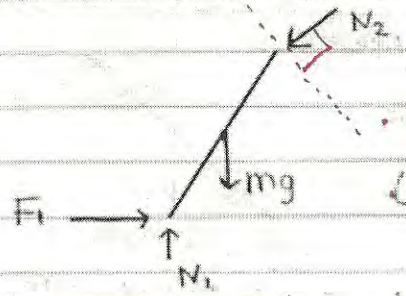
SOLUTION for 4.3.10

• first, start with a free body diagram

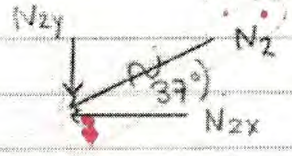


• the ladder's weight acts as a single force at the ladder's center.

each end has a normal

• force acting perpendicular to the surface (aka - the wall.)

• you can decompose the force N_2 into its x and y components



$N_{2x} = N_2 \cos(37°)$

$N_{2y} = N_2 \sin(37°)$

• Sum all the forces in the x and y-directions, set to zero

$\Sigma F_x = 0$ $\qquad$ $\Sigma F_y = 0$

• $F_1 - N_2 \cos(37°) = 0$ $\qquad$ $N_1 - mg - N_2 \sin(37°) = 0$

• Sum all the moments with respect to whichever point you want (I chose the bottom left corner (A) because it cancels out F_1 and N_1) and set to zero.

$\Sigma M_A = 0$

$0 = -mg(l/2)\sin(37°) - N_2 \sin(37°)(l \sin 37°) + (N_2 \cos 37)(l \cos 37)$

• solve for N_1 in terms of N_2

$N_2 \cos^2(37°) = mg/2 \sin(37°) + N_2 \sin^2(37°)$

$N_2 \cos^2(37°) - N_2 \sin^2(37°) = mg/2 \sin(37°)$

$N_2(\cos^2(37°) - \sin^2(37°)) = mg/2 \sin(37°)$

(plug in for mg from ΣF_y equation)

$N_2(\cos^2(37°) - \sin^2(37°)) = (1/2)(N_1 - N_2 \sin 37°)(\sin 37°)$

$2N_2(\cos^2(37°) - \sin^2(37°)) = N_1 \sin(37°) - N_2 \sin^2(37°)$

$2N_2 \cos^2(37°) - 2N_2 \sin^2(37°) + N_2 \sin^2(37°) = N_1 \sin(37°)$

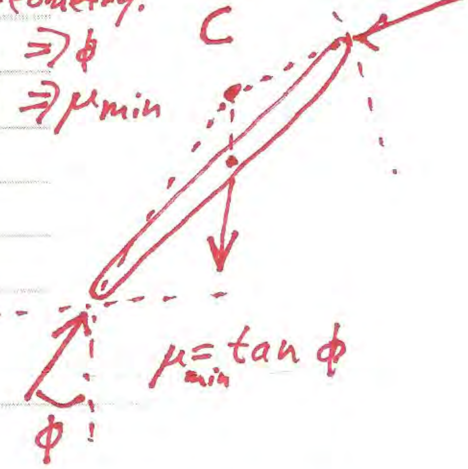
$$N_1 = N_2\left(\frac{2\cos^2(37°) - \sin(37)}{\sin 37}\right)$$

ALT. SOLUTION: 3-force body
locate C by
Geometry.
⇒ φ
⇒ μ_{min}

$\mu \geq \frac{F_1}{N_1}$ therefore $\mu = \dfrac{N_2 \cos(37°)}{N_2\left(\dfrac{2\cos^2(37°) - \sin(37°)}{\sin(37°)}\right)}$

Calculate the minimum value...

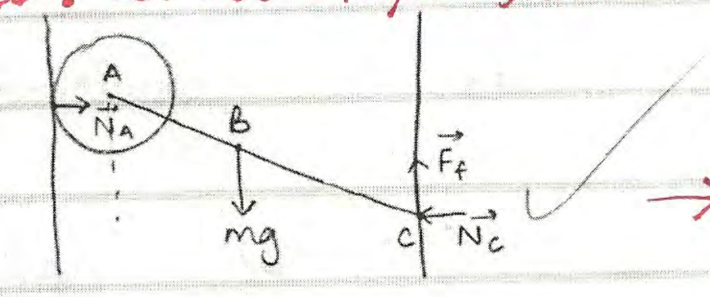
$\mu_{min} = .53$



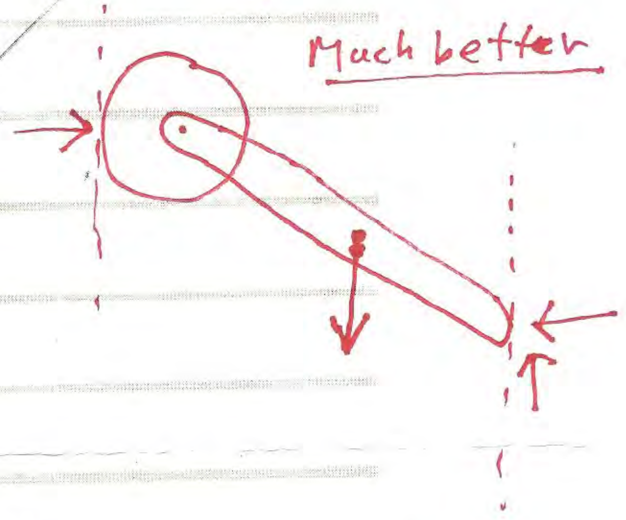
$\mu_{min} = \tan\phi$

FBD: Sort of O.K., not good

15



Much better



Since its in equilibrium,

$\Sigma F = 0.$

~~$\Sigma F_x = N_A - N_C = 0 \Rightarrow N_A = N_C$~~

$\Sigma F_y = \mu N_C - mg = 0 \Rightarrow \mu N_C = mg$

$\vec{M}_{/C} = 0$

$\vec{r}_{CA} \times (-N_A i) + (-9\hat{i} + \frac{27}{4}\hat{j})(-mg\hat{j}) = 0$

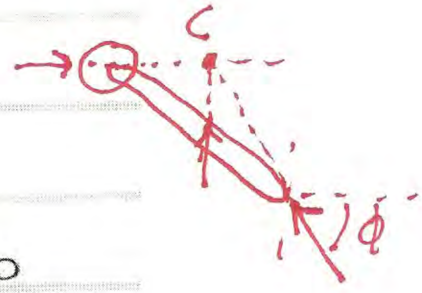
$-9 N_A \hat{k} + 9 mg\hat{k} = 0.$

$N_A = mg$

$N_C = N_A = mg.$

$\mu mg - mg = 0$

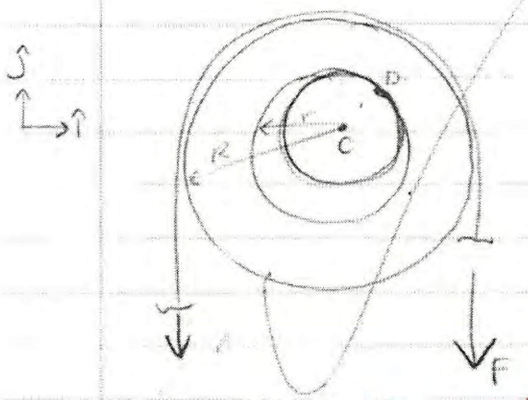
$\therefore \mu_{min} = 1.$



Alt. Soln: Locate C by geometry. Find φ
$\mu_{min} = \tan\phi.$

$\vec{F}_D \cdot \hat{i} = 0 \Rightarrow \theta = \phi - \pi/2$

4.3.20



$\mu = \tan\varphi$ $\quad F_f = \tan\varphi N$

a) Find F $\quad \tan\varphi = \frac{N}{F_f}$

Find θ: F_x: $N\cos\theta - F_f\sin\theta$

$N\cos\theta = \mu N\sin\theta$

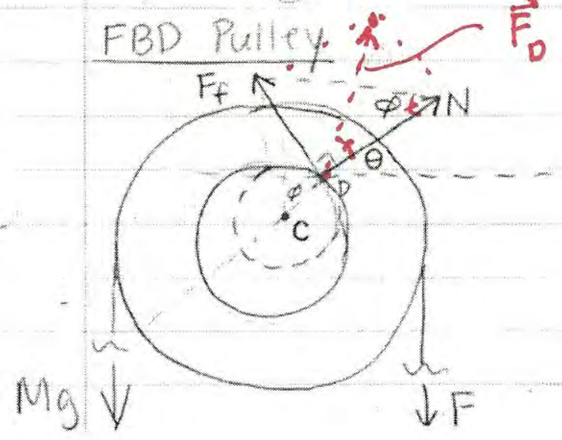
$\cos\theta = \tan\varphi\sin\theta$

$\cot\theta = \tan\varphi$

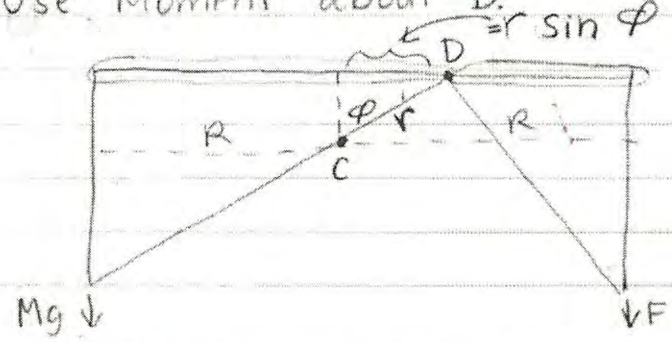
$90 - \varphi = \theta$ $\quad \theta = \varphi$ ✗

$\theta = \pi/2 - \phi$

FBD Pulley



Use Moment about D.



$\theta = \pi/2 - \phi \Rightarrow \cos\theta = \sin\phi$

$$\sum \vec{M}_{/D} = \vec{0} \qquad 0 = Mg(R + r\cos\theta) - F(R - r\cos\theta)$$

$$0 = MgR + Mgr\cos\theta - FR + Fr\cos\theta$$

$$FR - Fr\cos\theta = MgR + Mgr\cos\theta$$

$$F(R - r\cos\varphi) \times$$

$$F = \frac{Mg(R + r\sin\varphi)}{(R - r\sin\varphi)}$$

← the right answer [2 canceling errors]

(b) $M = 100\,kg$

$g = 10\ m/s^2$

$r = 1\ cm$

$R = 2\ cm$

$\mu = \sqrt{3}/3$

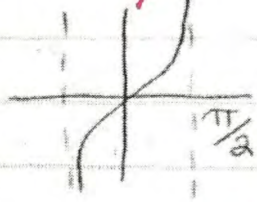
$\varphi = \pi/6$

$$F = \frac{(100)(10)(2 + 1(\frac{1}{2}))}{2 - 1(\frac{1}{2})}$$

$$\boxed{F = 1666.67\ N}$$

(c) as $\mu \to \infty$, $\tan\varphi \to \infty$ $\to$ $\varphi \to \pi/2$

$(\mu = \tan\varphi)$



$$F = \frac{Mg(R + r\sin(\pi/2))}{R - r\sin(\pi/2)}$$

$$\boxed{F \Rightarrow \frac{Mg(R+r)}{R-r}}$$

Note: say wheel is 10 times bigger than axle. then $r/R = \frac{1}{10}$ and $F \approx Mg \cdot 1.2$

That is, no matter how big μ, F is only 20% bigger than weight!