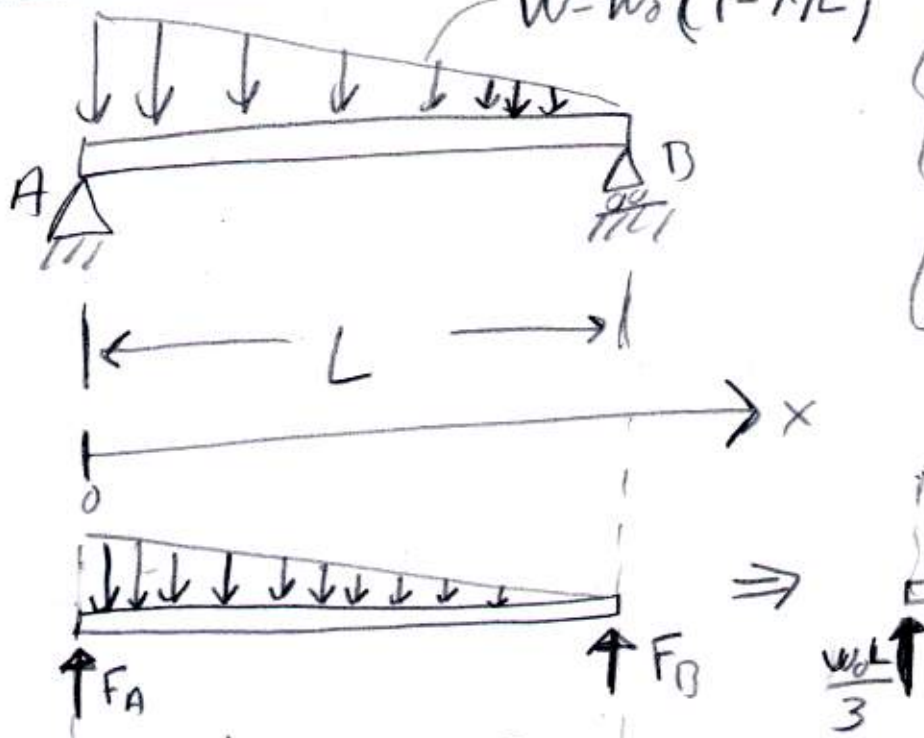
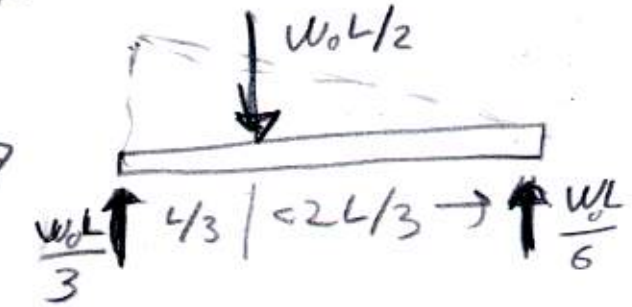
BJ 12.47

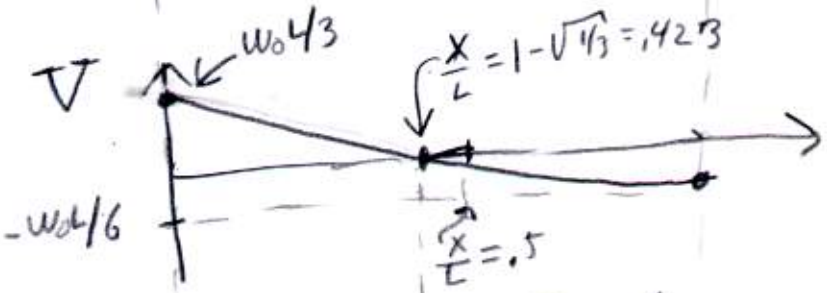


$V(x) = ?$

$M(x) = ?$

$M_{max} = ?$



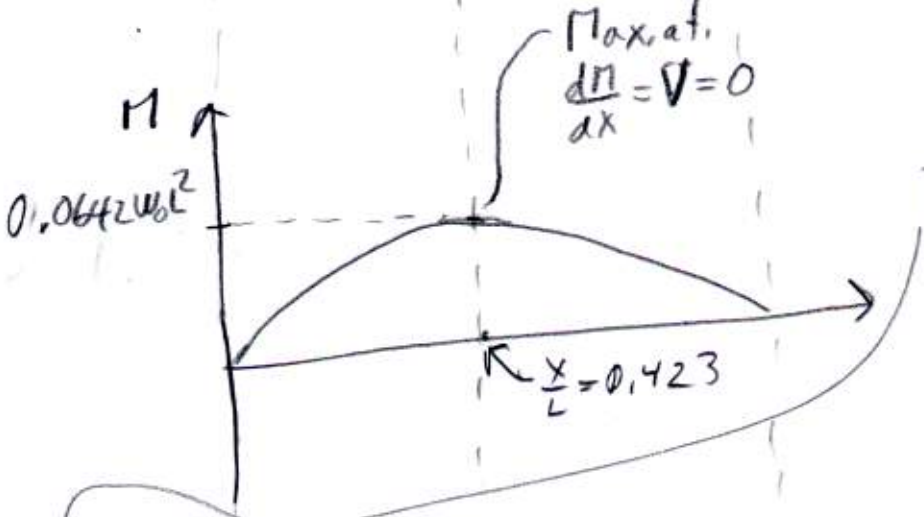


$$V = V_0 - \int_0^x w(x')\,dx'$$

$$V = \frac{w_0 L}{3} - \int_0^x w_0 (1 - x'/L)\,dx'$$

$$V = \frac{w_0 L}{3} - w_0 x + \frac{w x^2}{2L}$$

$$\boxed{V = w_0 L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]}$$



$V = 0 \Rightarrow \quad \frac{x}{L} = \frac{1 \pm \sqrt{1 - 2/3}}{1}$

+ root irrel. $\quad \frac{x}{L} = 1 - \sqrt{1/3} = .423$

$$M = M_0 + \int_0^x V(x')\,dx' = 0 + \int V\,dx$$

$$\boxed{M = w_0 L^2\left[\frac{1}{3}\frac{x}{L} - \frac{x^2}{2L^2} + \frac{1}{6}\left(\frac{x}{L}\right)^3\right]}$$

$$M_{max} = w_0 L^2 \left[\text{eval. at } \frac{x}{L} = 1 - \sqrt{1/3}\right] = \boxed{0.642 \cdot w_0 L^2 = M_{max}}$$