Solutions

Your Name: Manish Agarwal

Section time: ____________

T&AM 203 Prelim 1

Tuesday September 26, 2006

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3 problems, 25^+ points each, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problems.

b) Full credit if

- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance are used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- →| reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	1:	/25
Problem	2:	/25
Problem	3:	/25

1) **(25 pt) Pulleys.** In the problems below you are asked "What is the relation" between this and that. This means you should write the simplest possible equation in which this and that are the only unknowns.)

a) (1 point) Please read all the rules and hints at the front of the exam. Write here: "I read the cover page.":

____

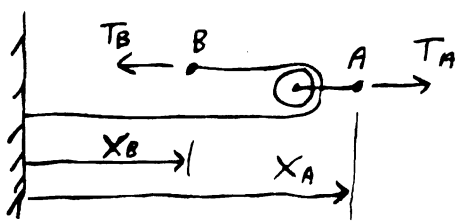
b) (3 points) The ideal pulley system (make the usual assumptions) in (b,c) below shown is part of a larger mechanism. What is the relation between T_A and T_B? Clearly justify your work from first principles.

c) (10 points) For the same pulley system what is the relation between a_A and a_B? Clearly justify your work from first principles.

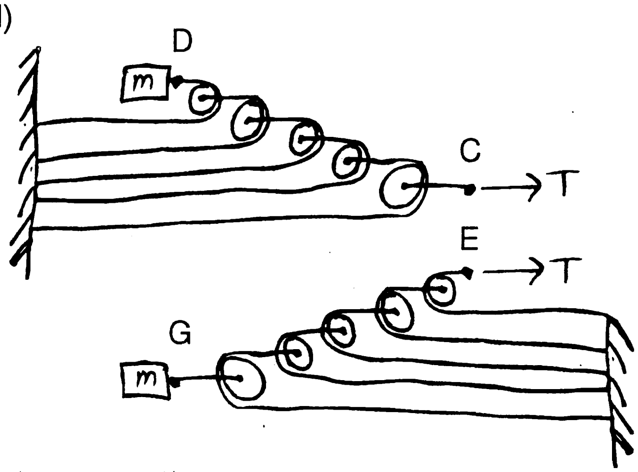
[Part (d) will only be graded if (b) and (c) above are correct.]

d) (11 points) The two pulley systems below (d) are treated as having all ideal components. What is the relation between a_C and a_E? You may use the results from parts (b) and (c) above without re-deriving them again and again. When comparing the systems use $m = m$ and $T = T$.

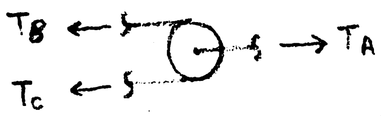
b,c)



d)



b)



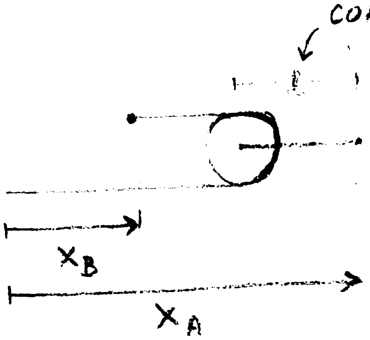
assuming massless pulley

$T_A - T_B - T_c = 0$

also $T_B = T_c$ assuming massless, inextensible string.

$\Rightarrow$ $\boxed{T_A = 2T_B}$

c)



length of the string

$= (x_A - \ell) + (x_A - \ell - x_B)$

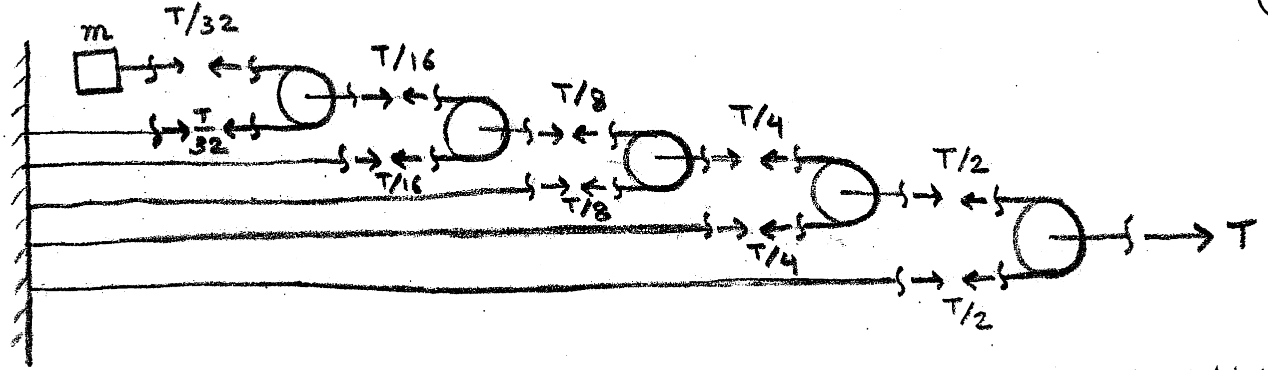
$=$ constant

differentiating wrt time

$\dot{x}_A + \dot{x}_A - \dot{x}_B = 0$ $\quad (\because \dot{\ell} = 0)$

differentiating again $\quad 2\ddot{x}_A = \ddot{x}_B \Rightarrow \boxed{2a_A = a_B}$

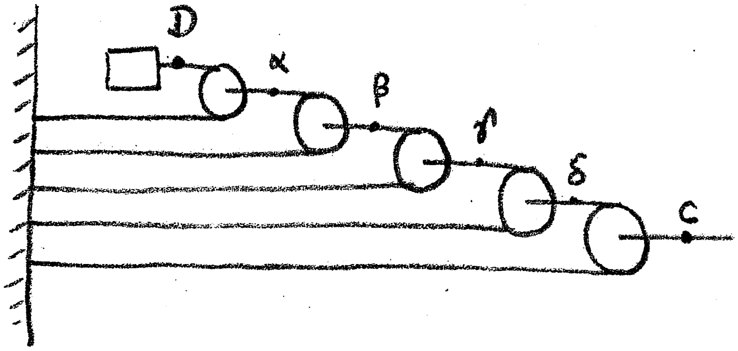
d)



using part b) the above FBDs can be established

N2L on m gives

$$\frac{T}{32} = m\,a_D \quad \Rightarrow \quad a_D = \frac{T}{32m}$$

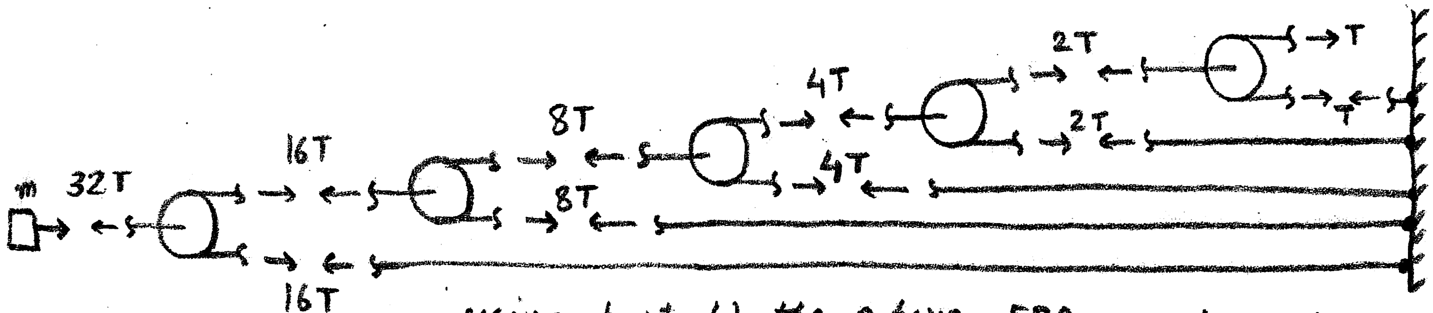


by part c)

$$2a_\alpha = a_D$$
$$2a_\beta = a_\alpha$$
$$2a_\gamma = a_\beta$$
$$2a_\delta = a_\gamma$$
$$2a_c = a_\delta$$

$$\Rightarrow \quad 2a_c = \frac{a_\gamma}{2} = \frac{a_\beta}{4} = \frac{a_\alpha}{8} = \frac{a_D}{16} = \frac{T}{32m}\cdot\frac{1}{16}$$

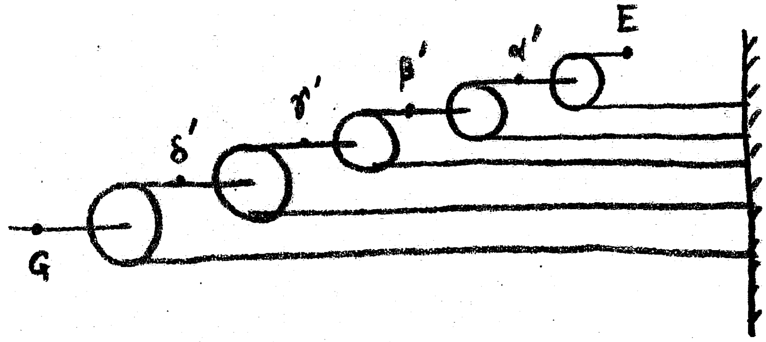
$$\Rightarrow \quad a_c = \frac{T}{m}\left(\frac{1}{32}\right)^2 \quad — \ (1)$$



using part b) the above FBDs can be established

N2L on m gives $\quad 32T = m\,a_G \quad a_G = 32\frac{T}{m}$

by part c)



$$2a_{\alpha'} = a_E$$
$$2a_{\beta'} = a_{\alpha'}$$
$$2a_{\gamma'} = a_{\beta'}$$
$$2a_{\delta'} = a_{\gamma'}$$
$$2a_G = a_{\delta'}$$

$$\Rightarrow \quad a_E = 2a_{\alpha'} = 4a_{\beta'} = 8a_{\gamma'} = 16a_{\delta'} = 32a_G$$

$$\Rightarrow \quad a_E = 32 \cdot \underbrace{32\frac{T}{m}}_{\text{substituting the value of } a_G} = (32)^2 \frac{T}{m} \quad — \quad 2$$

dividing 1 by 2

$$\frac{a_c}{a_E} = \left(\frac{1}{32}\right)^4 \qquad \boxed{a_E = (32)^4 a_c}$$

$$a_E \approx 1,000,000 \; a_c \quad !$$

⑤

2) (25 pt) **MATLAB etc.** The block of code shown calculates motions from a dynamics problem relevant to this course. It runs without error. (This code is no good outside a test, obviously, because the variable names are not suggestive, intermediate variables aren't used, and there is no commenting.)

a) (10 points) Write a mechanics question (with values, units, basic assumptions etc.) that the output of this code answers. Your question should make no reference to matlab or computers but should be in the language of mechanics.

[Grading of parts (b) and (c) below depend on the answer to (a) above being correct. So be confident before moving on.]

b) (10 points) Assume that the command `plot(t,z(:,2))` is added just below the command `t(end)`. Draw, as accurately as you can, the resulting plot. Label (give numerical values) key points and asymptotes which you find using your own pencil-and-paper analysis. Label the axes (even though the code does not do this).

c) (5 points) Get as far as you can towards finding a numerical value for `t(end)` without using the computer. Ultimately you will be stuck without a calculator. But get to a point where the job of the calculator is clear.

initial conditions

```
function prelim1q2
options=odeset('events',@fSally);
[t,z,tev, zev, i] = ode45(@fgeorge,[0 1000],[0 10],options);
t(end)
end

function zdot = fgeorge(t,z);
zdot = [z(2) -2*z(2)-10]';
end

function [value,done,dir] = fSally(t,z)
value = z(1) ; done = 1; dir= -1;
end
```

this is what the code prints. So the 'mechanics question' should demand time.

This suggests a differential equation of the form

$\dot{x}_1 = x_2$

$\dot{x}_2 = -2x_2 - 10$

Remember g!

requires to detect an event corresponding to $x_1 = 0$ when x_1 crosses from +ive to -ive

a) realizing $x_1 = x$ (position)

$x_2 = v$ (velocity)

we get the diff equ^n as

$$\dot{x} = v$$
$$\dot{v} = -2v - 10$$

or $\ddot{x} = -2\dot{x} - 10$

noting that its a one-dimensional motion and comparing it with the standard equ^n in mechanics

$$m\ddot{x} + c\dot{x} + kx = F$$

or $$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F}{m}$$

or $$\ddot{x} = -\frac{c}{m}\dot{x} - \frac{k}{m}x + \frac{F}{m}$$

we see $\frac{c}{m} = 2$ $\qquad k = 0$ $\qquad \frac{F}{m} = -10 = -g$!

linear damping

no stiffness

force opposing the motion in all probability should be gravity!

the mechanics question can, hence, be

"A projectile of mass 1 kg is projected vertically up with velocity 10 m/s. The air applies a linear drag proportional to the velocity (drag constant 2 N/(m/s)). Find the time projectile take to hit the ground again."

b) plot (t, z(:,2)) wants a plot of v vs t

now $\quad \dot{v} = -2v - 10$

$$\int_{10}^{v} \frac{dv}{2v+10} = -\int_0^t dt \quad \Rightarrow \quad \frac{1}{2} \ln(2v+10) \Big]_{10}^{v} = -t$$

initial condition $\longrightarrow$ 10

$$\Rightarrow \ln\left(\frac{2v+10}{30}\right) = -2t$$

$$\Rightarrow \quad 2v + 10 = 30 e^{-2t}$$

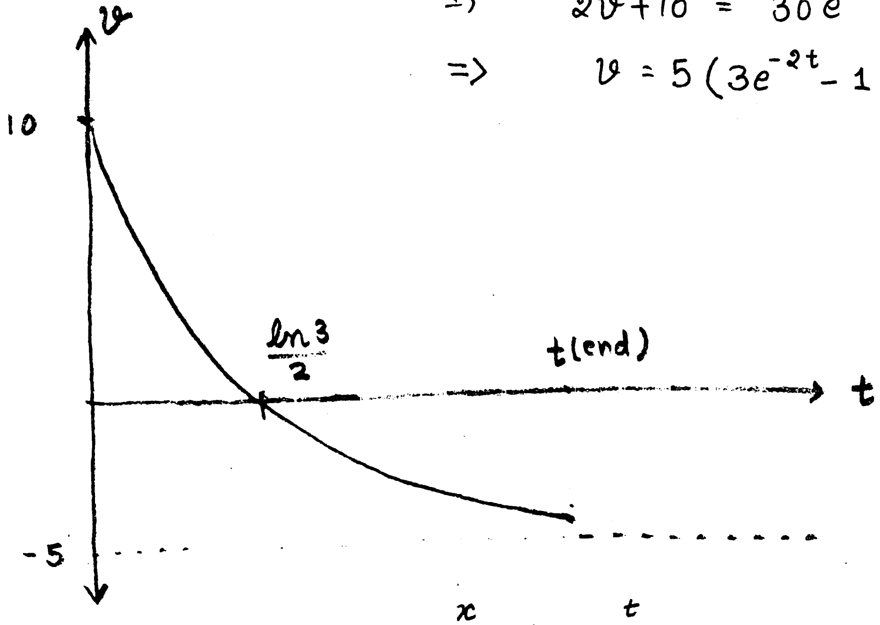
$$\Rightarrow \quad v = 5(3e^{-2t} - 1)$$

$v = 0$

$\Rightarrow e^{-2t} = 1/3$

$2t = \ln 3$

$t = \frac{1}{2} \ln 3$



c) $\quad \dfrac{dx}{dt} = 5(3e^{-2t} - 1) \quad \Rightarrow \quad \displaystyle\int_0^x dx = 5\int_0^t (3e^{-2t} - 1)\, dt$

$$\Rightarrow \quad x = 5 \left\{ \frac{3e^{-2t}}{-2} - t \right\}_0^t \quad \Rightarrow \quad x = 5\left[-\frac{3}{2} e^{-2t} - t + \frac{3}{2} \right]$$

$$x = 5\left[\frac{3}{2}(1 - e^{-2t}) - t \right]$$

now t(end) occurs at $x = 0$

$$\Rightarrow \quad \boxed{\frac{3}{2}(1 - e^{-2\,t(end)}) - t(end) = 0}$$

is to be solved for t(end)

now use the calculator!

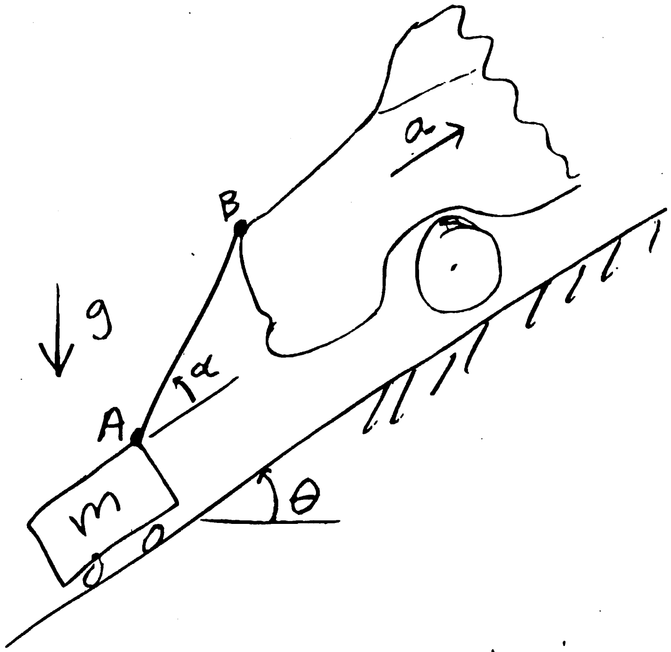
⑦

3) **(25 pt)** A car drags a cart. A car with <u>known acceleration a</u> accelerates up a hill dragging a cart.

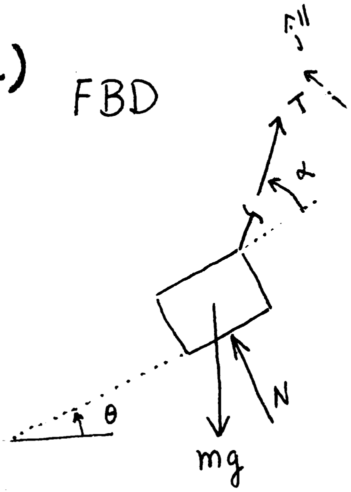
a) (15 points) Assume no friction. Find the force of the ground on the cart. Answer in terms of some or all of m, g, a, θ, α and L_{AB}.

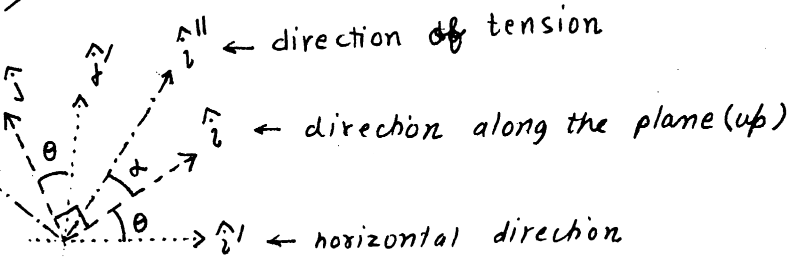
[Your score is the better of the two scores from part (a) and part (b).]

b) (25 points) Assume there is friction between the cart and the ground. Find the tension in the cable AB. Answer in terms of some or all of m, g, a, θ, α, L_{AB} and the friction coefficient μ (or the friction angle ϕ, defined as $\tan\phi = \mu$).



a) FBD





applying N2L on mass m gives

$$\vec{F}_{ext} = m\vec{a}$$

$$T\hat{i}'' - mg\hat{j}' + N\hat{j} = m(\underbrace{a_t}_{\text{tangential acceleration}}\hat{i} + \underbrace{a_n}_{\text{normal acceleration} = 0}\hat{j})$$

$$\{\}\cdot\hat{i} \Rightarrow T\underbrace{(\hat{i}''\cdot\hat{i})}_{\cos\alpha} - mg\underbrace{(\hat{j}'\cdot\hat{i})}_{\cos(\frac{\pi}{2}-\theta)} + N\underbrace{(\hat{j}\cdot\hat{i})}_{0} = m a_t$$

$$\Rightarrow \quad T\cos\alpha - mg\sin\theta = m a_t = m a \qquad \text{— ①}$$

($a_t = a$ assuming the string remains tight.)

$$\{\}\cdot\hat{j} \Rightarrow \quad T\underbrace{(\hat{i}''\cdot\hat{j})}_{\cos(\frac{\pi}{2}-\alpha)} - mg\underbrace{(\hat{j}'\cdot\hat{j})}_{\cos\theta} + N\underbrace{(\hat{j}\cdot\hat{j})}_{1} = 0$$

$$T\sin\alpha - mg\cos\theta + N = 0 \quad — (2)$$

from (2)

$$N = mg\cos\theta - T\sin\alpha \quad — (3)$$

from (1)

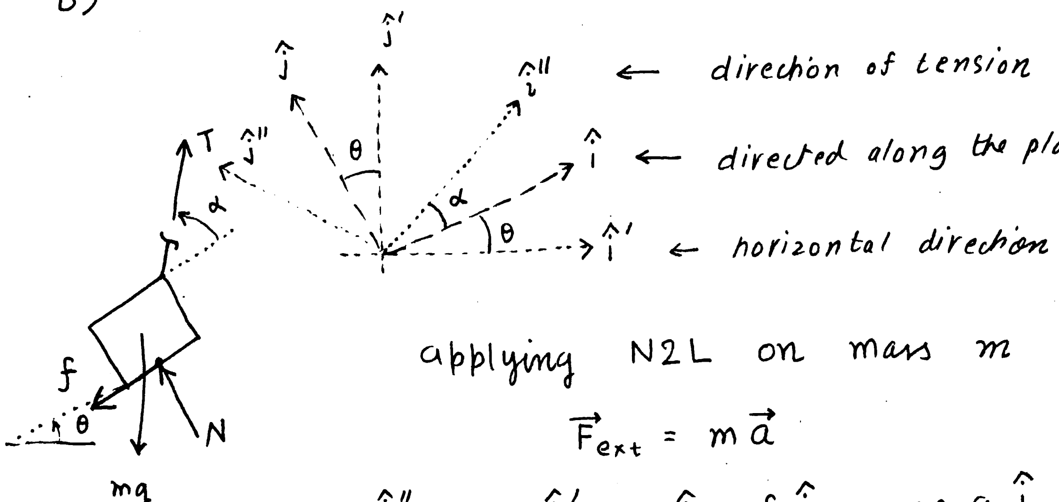
$$T = \frac{mg\sin\theta + ma}{\cos\alpha}$$

Substituting in (3)

$$N = mg\cos\theta - \frac{\sin\alpha}{\cos\alpha}(mg\sin\theta + ma)$$

$$\boxed{N = m\left[g(\cos\theta - \tan\alpha\sin\theta) - a\tan\alpha\right]}$$

b) AN extra frictional force appear in this part !



applying N2L on mass m gives

$$\vec{F}_{ext} = m\vec{a}$$

$$T\hat{i}'' - mg\hat{j}' + N\hat{j} - f\hat{i} = ma\hat{i} \qquad \text{noting } f = \mu N$$

$$\{\}\cdot\hat{i} \quad T\underbrace{(\hat{i}''\cdot\hat{i})}_{\cos\alpha} - mg\underbrace{(\hat{j}'\cdot\hat{i})}_{\cos(\frac{\pi}{2}-\theta)} + N\underbrace{(\hat{j}\cdot\hat{i})}_{0} - \mu N\underbrace{(\hat{i}\cdot\hat{i})}_{1} = ma\underbrace{(\hat{i}\cdot\hat{i})}_{1}$$

$$\Rightarrow \quad T\cos\alpha - mg\sin\theta - \mu N = ma \quad — (1)$$

$$\{\}\cdot\hat{j} \quad T\underbrace{(\hat{i}''\cdot\hat{j})}_{\cos(\frac{\pi}{2}-\alpha)} - mg\underbrace{(\hat{j}'\cdot\hat{j})}_{\cos\theta} + N\underbrace{(\hat{j}\cdot\hat{j})}_{1} - \mu N\underbrace{(\hat{i}\cdot\hat{j})}_{0} = ma\underbrace{(\hat{i}\cdot\hat{j})}_{0}$$

$$\Rightarrow \quad T\sin\alpha - mg\cos\theta + N = 0 \quad — (2)$$

from (2)

$$N = mg\cos\theta - T\sin\alpha$$

Substituting the value of N in 1

$$T\cos\alpha - mg\sin\theta - \mu(mg\cos\theta - T\sin\alpha) = ma$$

$$\Rightarrow \quad T(\cos\alpha + \mu\sin\alpha) = ma + mg\sin\theta + \mu mg\cos\theta$$

$$\boxed{T = \frac{m(a + g\{\sin\theta + \mu\cos\theta\})}{\cos\alpha + \mu\sin\alpha}}$$

Note :- from 2 and answer above

$$N = mg\cos\theta - T\sin\alpha$$
$$= mg\cos\theta - \frac{m\sin\alpha(a + g\{\sin\theta + \mu\cos\theta\})}{\cos\alpha + \mu\sin\alpha}$$

put $\mu = 0$ to get the answer to part a)

$$N(\mu=0) = mg\cos\theta - m\tan\alpha(a + g\sin\theta)$$
$$= m[g(\cos\theta - \tan\alpha\sin\theta) - a\tan\alpha]$$

which is what we found in a) !