.33 Two solid steel shafts ($G = 77$ GPa) are connected by the gears shown. Knowing that the radius of gear B is $r_B = 20$ mm, determine the angle through which end A rotates when $T_A = 75$ N · m.

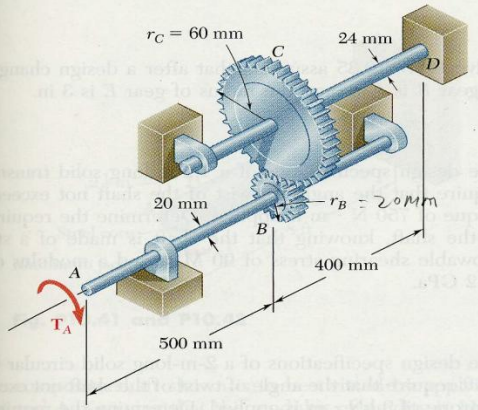


Fig. P10.33

10.33)

$T_B = T_A = 75\ N\cdot m$

$\Sigma M_B = 0$ (Based on FBD of the smaller gear)

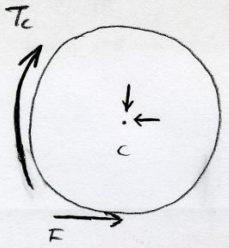
$\Rightarrow F(r_B) - T_B = 0$

$F = \frac{T_B}{r_B} = \frac{75\ N\cdot m}{0.02\ m} = 3750\ N$

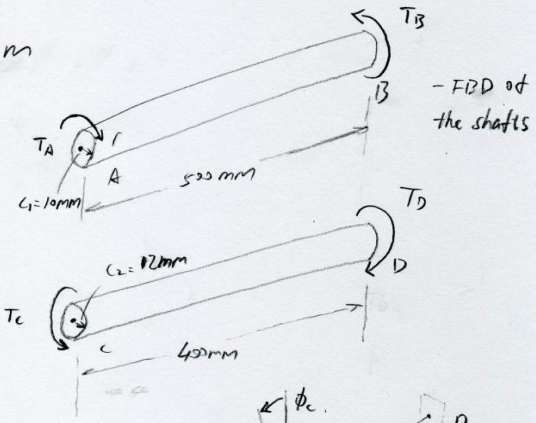
$\Sigma M_C = 0$ (Based on FBD of the larger gear)

$\Rightarrow F(r_C) - T_C = 0$

$T_C = 3750\ N\ (0.06\ m) = 225\ N\cdot m$



- FBD of the gears



Shaft AB

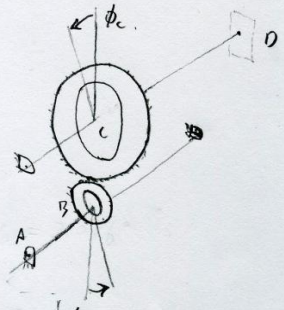
$T_A = T_B = 75\ N\cdot m, \quad \phi_{A/B} = \frac{T_A L}{J G}$

$\Rightarrow \phi_{A/B} = \frac{(75\ N\cdot m)(0.5\ m)}{\frac{1}{2}\pi (0.01\ m)^4 (77\times 10^9\ \frac{N}{m^2})} = 0.031\ rad$

Shaft CD

$T_C = T_D = 225\ N\cdot m, \quad \phi_{C/D} = \frac{T_C L}{J G}$

$\phi_{C/D} = \frac{(225\ N\cdot m)(0.4\ m)}{\frac{1}{2}\pi (0.012\ m)^4 (77\times 10^9\ \frac{N}{m^2})} = 0.0359\ rad$



Since end D of shaft CD is fixed,

$\phi_C = \phi_{C/D} = 0.0359\ rad$

Since $r_B \phi_B = r_C \phi_C \Rightarrow \phi_B = \frac{r_C \phi_C}{r_B} = \frac{(0.06\ m)(0.0359\ rad)}{(0.02)\ m} = 0.1077\ rad$

$\phi_A = \phi_B + \phi_{A/B} = 0.1077\ rad + 0.031\ rad = 0.1387\ rad$

$= 7.94°$