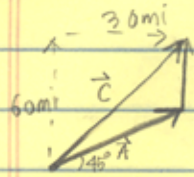
TAM 2020

Homework 2

Solution

2.5.9



Find $\vec{A}$

$$\vec{A} + \vec{B} = \vec{C}$$

$$A\hat{\lambda}_A + B\hat{\lambda}_B = \vec{C}$$

Component Breakdown:

$$A\hat{\lambda}_{Ax} + B\hat{\lambda}_{Bx} = C_x = 30\,mi$$

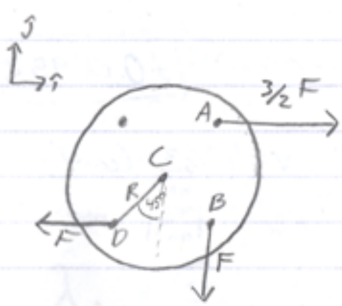
$$A\hat{\lambda}_{Ay} + B\hat{\lambda}_{By} = C_y = 60\,mi$$

where $\hat{\lambda}_A = \frac{\sqrt{2}}{2}\hat{\imath} + \frac{\sqrt{2}}{2}\hat{\jmath}$ $\qquad \hat{\lambda}_B = \hat{\jmath}$

$$A\left(\frac{\sqrt{2}}{2}\right) + \cancel{B(0)} = 30\,mi \longrightarrow A = \frac{60}{\sqrt{2}}\,mi = 30\sqrt{2}\,mi$$

$$A\left(\frac{\sqrt{2}}{2}\right) + B(1) = 60\,mi$$

2.6.8



$$\sum \vec{F}_{\hat{\imath}} = (3/2\,F - F)N\hat{\imath}$$

$$\sum \vec{F}_{\hat{\jmath}} = (-F)N\hat{\jmath}$$

$$\boxed{\vec{F}_{net} = \tfrac{1}{2}FN\hat{\imath} - FN\hat{\jmath}}$$

$a \times b = ab\sin\theta\,\hat{n}$

Karen Duffy explained moments to me.

$$\vec{M}_C = \vec{r}_{A/C} \times 3/2\,\vec{F} + \vec{r}_{B/C} \times \vec{F} + r_{D/C} \times \vec{F}$$

$$= R \cdot \tfrac{3}{2}F \cdot \sin 135 \cdot -\hat{k} + R \cdot F \cdot \sin 135 \cdot -\hat{k}$$

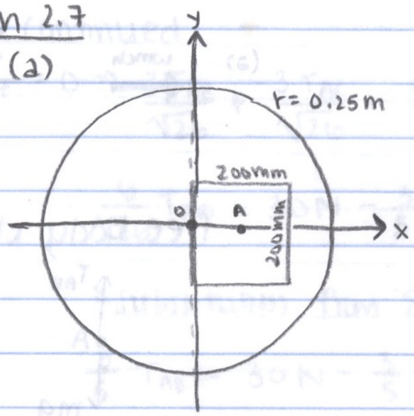
$$+ R \cdot F \cdot \sin 135 \cdot -\hat{k}$$

$$= \left(\frac{3RF}{2} \cdot \frac{\sqrt{2}}{2} + \frac{RF \cdot \sqrt{2}}{2} + \frac{RF \cdot \sqrt{2}}{2}\right)(-\hat{k})$$

$$= -\frac{RF\sqrt{2}}{2}\left(\frac{3}{2} + 2\right)\hat{k} = \boxed{-\frac{7RF\sqrt{2}}{4}\hat{k}} = \vec{M}_C$$

⇒ Section 2.7

(11) (a)



Let m_1 = mass of plate with cutout

Let m_2 = mass of cutout

Let m = mass of the whole plate = 1kg

Let x_{cm} = center of mass with cutout

Let x_A = center of mass of cutout = 100mm

x_0 = center of mass of whole plate

$$m_1 x_{cm} + m_2 x_A = m x_0 = 0$$

$$x_{cm} = -\frac{m_2 x_A}{m_1}$$

$$\frac{m_2}{m_1} = \frac{(0.2m)^2}{\pi(0.25m)^2 - (0.2m)^2}$$

$$= \frac{0.04m^2}{0.156m^2}$$

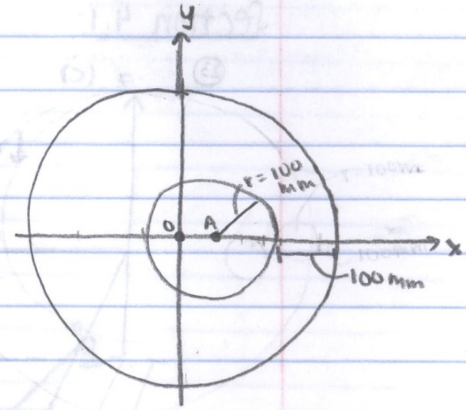
$$x_{cm} = -0.256(100mm)$$

$$= -25.58 mm$$

center of mass at

(−25.58 mm, 0mm)

(b)



Let m_1 = mass of plate with cutout

Let m_2 = mass of cutout

m = 1kg = mass of the whole plate

Let x_{cm} = center of mass with cutout

Let x_A = center of mass of cutout = 0.05 m

Let x_0 = center of mass of whole plate

$$x_{cm} = -\frac{m_2 x_A}{m_1}$$

$$\frac{m_2}{m_1} = \frac{\pi(0.1m)^2}{\pi(0.25m)^2 - \pi(0.1m)^2}$$

$$= \frac{0.0314 m^2}{0.1649 m^2}$$

$$x_{cm} = -0.1904(0.05m)$$

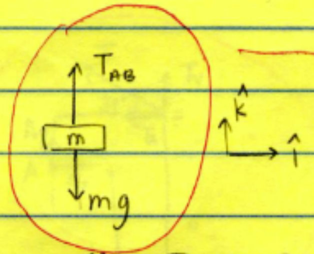
$$= -9.52 mm$$

center of mass at

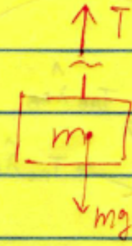
(−9.52 mm, 0 mm)

4.1.23 $\Rightarrow$

a.



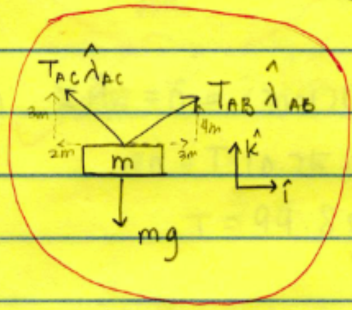
show strings on FBD



see examples on textbook

$$mg = T_{AB} = 3\text{kg}(10\,\text{m/s}^2) = 30\text{N}.$$

b



$$T_{AC}\hat{\lambda}_{AC} + T_{AB}\hat{\lambda}_{AB} - mg\,\hat{k} = \vec{0}$$

$$\hat{\lambda}_{AC} = \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} = \frac{-2m\hat{\imath} + 3m\hat{k}}{\sqrt{13}\,m}$$

$$\hat{\lambda}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{3m\hat{\imath} + 4m\hat{k}}{5m}$$

$$\left(\frac{-2T_{AC}}{\sqrt{13}} + \frac{3T_{AB}}{5}\right)\hat{\imath} + \left(\frac{3T_{AC}}{\sqrt{13}} + \frac{4T_{AB}}{5} - mg\right)\hat{k} = \vec{0}$$

$T_{AB} = ?$

$$\hat{n}_{AC}\cdot(T_{AC}\hat{\lambda}_{AC} + T_{AB}\hat{\lambda}_{AB} - mg\hat{k}) = \vec{0}\cdot\hat{n}_{AC}$$

$$(\hat{n}_{AC}\cdot\hat{\lambda}_{AB})T_{AB} - (\hat{n}_{AC}\cdot mg\hat{k}) = 0$$

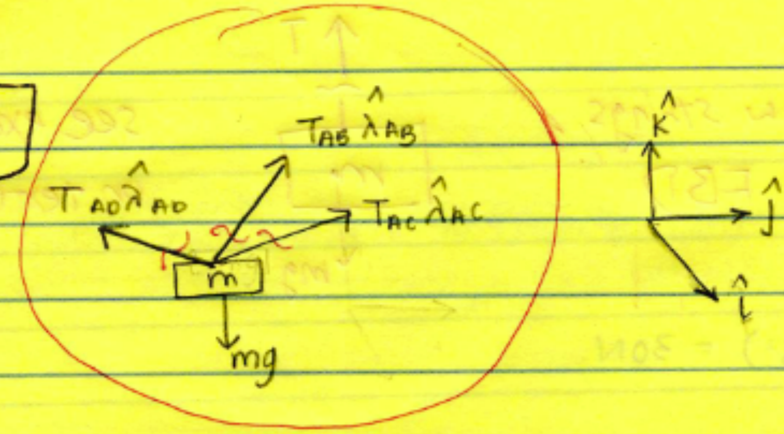
$$T_{AB} = \frac{\hat{n}_{AC}\cdot mg\hat{k}}{\hat{n}_{AC}\cdot\hat{\lambda}_{AB}}$$

where $\hat{n}_{AC} = \hat{\jmath}\times\hat{\lambda}_{AC} = \frac{3}{\sqrt{13}}\hat{\imath} + \frac{2}{\sqrt{13}}\hat{k}$

$$T_{AB} = \frac{\left(\frac{3}{\sqrt{13}}\hat{\imath} + \frac{2}{\sqrt{13}}\hat{k}\right)\cdot(30\hat{k})}{\left(\frac{3}{\sqrt{13}}\hat{\imath} + \frac{2}{\sqrt{13}}\hat{k}\right)\cdot\left(\frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{k}\right)} = \frac{60/\sqrt{13}}{17/5\sqrt{13}}$$

$$T_{AB} = \frac{300}{17}\,\text{N}.$$

$\boxed{c.}$



$$T_{AD}\hat{\lambda}_{AD} + T_{AB}\hat{\lambda}_{AB} + T_{AC}\hat{\lambda}_{AC} - mg\hat{k} = \vec{0}$$

$$\hat{\lambda}_{AD} = \frac{\vec{r}_{AD}}{|\vec{r}_{AD}|} = \frac{-4m\,\hat{j} + 3m\hat{k}}{5}$$

$$\hat{\lambda}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{-1m\,\hat{i} + 4m\hat{j} + 3m\hat{k}}{\sqrt{26}}$$

$$\hat{\lambda}_{AC} = \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} = \frac{1m\hat{i} + 4m\hat{j} + 3m\hat{k}}{\sqrt{26}}$$

$$x: \frac{-1}{\sqrt{26}}T_{AB} + \frac{1}{\sqrt{26}}T_{AC} = 0$$

$$y: \frac{-4}{5}T_{AD} + \frac{4}{\sqrt{26}}T_{AB} + \frac{4}{\sqrt{26}}T_{AC} = 0$$

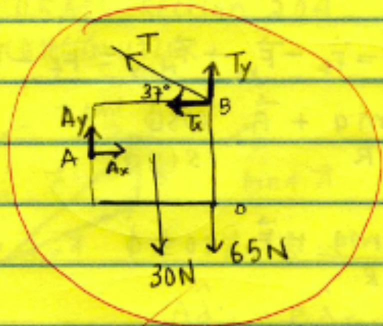
$$z: \frac{3}{5}T_{AD} + \frac{3}{\sqrt{26}}T_{AB} + \frac{3}{\sqrt{26}}T_{AC} = 30$$

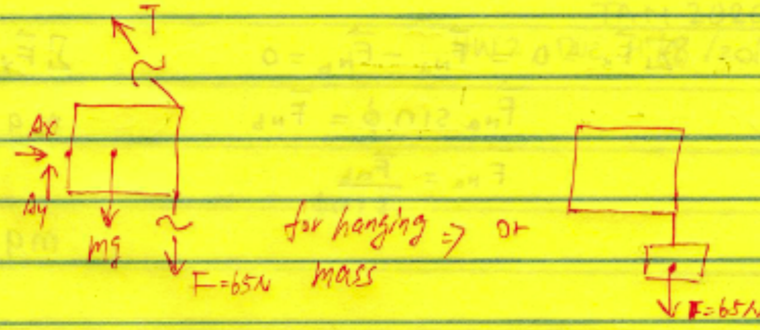
$$\begin{array}{ccc} T_{AD} & T_{AB} & T_{AC} \end{array}$$
$$\begin{bmatrix} 0 & -1/\sqrt{26} & 1/\sqrt{26} \\ -4/5 & 4/\sqrt{26} & 4/\sqrt{26} \\ 3/5 & 3/\sqrt{26} & 3/\sqrt{26} \end{bmatrix} \begin{bmatrix} T_{AD} \\ T_{AB} \\ T_{AC} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = 30$$

used a calculator to find

$$T_{AB} = 12.75\text{ N} = \frac{5\sqrt{26}}{2}\text{ N}$$

4.2.14





a. $\sum \vec{M}_{/A} = \vec{0} = (30N)(0.5m) + (65N)(1m) - T\cos 37°(0.25m) - T\sin 37°(1m)$

$80 = T(0.25\cos 37° + \sin 37°)$

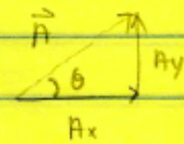
$T = 99.8N$

b. $A_x = T_x = T\cos 37° = 99.8\cos 37°$

$A_y = 30N + 65N - T\sin 37°$

$A_x = 79.8N$

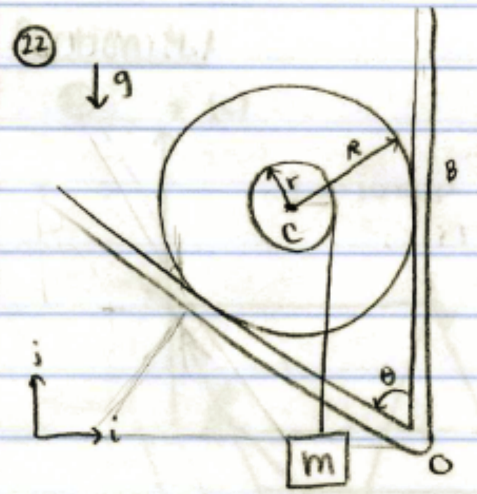
$A_y = 34.94N$



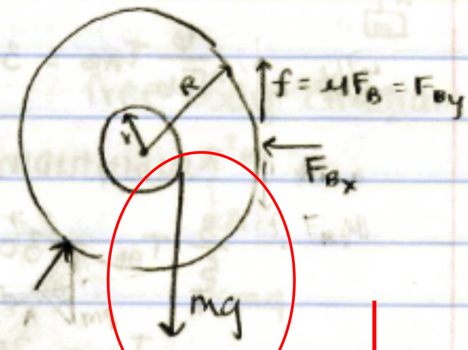
$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = 23.6°$

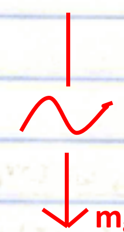
$A = \sqrt{A_x^2 + A_y^2} = 87.1N$

(22)



Free body diagram:





* since surface A is frictionless.

$f_1 = 0$

Forces: weight $= -mg\,\hat{j}$

at B $= -F_{Bx}\hat{i} + F_{By}\hat{j}$

friction at B $= \mu F_B \hat{j} = F_{By}$

at A $= F_{A,x}\hat{i} + F_{A,y}\hat{j}$

$\Sigma F_x = 0 \Rightarrow A\cos\theta - F_{Bx} = 0$

$\Sigma F_y = 0 \Rightarrow A\sin\theta + F_{By} - mg = 0$

$$A = \frac{mg\left(1-\frac{r}{R}\right)}{\sin\theta}$$

Use moments:

$M_C = mgr(-\hat{k})$

$M_{B/C} = Rf = RF_{By}\hat{k}$

$M_C + M_{B/C} = 0$

$F_{By} = \dfrac{mgr}{R}$

$$F_{Bx} = \frac{mg\left(1-\frac{r}{R}\right)}{\sin\theta}\cdot\cos\theta$$

$$= mg\left(1-\frac{r}{R}\right)\cot\theta$$

$$F = \left[mg\left(1-\frac{r}{R}\right)\cot\theta\right]\hat{i} + \frac{mgr}{R}\hat{j}$$