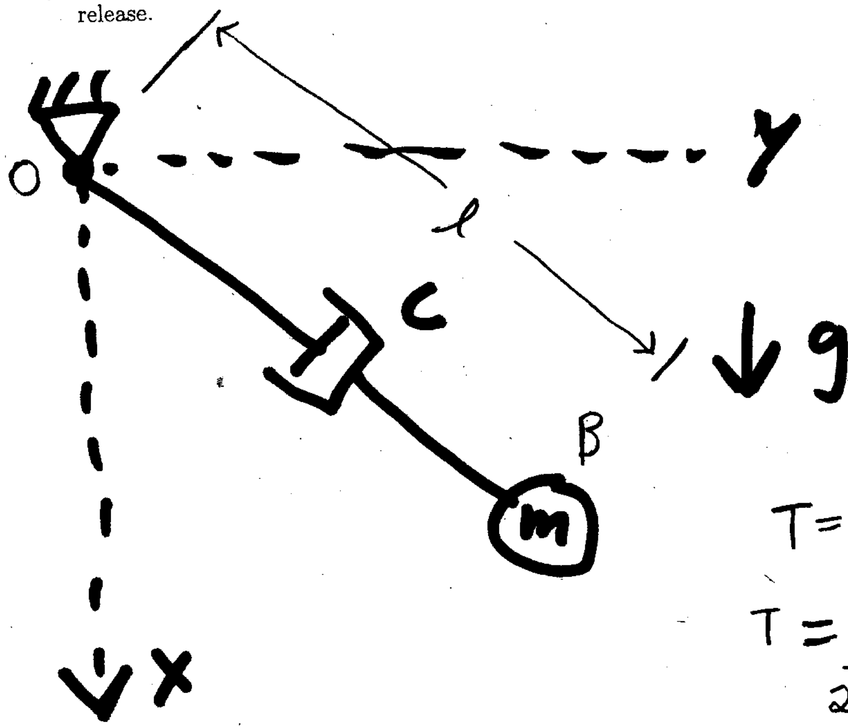
Note: all 6 tensions can be found one at a time, never using values of other tensions.

2) **(25 pt)** In the new sport of spongy jumping a spring is replaced by a dashpot c. Assume $m = 7\,\text{kg}$, $g = 10\,\text{m/s}^2$, and $c = 13\,\text{kg/s}$. The mass is released from rest at $x = 0\,\text{m}$, $y = 5\,\text{m}$.

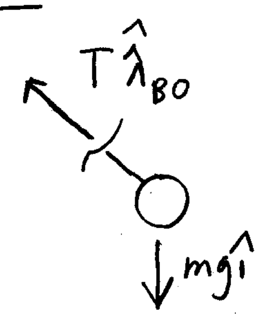
a) Just after (one milli-micro second after) release what is the acceleration of the mass?

b) What are the equations of motion for this system (differential equations involving x and y and their derivatives)?

c) Write Matlab code that would give the x coordinate of the center of mass 16 seconds after release.



FBD



$$T = c\,\dot{\ell}_{OB} = c\frac{d}{dt}\sqrt{x^2+y^2}$$

$$T = \frac{c\,(2x\dot{x}+2y\dot{y})}{2\sqrt{x^2+y^2}}$$

$$T = c\,(x\dot{x}+y\dot{y})/\ell$$

$$\hat{\lambda}_{MB} = \underline{r}_{BO}/|\underline{r}_{BO}| = (-x\,\hat{\imath} - y\,\hat{\jmath})/\ell$$

$$T\hat{\lambda}_{BO} = \frac{-c(x\dot{x}+y\dot{y})}{\ell^2}\,(x\hat{\imath}+y\hat{\jmath})$$

LMB $\qquad \sum \underline{F}_i = m\,\underline{a}$

$$\left\{ mg\,\hat{\imath} - \frac{c(x\dot{x}+y\dot{y})}{\ell^2}(x\hat{\imath}+y\hat{\jmath}) = m(\ddot{x}\,\hat{\imath} + \ddot{y}\,\hat{\jmath}) \right\}$$

$$\{\,\}\cdot\hat{\imath} \Rightarrow \quad \boxed{mg - \frac{c(x^2\dot{x} + xy\dot{y})}{\ell^2} = m\ddot{x}} \quad (a)$$

$$\{\,\}\cdot\hat{\jmath} \Rightarrow \quad \boxed{-\frac{cxy\dot{x} + y^2\dot{y}}{\ell^2} = m\ddot{y}}$$

define $V_x = \dot{x}$, $V_y = \dot{y}$

$$\Rightarrow \begin{aligned} \dot{z}_1 &= \dot{x} = V_x \\ \dot{z}_2 &= \dot{y} = V_y \\ \dot{z}_3 &= \dot{V}_x = g - c(x^2 V_x + xy V_y)/(\ell^2 m) \\ \dot{z}_4 &= \dot{V}_y = -c(xy V_x + y^2 V_y)/(\ell^2 m) \end{aligned} \quad *$$

(a) in first order form

(b) at $t = 0^+$ $V_x = 0$, $V_y = 0 \Rightarrow \dot{V}_x = g$, $\dot{V}_y = 0$

eqn *

$$\Rightarrow \boxed{\underline{a}(0^+) = g\hat{\imath}} \text{ (b)}$$

$(= 10\,\text{m/s}^2\,\hat{\imath})$

```
tspan=[0 16];
z0 = [0 5 0 0]';
[t z]=ode23('sponge', tspan, z0);
xe = z(end,1)
```

useful Matlab command

(c)

run this after saving sponge.m

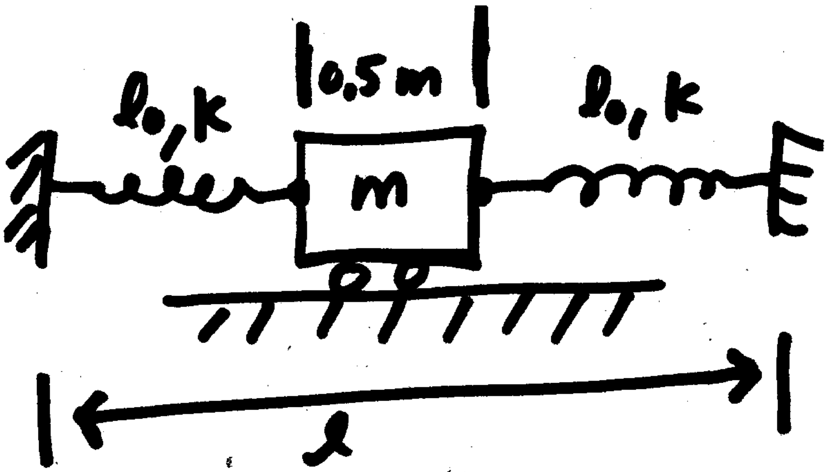
sponge.m

```
function zdot= sponge(t,z)
m=7; g=10; c=13
x= z(1); y= z(2); Vx= z(3); Vy= z(4);
xdot= Vx ;
ydot= Vy;
l2= x^2 + y^2 ;
Vxdot = g - c*(x^2 * Vx + x*y* Vy)/(l2*m);
Vydot = -c*(x*y* Vx + y^2 * Vy)/( l2 * m);
zdot= [xdot ydot Vxdot Vydot]';
```

3) **(25 pt)** A mass $m = 7\,\mathrm{kg}$ is held in place by two equal springs with $k = 5\,\mathrm{N/m}$ and $\ell_0 = 3\,\mathrm{m}$.

a) How long does one oscillation take if $\ell = 6.5\,\mathrm{m}$?

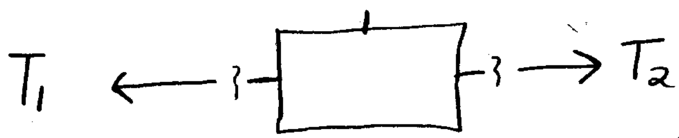
b) How long does one oscillation take if $\ell = 12.5\,\mathrm{m}$?



$\rightarrow x$ = displacement from middle

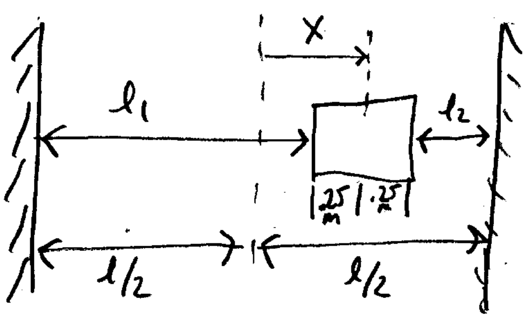
FBD

$\rightarrow \hat{i}$



$T_2 = (\ell_2 - \ell_0)\,k$

$T_1 = (\ell_1 - \ell_0)\,k$



$\Rightarrow \ell_1 = \frac{\ell}{2} + x - 0.25\,m$

$\ell_2 = \frac{\ell}{2} - x - 0.25\,m$

LMB $\quad \{\sum \underline{F}_i = m\underline{a}\}\cdot\hat{i}$

$T_2 - T_1 = m\ddot{x}$

$(\ell_2 - \ell_0)\,k - (\ell_1 - \ell_0)\,k = m\ddot{x}$

(cont'd)

$$\left[\underbrace{\left(\frac{l}{2} - x - .25m\right)}_{l_2} - l_0\right]k - \left[\underbrace{\left(\frac{l}{2} + x - 0.25m\right)}_{l_1} - l_0\right]k = m\ddot{x}$$

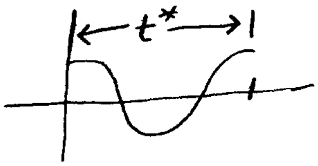
(l, l_0, .25m all drop out!)*

$$\Rightarrow \quad -2kx = m\ddot{x}$$

$$\boxed{\ddot{x} + \left(\frac{2k}{m}\right)x = 0}$$

Classic Harmonic Oscillator eqn.

ODE Soln: $x = A\cos\left(\underbrace{\sqrt{2k/m}\,t}\right) + B\sin\left(\underbrace{\sqrt{\frac{2k}{m}}\,t}\right)$



at one oscillation this whole $() = 2\pi$

$$\sqrt{2k/m}\;t^* = 2\pi \Rightarrow t^* = \frac{2\pi}{\sqrt{2k/m}}$$

$$t^* = \frac{2\pi}{\sqrt{2\cdot(5N/m)/(7kg)}}$$

$1N = 1\,kg\cdot m/s^2$

$$t^* = \frac{2\pi}{\sqrt{(10/7)/s^2}}$$

$$\boxed{\boxed{t^* = \frac{2\pi}{\sqrt{10/7}} \text{ seconds}}}$$ (a) & (b)

*Note: stretching a linear spring increases the force but not the stiffness.