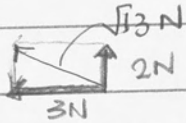
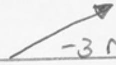
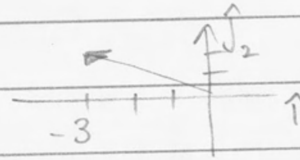
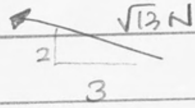
2.1.6 a. 

resultant vector:  √13 N $-3N\hat{i} + 2N\hat{j}$

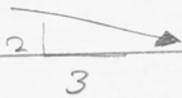
b.  $-3N\hat{i} + 2N\hat{j}$



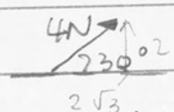
$mag = \sqrt{(-3)^2 + (2^2)} = \sqrt{13} N$

c. 

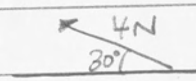
correct mag.
direc: $-3N\hat{i} + 2N\hat{j}$

(*) d 

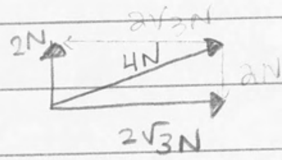
correct magnitude but WRONG direction
$3N\hat{i} - 2N\hat{j}$

2.1.7 a. 

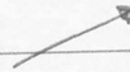
$2\sqrt{3}N\hat{i} + 2N\hat{j}$

b. 

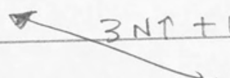
$-2\sqrt{3}N\hat{i} + 2N\hat{j}$

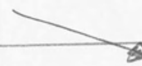
c. 

$2\sqrt{3}N\hat{i} + 2N\hat{j}$

d. 

$2N(-\hat{i} + \sqrt{3}\hat{j})$
$= -2N\hat{i} + 2\sqrt{3}N\hat{j}$
$\text{magnitude} = \sqrt{4 + 4(3)}$
$= 4N.$

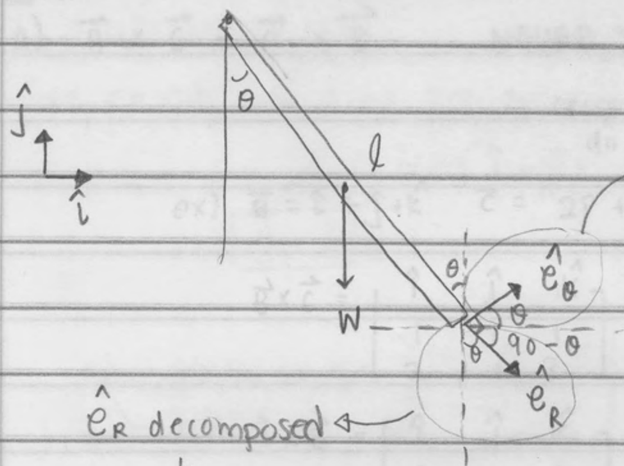
e.  $3N\hat{i} + 1N\hat{j}$
mag: $\sqrt{10}$ N

f. 

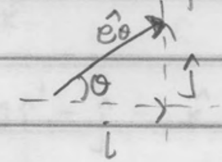
$3N(\frac{1}{3}\hat{i} + \hat{j})$
$= \hat{i} + 3\hat{j}$
$\text{magnitude} = \sqrt{10}$ N.

$\boxed{\therefore \text{same vectors} = a \,\&\, c}$

2.2.20 SOLUTION

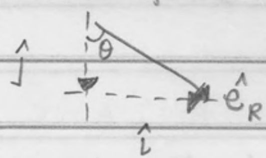


$\hat{e}_\theta$ decomposed:



$\hat{e}_\theta = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\hat{e}_R$ decomposed



$\hat{e}_R = \sin\theta\,\hat{i} - \cos\theta\,\hat{j}$

Components of W along $\hat{e}_R$ and $\hat{e}_\theta$?

$$W_{\hat{e}_R} = -W\hat{j} \cdot (\sin\theta\,\hat{i} - \cos\theta\,\hat{j}) = W\cos\theta$$

$$W_{\hat{e}_\theta} = -W\hat{j} \cdot (\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) = -W\sin\theta$$

FOR PARTS 'C' & 'D' of 2.3.2, I googled the properties of cross products & came across the scalar/vector triple product

2.3.2

a) $\vec{B} \times \vec{C} = \vec{C} \times \vec{B}$ NEVER TRUE b/c cross product of vectors is not communtative. direction of the cross product change, depending on whether you do $\vec{B} \times \vec{C}$ or $\vec{C} \times \vec{B}$

ex) $\vec{B} = \hat{\imath} + \hat{\jmath} + \hat{k}$ $\vec{C} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$$

$$\vec{C} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = -2\hat{\imath} + \hat{\jmath} + \hat{k}$$

opposite direc. so $\vec{B} \times \vec{C} \neq \vec{C} \times \vec{B}$

b) $\vec{B} \times \vec{C} = \vec{C} \cdot \vec{B}$ NONSENSE cross products yield vectors while dot products yield scalars. And Vectors do not equate scalars (vectors $\neq$ scalars)

c) $\vec{C} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{C} \times \vec{A})$ ALWAYS TRUE $(\vec{C} \cdot (\vec{A} \times \vec{B}))$ & $(\vec{B} \cdot (\vec{C} \times \vec{A}))$ are identities of the scalar triple product

ex) $\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{B} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{C} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

$$(\vec{A} \times \vec{B}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \hat{\jmath} - \hat{k}$$

$$\vec{C} \cdot (\vec{A} \times \vec{B}) = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) \cdot (\hat{\jmath} - \hat{k}) = 2 - 1 = 1$$

$$\vec{C} \times \vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \hat{\imath} - \hat{k}$$

$$\vec{B} \cdot (\vec{C} \times \vec{A}) = (2\hat{\imath} + \hat{\jmath} + \hat{k}) \cdot (\hat{\imath} - \hat{k}) = 2 - 1 = 1$$

equal ✓

d) $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$ ALWAYS TRUE property of the vector triple product

ex) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{B} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{C} = \hat{i} + 2\hat{j} + \hat{k}$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & -1 & 3 \end{vmatrix} = 4\hat{i} - 4\hat{j}$$

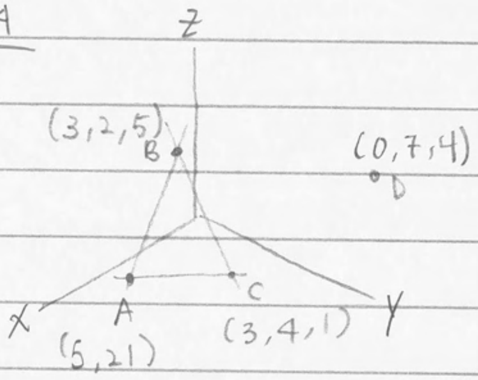
same ✓

$(\vec{A} \cdot \vec{C}) = 1 + 2 + 1 = 4$ $\quad (\vec{A} \cdot \vec{C})\vec{B} = 8\hat{i} + 4\hat{j} + 4\hat{k}$

$(\vec{A} \cdot \vec{B}) = 2 + 1 + 1 = 4$ $\quad (\vec{A} \cdot \vec{B})\vec{C} = 4\hat{i} + 8\hat{j} + 4\hat{k}$

$(\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C} = 4\hat{i} - 4\hat{j}$

2.3.14



a) unit normal vector to plane

$$\vec{n} = \vec{r}_{B/A} \times \vec{r}_{B/C} = (-2\hat{i} + 4\hat{k}) \times (-2\hat{j} + 4\hat{k})$$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 4 \\ 0 & -2 & 4 \end{bmatrix} = 8\hat{i} + 8\hat{j} + 8\hat{k}$$

$\vec{n} = (2, 2, 1)$; $|\vec{n}| = \sqrt{9}$

$$\hat{n} = \frac{2\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{9}} = \frac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$$

$$\vec{r}_{DE} = \vec{r}_D - \vec{r}_E$$

b) $\vec{r}_{D/C} = -3\hat{i} + 3\hat{j} + 3\hat{k}$

(1) distance from D to plane = $\vec{r}_{D/C} \cdot \vec{n}$

$d = \vec{r}_{D/C} \cdot \vec{n} = (-3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot \frac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$

$d = \frac{1}{3}(\cancel{-6} + \cancel{6} + 3)$

$\boxed{d = 1}$

c) point on the plane closest to point D = E

$\vec{r}_E = \vec{r}_D - \vec{r}_{DE}$ where $\vec{r}_{DE}$ is equal the $\hat{n}$ since the closest line would be on a line thru point D & perpendicular to plane

$\vec{r}_{DE} = \frac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$

$\vec{r}_E = (0, 7, 4) - \frac{1}{3}(2, 2, 1)$

$= \left(-\frac{2}{3}, \frac{19}{3}, \frac{11}{3}\right) = \frac{1}{3}(-2, 19, 11)$

d) Point E = $\left(\frac{-2}{3}, \frac{19}{3}, \frac{11}{3}\right)$

The triangular plane given does not contain any points in the negative x direction since the x-component of point E is negative, it cannot lie on the plane

2.2.11 (a) $\vec{A}+\vec{B}=\vec{B}+\vec{A}$ This is always true. Example: $\vec{A}=(0,17,-2)$ $\vec{B}=(-1,3,2)$
$\vec{A}+\vec{B}=(-1,20,0)$ $\vec{B}+\vec{A}=(-1,20,0)$ easily verified by using commutative property of real #'s on components

(b) $\vec{A}+b=b+\vec{A}$ Nonsensical. The concept of adding a vector to a scalar does not make sense.

(c) $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$ Always true. let $\vec{A}=(a_x,a_y,a_z)$ $\vec{B}=(b_x,b_y,b_z)$
$\vec{A}\cdot\vec{B}=a_xb_x+a_yb_y+a_zb_z=b_xa_x+b_ya_y+b_za_z=\vec{B}\cdot\vec{A}$

(d) $\vec{B}/\vec{C}=B/C$ Nonsensical. There is no defined concept of division of vectors.

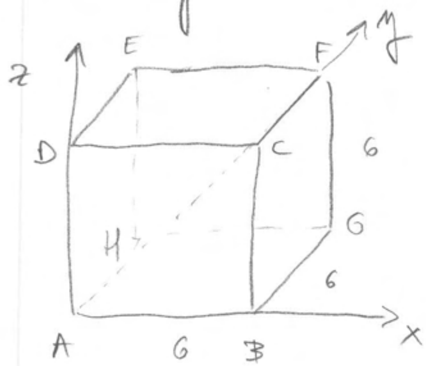
(e) $b/\vec{A}=b/A$ Nonsensical. You cannot divide a scalar by a vector.

(f) $\vec{A}=(\vec{A}\cdot\vec{B})\vec{B}+(\vec{A}\cdot\vec{C})\vec{C}+(\vec{A}\cdot\vec{D})\vec{D}$ This is at least sometimes true (for example it is True if $\vec{A}$ is the zero vector)

if $\vec{A}=\vec{B}=\vec{C}=\vec{D}=(1,1,1)$
then $(\vec{A}\cdot\vec{B})\vec{B}+(\vec{A}\cdot\vec{C})\vec{C}+(\vec{A}\cdot\vec{D})\vec{D}=(9,9,9)\neq(1,1,1)=\vec{A}$ so the statement is sometimes true

possibly (f) is only true if $\vec{B},\vec{C},\vec{D}$ are a set of orthonormal basis vectors?

(20)



① $\vec{r}_F = \vec{r}_D + \vec{r}_{C/D} + \vec{r}_{F/C}$

where $\vec{r}_D = 6\vec{k}$

$\vec{r}_{C/D}$ is a position of C relative to D

$= 6i$

$\vec{r}_{F/C}$ is a position of F relative to D

$= 6j$

Hence, $\vec{r}_F = 6k + 6i + 6j$

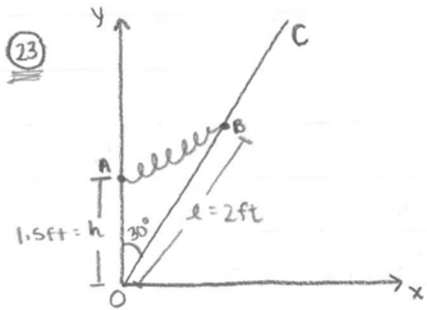
② $|\vec{r}_F| = \sqrt{r_{Fi}^2 + r_{Fj}^2 + r_{Fk}^2} = \sqrt{6^2+6^2+6^2} = 6\sqrt{3}$

③ We can find $\vec{r}_G$ as a sum of $\vec{r}_F$ and $\vec{r}_{G/F}$.

Hence, $\vec{r}_G = 6k+6i+6j + (-6k) = 6i + 6j$

Note that $\vec{r}_{G/F}$ represents a position of G relative to F. Graphically, this makes sense because as we "move" from F to G, we are only sliding down on the z-axis (6 units down).

(23)



A (0, 1.5)

$\sin(60) = \frac{B_y}{2}$

$\cos(60) = \frac{B_x}{2}$

B (1, 1.732)

$\vec{r}_A = 1.5ft\,\hat{j}$

$\vec{r}_B = 1ft\,\hat{i} + 1.732ft\,\hat{j}$

$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = 1ft\,\hat{i} + 0.232ft\,\hat{j}$

$|\vec{r}_{B/A}| = \sqrt{1^2 + (0.232)^2}$

$= 1.027ft$

$\lambda_{AB} = \frac{\hat{i} + 0.232\hat{j}}{1.027}$

$\vec{F} = F\lambda_{AB} = k\vec{r}_{B/A}$

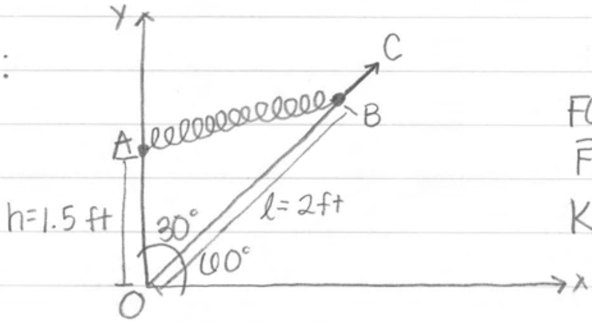
$= (100\ lbf/ft)(1ft\,\hat{i} + 0.232ft\,\hat{j})$

$= 100lbf\,\hat{i} + 23.2\,lbf\,\hat{j}$

SOLUTION 2.1.23

Known: A structure containg a spring is given.

Schematic & Given Data:



Force in the spring:
$\vec{F} = K\vec{r}_{AB}$
$K = 100$ lbf/ft

Find: (a) a unit vector $\hat{\lambda}_{AB}$ along AB
(b) calculate the spring force $\vec{F} = F\hat{\lambda}_{AB}$

Analysis:

(a) $\vec{r}_A = 1.5\cos(90°)\hat{i} + 1.5\sin(90°)\hat{j} = 1.5\hat{j}$

$\vec{r}_B = 2\cos(60°)\hat{i} + 2\sin(60°)\hat{j} = 1\hat{i} + \sqrt{3}\hat{j}$

$\vec{r}_A + \vec{r}_{AB} = \vec{r}_B \rightarrow \vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

$\vec{r}_{AB} = 1\hat{i} + \left(\frac{2\sqrt{3}-3}{2}\right)\hat{j}$

$|\vec{r}_{AB}| = \sqrt{1^2 + 0.23^2} = \sqrt{1.054} = 1.027$

$\hat{\lambda}_{AB} = \frac{1}{1.027}(\hat{i} + 0.232\hat{j}) = \boxed{0.974 \text{ ft}\,\hat{i} + 0.226 \text{ ft}\,\hat{j}}$

(b) $\vec{F} = K\vec{r}_{AB} = \underbrace{K|\vec{r}_{AB}|}_{F}\hat{\lambda}_{AB} = F\hat{\lambda}_{AB}$

$F = (100 \text{ lbf/ft})(1.027) = \boxed{102.7 \text{ lbf/ft}}$