Basic Statics

How to do statics: Draw FBDs. Use force and moment balance.

Free Body Diagram (FBD)
A picture of a system and all the external forces and torques acting on it. At every cut there is a force from the thing it was cut from. For every motion caused or prevented there is a force or moment component. No FBD ⇒ no mechanics.

Action & Reaction on FBDs of A and B
If A feels force $\vec{F}$ and couple $\vec{M}$ from B, then B feels force $-\vec{F}$ and couple $-\vec{M}$ from A. (With $\vec{F}$ and $-\vec{F}$ acting on the same line of action.)

Force and Moment Balance
For every FBD in equilibrium:

\[
\sum \vec{F} = \vec{0} \\
\sum \vec{M}_C = \vec{0}
\]

- The torque $\vec{M}_C$ of a force depends on the reference point C. But, for a body in equilibrium, and for any point C, the sum of all the torques relative to point C must add to zero.

- Dotting the force balance equation with a unit vector gives a scalar equation, e.g. $\{\sum \vec{F}\} \cdot \hat{n} = 0 \Rightarrow \sum F_y = 0$.

- Dotting the moment balance equation with a unit vector gives a scalar equation, e.g., $\{\sum \vec{M}_C\} \cdot \hat{\lambda} = 0 \Rightarrow$ net moment about axis in direction $\hat{\lambda}$ through $C = 0$.

Facts, definitions & miscellaneous
- The moment of a force is unchanged if the force is slid along its line of action.
- For many purposes the words 'moment', 'torque', and 'couple' have the same meaning.

Two-force body. If a body in equilibrium has only two forces acting on it then the two forces must be equal and opposite and have a common line of action.

Caution: Machine and frame components are often not two-force bodies (e.g. transmitted force is not along a bar).

Three-force body. If a body in equilibrium has only three forces acting on it then the three forces must be coplanar and have lines of action that intersect at one point.

Truss: A collection of weightless two-force bodies connected with hinges (2D) or ball and socket joints (3D).

Method of joints. Draw free body diagrams of each joint in a truss.

Zero force member. A bar in a truss with zero tension.

Method of sections. Draw free body diagrams of various regions of a truss. 2D/3D: Try to make the FBD cuts for the sections go through only three/six bars with unknown forces.

Hydrostatics: $p = \rho gh$. $F = \int \rho dA$

Power in a shaft: $P = Tw$

Saint Venant's Principle: Far from the region of loading, the stresses in a structure would only change slightly if a load system were replaced with any other load system having the same net force and moment.

Cross section geometry

<table>
<thead>
<tr>
<th>Definition</th>
<th>Composite</th>
<th>annulus (circle: $c_b = 0$)</th>
<th>thin-wall annulus (approx)</th>
<th>rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$\int dA$</td>
<td>$\sum A_i$</td>
<td>$\pi(c_2^2 - c_1^2)$</td>
<td>$2\pi c_t$</td>
</tr>
<tr>
<td>$J$</td>
<td>$I_{zz}$</td>
<td>$\int r^2 dA$</td>
<td>$\frac{2}{3}(c_2^2 - c_1^2)$</td>
<td>$2\pi c^3 t$</td>
</tr>
<tr>
<td>$I$</td>
<td>$I_{yy}$</td>
<td>$\int (r^2 + d^2 A_i)$</td>
<td>$\frac{4}{3}(c_2^2 - c_1^2)$</td>
<td>$\pi c^3 t$</td>
</tr>
</tbody>
</table>

Stress, strain, and Hooke’s Law

<table>
<thead>
<tr>
<th>Stress</th>
<th>Strain</th>
<th>Hooke’s Law</th>
</tr>
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<tbody>
<tr>
<td>Normal:</td>
<td>$\sigma = P/A$</td>
<td>$\epsilon = \delta/L$</td>
</tr>
</tbody>
</table>

Shear: $\tau = P/A$ $\gamma =$ change of formerly right angle $\tau = G\gamma$ $2G = \frac{T}{2\pi r}$

Stress and deformation of some things

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$P$</th>
<th>$\delta$</th>
<th>$\phi$</th>
<th>$\psi$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F = \sigma A$</td>
<td>$\delta = \frac{P}{A}$</td>
<td>$\phi = \frac{T}{I_c}$</td>
<td>$\psi = \frac{M}{I_c}$</td>
<td>$\theta = \frac{V}{A}$</td>
<td></td>
</tr>
</tbody>
</table>

Symbols
- A,B,C,D,G,… Points on pictures. 
- $A =$ Cross sectional area
- $a, b, c, d, h, l, L, r, R, \ldots$ Distances on pictures
- $f, f, L, \lambda, \phi$ and $x, y, z$ Unit vectors and coordinates
- $c$ Max distance from centerline (torsion) or neutral axis (bending) 
- $E =$ Young’s modulus, $E_{steel} \approx 30 \times 10^6$ psi $\approx 200$ GPa $\approx 2 \times 10^6$ Atm.

F, $F$ Force
- $g,$ $G$ Acceleration of gravity [force/mass] and Shear modulus [stress], respectively
- $I, I$ Area moments of inertia (2nd moments of area). Polar and $x,y, \ldots$
- $M, \vec{M}$ Torque, moment [distance x force].
- $P, T$ Tension [force]

$Q, q$ In beams, $t(y) =$ thickness at $y, Q(y) =$ first moment of area above $y$. $u, v$ Displacement of beam [distance]
- $y$ Distance up from neutral axis on a beam
- $w$ Downwards loading per unit length for beams. E.g., $w = y$
- $a, b, c, \ldots$ Angles
- $d$ Coefficient of thermal expansion, $a_{steel} \approx 12 \times 10^{-6}/^\circ C$
- $\gamma$ Density, mass per unit volume, area or length. E.g., $\gamma_{water} \approx 10,000$ N/m$^3$
- $\delta$ Elongation or displacement [distance]
- $\epsilon, \eta$ Elongation strain [dimensionless], Shear strain [dimensionless], respectively
- $\rho$ Poisson’s ratio
- $\rho$ Radius of curvature (in bending), distance from centerline (in torsion), density [mass/volume], e.g., $\rho_{water} \approx 1000$ kg/m$^3$

$\sigma$ Normal stress, tension stress. Subscripts: $y =$ yield, $y =$ direction, $x = x$ direction, $u =$ ultimate, all =$allowable$, max = maximum

$\tau$ Shear stress
- $\phi, \theta$ Rotation of a shaft, slope of a beam