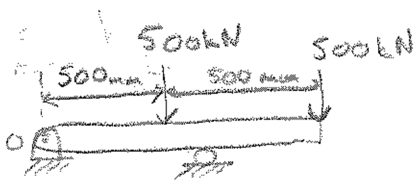
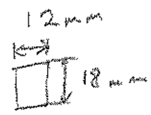
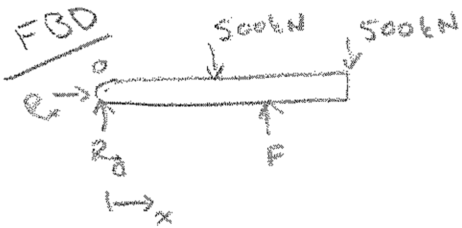
12.261

a) Determine the distance 'a' for which the maximum abs. value of the bending moment in the beam is as small as possibl







ΣF_x: $R_x = 0$

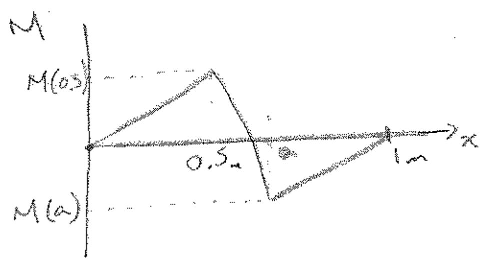
$\Sigma M_{/o}$: $aF = 500\,kN(0.5m) + 500\,kN(1m)$

$F = \dfrac{750\,kN\cdot m}{a}$

ΣF_y: $R_y = 1000\,kN - F$

$R_y = 1000\,kN - \dfrac{750\,kN\cdot m}{a}$

See page 3

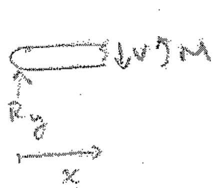


The maximum M is minimized when the magnitude of the 'peaks' are equal.

$\Rightarrow |M(0.5m)| = |M(a)|$

$\Rightarrow M(0.5m) = -M(a)$ ⊛

① if $0 < x < 0.5$



$M = R_y x$

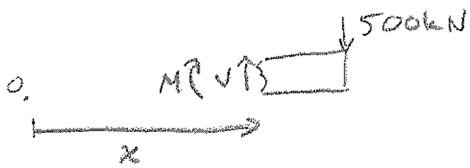
$= \left(1000\,kN - \dfrac{750\,kN\cdot m}{a}\right) x$

M is continuous $\Rightarrow M(0.5m) = \left(1000\,kN - \dfrac{750\,kN\cdot m}{a}\right)(0.5m)$

$= 500\,kN\,m - \dfrac{375\,kN\cdot m^2}{a}$

12.26 cont'd

② if $a < x < 1m$



$$M = -500kN(1m - x)$$
$$= 500kN(x) - 500kN\cdot m$$

$$M(a) = 500kN(a - 1m)$$

$$(*) \quad M(0.5m) = -M(a)$$

$$500\,kN\cdot m - \frac{375\,kN\cdot m^2}{a} = 500\,kN\cdot(1m - a)$$

$$\cancel{125\,kN}\left[4m - \frac{3m^2}{a}\right] = \cancel{125\,kN}\left[4(1m - a)\right]$$

$$4m - \frac{3m^2}{a} = 4m - 4a$$

$$4a^2 = 3m^2$$

$$\boxed{a = \frac{\sqrt{3}}{2}\,m}$$

b) $\sigma_{max} = \frac{-M_{max}c}{I} = \frac{-M_{max}c}{\frac{1}{12}bh^3}$

$$M_{max} = M\left(a = \frac{\sqrt{3}}{2}m\right) = -250(2 - \sqrt{3})\,kN\cdot m$$

$$\sigma_{max} = \frac{250(2-\sqrt{3})(.009)}{\frac{1}{12}(0.012)(0.018)^3}\ kN/m^2$$

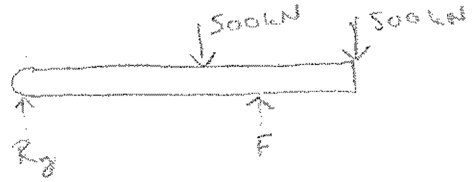
$$\boxed{\sigma_{max} = 103.4\,GPa}$$

Different answer than book!

A force of 500 kN on a beam with an I of $\frac{1}{12}(.012)(.018)^3 = 5.8\times10^{-9}\,m^4$ ⇒ unrealistically large stresses.

Note: the ultimate tensile strength of steel is $\sigma_{UTS} \approx 2.2\,GPa$ much less than σ_{max} found.

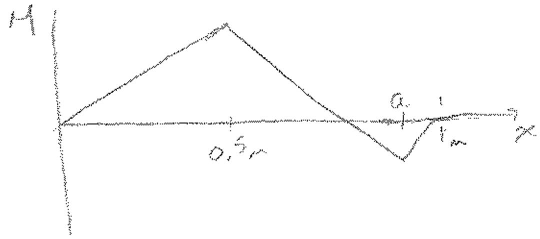
12.26 cont'd



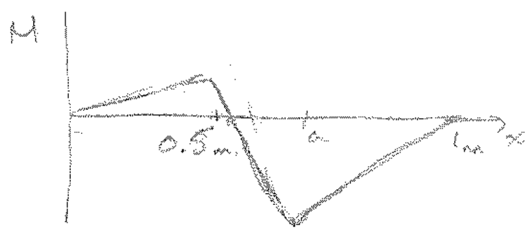
$$F = \frac{750 \text{ kN·m}}{a}$$

$$R_y = 1000 \text{ kN} - \frac{750 \text{ kN·m}}{a}$$

for a close to 1m



for a close to 0.75m



As 'a' decreases, the peak at $x = a$ increases in magnitude while the peak at $x = 0.5$m decreases.

The minimum $|M_{max}|$ occurs when the peaks are equal.