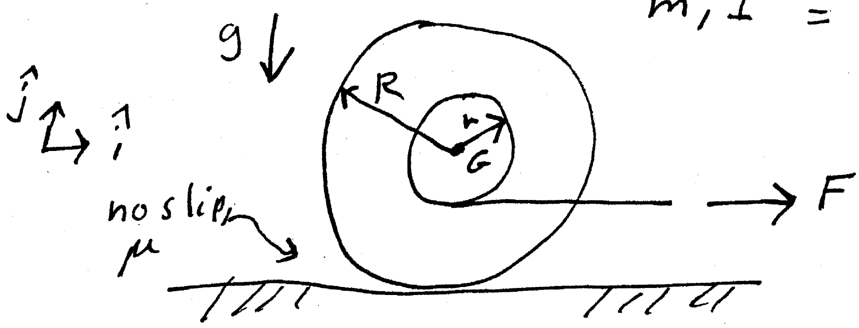
7) A film spool rolls without slip.
In terms of the variables shown find the force (vector) of the ground on the spool.

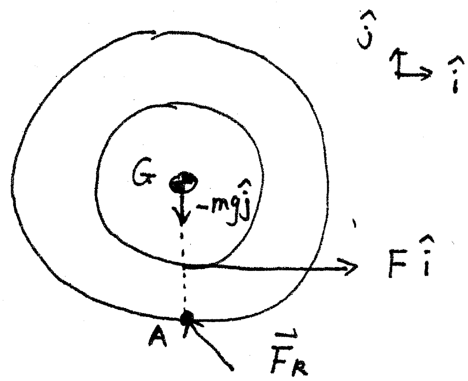
given

$m, I^G = \int r_{/G}^2 \, dm$



Solution:

FBD:



Assume A is the contact point between the spool and the ground.

Let $\vec{F}_R$ be the force vector of the ground on the spool.

$$\vec{F}_R = \underbrace{f}_{\text{friction force.}} \hat{i} + \underbrace{N}_{\text{normal supporting force}} \hat{j}$$

Kinematics:

Assume velocity of ceter of mass is $\vec{V}_G$,

obviously, $\vec{V}_G = V_G \hat{i}$

Assume angular velocity of the spool is $\vec{\omega} = \omega \hat{k}$

No slip condition $\Rightarrow \quad V_G + \omega R = 0$

$\Rightarrow \quad V_G = -\omega R$

This is true all the time, We can take time derivative of this equation.

$\Rightarrow \quad \dot{V}_G = -\dot{\omega} R$

$\therefore$ Acceleration of G : $\vec{a}_G = \dot{\vec{V}}_G = \dot{V}_G \hat{i} = a_G \hat{i}$

Angular acceleration of spool: $\vec{\alpha} = \dot{\vec{\omega}} = \dot{\omega} \hat{k} = \alpha \hat{k}$

$$\boxed{a_G = -\alpha R} \quad ①$$

LMB of spool $\Rightarrow$

$$\{ F\hat{i} - mg\hat{j} + \vec{F}_R = m\vec{a}_G \} \qquad \left(\begin{array}{l} \vec{F}_R = f\hat{i} + N\hat{j} \\ \vec{a}_G = a_G \hat{i} \end{array} \right)$$

$\{\} \cdot \hat{i} \Rightarrow \quad \boxed{F + f = m a_G} \quad ②$

$\{\} \cdot \hat{j} \Rightarrow \quad -mg + N = 0 \quad \Rightarrow \quad \boxed{N = mg} \quad ③$

AMB of spool about G $\Rightarrow$

$$\Sigma M_{/G} = \dot{\vec{H}}_{/G} \Rightarrow \vec{r}_{A/G} \times \vec{F}_R + Fr\hat{k} = I_G \vec{\alpha}$$

$$\Rightarrow \{ fR\hat{k} + Fr\hat{k} = I^G \alpha \hat{k} \}$$

$\{\} \cdot \hat{k} \Rightarrow \quad \boxed{fR + Fr = I^G \alpha} \quad ④$

The force vector

$$\vec{F}_R = -\frac{I^G + mRr}{mR^2 + I^G}\hat{i} + mg\hat{j}$$

Using ①; ②, ④ become

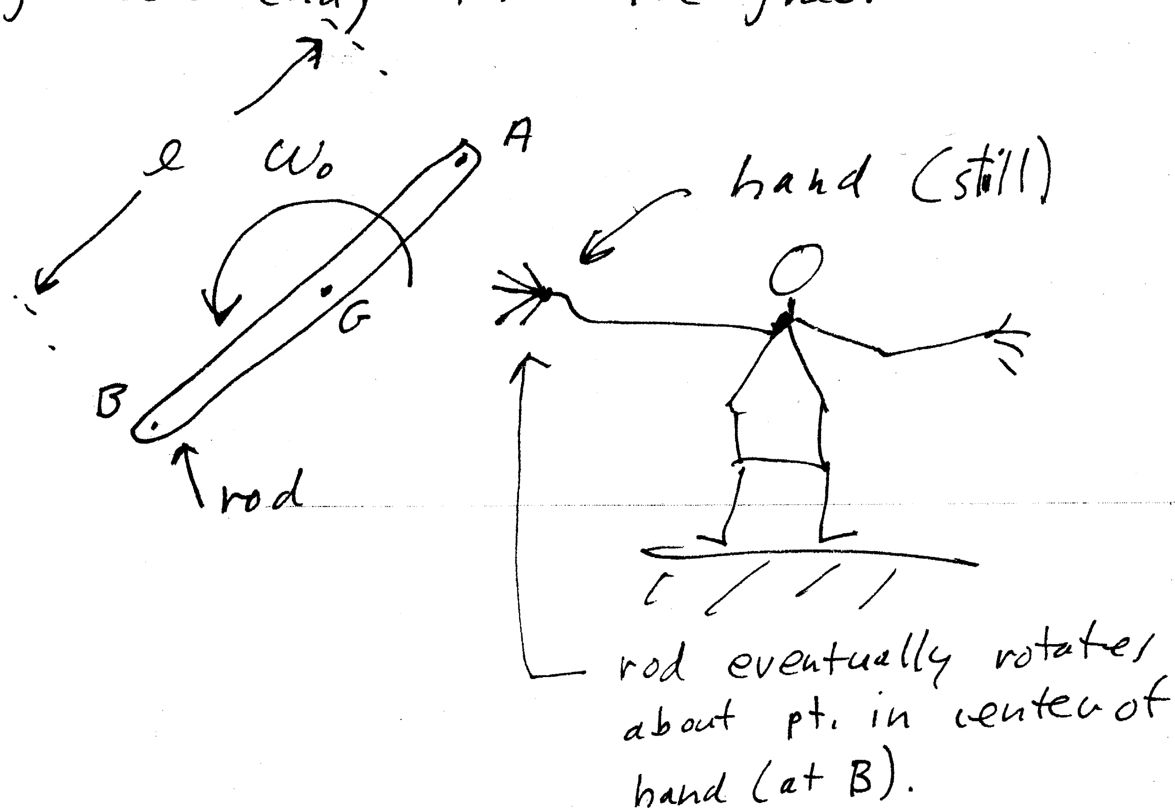
$$\begin{cases} F + f = m a_G \\ Fr + fR = -I^G \dfrac{a_G}{R} \end{cases}$$

$$\Rightarrow \cancel{f = -\frac{m a_G r \mp I}{(R-r)}} \qquad \boxed{f = -\frac{I^G + mRr}{mR^2 + I^G} F}$$

8) A rod is spinning (freely) in place in space at ω_0. It has length ℓ, inertial I^G, mass m and C.O.M. at G at its center.

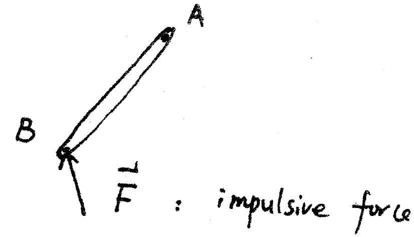
A hand suddenly grabs one end imparting an impulsive force (but no impulsive torque). The hand holds the end still.

What is the speed V_A of the ungrabbed end, after the grab.



Solution: Since the time of "grabbing" is very short and the impulsive force is much larger than other forces, all forces except the impulsive force can be negelected.

FBD (during "grabbing"):



$\therefore \quad \Sigma M_{/B} = 0$ during grabbing

$\Rightarrow \quad \dot{\vec{H}}_{/B} = 0$ during grabbing

$\therefore$ Angular momentum $\vec{H}_{/B}$ is conserved before grabbing the rod and after:

$$\vec{H}^{+}_{/B} = \vec{H}^{-}_{/B}$$

Before "grabbing":

$$\vec{H}^{-}_{/B} = \vec{r}_{G/B} \times \cancelto{0}{\vec{V}_G} + I^G \omega_o \hat{k}$$

$$= I^G \omega_o \hat{k}$$

After "grabbing":

The rod rotates about B with angular velocity $\vec{\omega}_a = \omega_a \hat{k}$
since B is fixed in this case,

$$\therefore \quad \vec{H}^{+}_{/B} = I^B \vec{\omega}_a$$

$$= \left(I^G + m\left(\frac{l}{2}\right)^2 \right) \omega_a \hat{k}$$

$$= \left(I^G + \frac{ml^2}{4} \right) \omega_a \hat{k}$$

$\vec{H}^{+}_{/B} = \vec{H}^{-}_{/B} \Rightarrow \qquad I^G \omega_o = \left(I^G + \frac{ml^2}{4}\right) \omega_a$

$$\Rightarrow \qquad \omega_a = \frac{4 I^G}{4I^G + ml^2} \omega_o$$

The speed $\vec{V}_A$ after the grab is $\vec{V}_A = \vec{V}_B + \cancel{\vec{r}_A}\, \vec{\omega}_a \times \vec{r}_{A/B}$

$$\Rightarrow \quad |\vec{V}_A| = V_A = \omega_a l = \boxed{\frac{4 I^G l}{4I^G + ml^2} \omega_o}$$

9) A mass m is going to be pulled by a rope & pulley mechanism with an applied force F at some point.

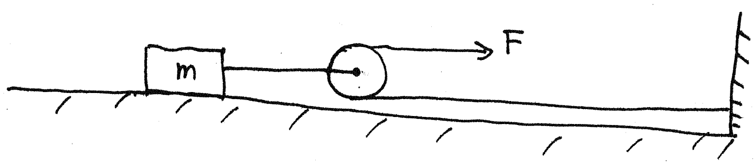
a) design a mechanism so the mass has an accelation of $a_A = 2F/m$.

b) " with $a_A = F/2m$

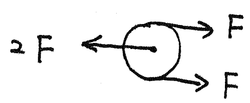
c) " with $a_A = F/8m$

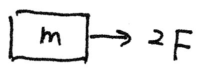
Solutions : (All pulleys are frictionless and massless)

a).



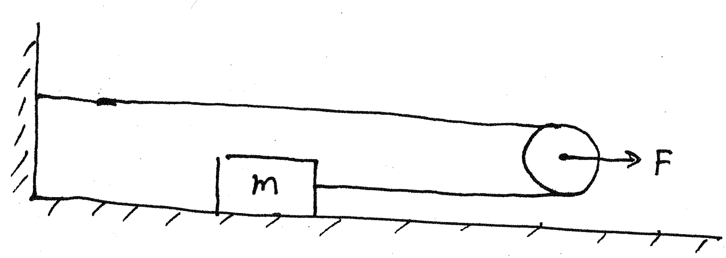
FBD:



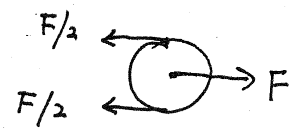


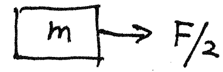
$$\therefore \quad a_A = \frac{2F}{m}$$

b).



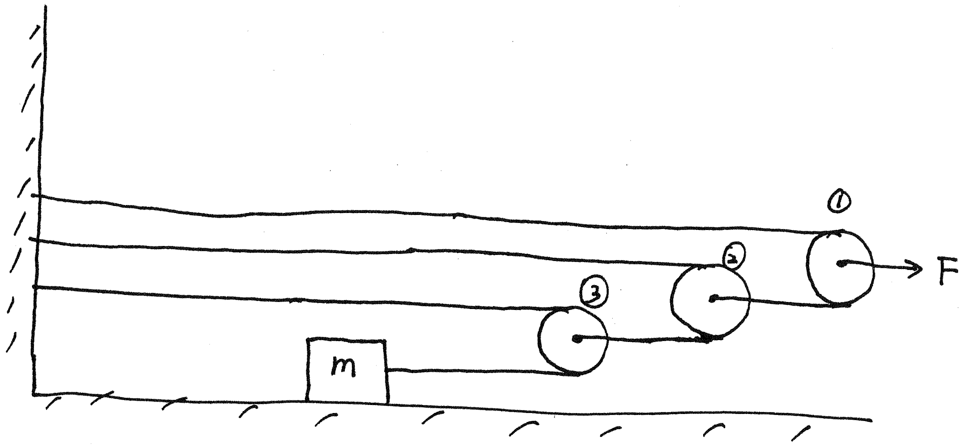
FBD:





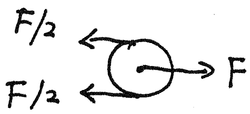
$$\therefore \quad a_A = \frac{F/2}{m} = \frac{F}{2m}$$

c).

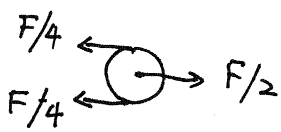


FBD:

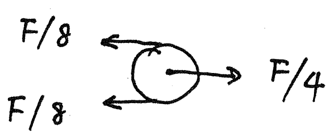
pulley ①

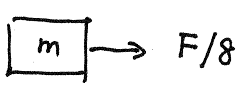


pulley ②



pulley ③





$$\therefore \quad a_A = \frac{F/8}{m} = \frac{F}{8m}$$