Statics and Strength of Materials: fact sheet

(12/12/94, revised 5/10/01, 12/14/02, 12/9/2010 — A. Ruina)

Basic Statics

How to do statics: Draw FBDs. Use force and moment balance.

Free Body Diagram (FBD)

A picture of a system and all the external forces and torques acting on it. At every cut there is a force from the thing it was cut from. For every motion caused or prevented there is a force or moment component. No FBD ⇒ no mechanics.

Action & Reaction on FBDs of $\mathcal{A}$ and $\mathcal{B}$

If	$\mathcal{A}$	feels force	$\vec{F}$	and couple	$\vec{M}$	from $\mathcal{B}$.
then	$\mathcal{B}$	feels force	$-\vec{F}$	and couple	$-\vec{M}$	from $\mathcal{A}$.

(With $\vec{F}$ and $-\vec{F}$ acting on the same line of action.)

Force and Moment Balance

For every FBD in equilibrium:

$$\overbrace{\sum_{\substack{\text{All external}\\ \text{forces}}} \vec{F} = \vec{0}}^{\text{Force Balance}} \qquad \overbrace{\sum_{\substack{\text{All external}\\ \text{torques}}} \vec{M}_{/C} = \vec{0}}^{\text{Moment Balance about pt C}}$$

- The torque $\vec{M}_{/C}$ of a force depends on the reference point C. But, for a body in equilibrium, and for any point C, the sum of *all* the torques relative to point C must add to zero).
- Dotting the force balance equation with a unit vector gives a scalar equation, e.g. $\left\{\sum \vec{F}\right\} \cdot \hat{\imath} = 0 \quad \Rightarrow \quad \sum F_x = 0.$
- Dotting the moment balance equation with a unit vector gives a scalar equation, e.g.,

  $\left\{\sum \vec{M}_{/C}\right\} \cdot \hat{\boldsymbol{\lambda}} = 0$

  ⇒ net moment about axis in direction $\hat{\boldsymbol{\lambda}}$ through $\boldsymbol{C} = 0$.

Facts, definitions & miscellaneous

- The moment of a force is unchanged if the force is slid along its line of action.
- For many purposes the words 'moment', 'torque', and 'couple' have the same meaning.
- **Two-force body.** If a body in equilibrium has only two forces acting on it then the two forces must be equal and opposite and have a common line of action.
- **Caution:** Machine and frame components are often **not** two-force bodies (e.g, transmitted force is not along a bar).
- **Three-force body.** If a body in equilibrium has only three forces acting on it then the three forces must be coplanar and have lines of action that intersect at one point.
- **Truss:** A collection of weightless two-force bodies connected with hinges (2D) or ball and socket joints (3D).
- **Method of joints.** Draw free body diagrams of each joint in a truss.
- **Zero force member.** A bar in a truss with zero tension.
- **Method of sections.** Draw free body diagrams of various regions of a truss. 2D/3D: Try to make the FBD cuts for the sections go through only three/six bars with unknown forces.
- **Hydrostatics:** $p = \rho g h, \quad F = \int p\, dA$
- **Power in a shaft:** $P = T\omega$.
- **Saint Venant's Principle:** Far from the region of loading, the stresses in a structure would only change slightly if a load system were replaced with any other load system having the same net force and moment.

Cross section geometry

	Definition	Composite	annulus (circle: $c_1 = 0$)	thin-wall annulus (approx)	rectangle
A	$\int dA$	$\sum A_i$	$\pi(c_2^2 - c_1^2)$	$2\pi c t$	bh
$J = I_{zz}$	$\int \rho^2\, dA$		$\frac{\pi}{2}(c_2^4 - c_1^4)$	$2\pi c^3 t$	
I	$\int y^2\, dA$	$\sum(I_i + d_i^2 A_i)$	$\frac{\pi}{4}(c_2^4 - c_1^4)$	$\pi c^3 t$	$bh^3/12$
$\bar{y}$	$\frac{\int y\, dA}{\int dA}$	$\frac{\sum y_i A_i}{\sum A_i}$	center	center	center

Stress, strain, and Hooke's Law

	Stress	Strain	Hooke's Law
Normal:	$\sigma = P_\perp / A$	$\epsilon = \delta / L_0 = \frac{L - L_0}{L_0}$	$\sigma = E\epsilon$ $[\epsilon = \sigma/E + \alpha \Delta T]$ $\epsilon_{tran} = -\nu \epsilon_{long}$
Shear:	$\tau = P_\parallel / A$	$\gamma =$ change of formerly right angle	$\tau = G\gamma$ $2G = \frac{E}{1+\nu}$

Stress and deformation of some things

	Equilibrium	Geometry	Results
Tension	$P = \sigma A$	$\epsilon = \delta / L$	$\delta = \frac{PL}{AE}$ $\delta = \frac{PL}{AE} + \alpha L \Delta T$
Torsion	$T = \int \rho \tau\, dA$	$\gamma = \rho \phi / L$	$\phi = \frac{TL}{JG}$ $\tau = \frac{T\rho}{J}$
Bending in Beams	$M = -\int y \sigma\, dA$ $\frac{dM}{dx} = V = \int \tau\, dA$ $\frac{dV}{dx} = -w$	$\epsilon = -y/\rho = -y\kappa$ $v'' = \frac{d^2}{dx^2} v = \frac{1}{\rho} = \kappa$ $= d\theta/dx$	$v'' = \frac{M}{EI}$ $\sigma = \frac{-My}{I}$ $\tau(y) = \frac{VQ(y)}{It(y)}$

Symbols

A,B,C,D,G,... Points on pictures.
A = Cross sectional area
$a, b, c, d, h, \ell, L, r, R, w, \ldots$ Distances on pictures
$\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{\lambda}, \hat{n}$ and x, y, z Unit vectors and coordinates
c Max distance from centerline (torsion) or neutral axis (bending)
E = Young's modulus, $E_{\text{steel}} \approx 30 * 10^6$ psi $\approx$ 200 GPa $\approx 2 * 10^6$ Atm.
$F, \vec{F}$ Force
g, G Acceleration of gravity [force/mass] and Shear modulus [stress], respectively
J, I Area moments of inertia (2nd moments of area). Polar and xx, respectively.
$T, M, \vec{M}$ Torque, moment [distance × force]
P, T Tension [force]
Q, t In beams, $t(y)$ = thickness at y, $Q(y)$ = first moment of area above y.
u, v Displacement of beam [distance]
y Distance up from neutral axis on a beam
w Downwards loading per unit length for beams. E.g. $w = \gamma$
$\alpha, \beta, \gamma, \phi, \theta, \ldots$ Angles
α = Coefficient of thermal expansion, $\alpha_{\text{steel}} \approx 12 * 10^{-6}/^\circ C$
γ Density, mass per unit volume, area or length. E.g., $\gamma_{\text{water}} \approx 10,000\,\text{N/m}^3$
δ Elongation or displacement [distance]
ϵ, γ Elongation strain [dimensionless], Shear strain [dimensionless], respectively
ν Poisson's ratio
ρ Radius of curvature (in bending), distance from centerline (in torsion), density [mass/volume], e.g., $\rho_{\text{water}} \approx 1000\,\text{kg/m}^3$
σ Normal stress, tension stress. Subscripts: y= yield, $y = y$ direction, $x = x$ direction, u=ultimate, all=allowable, max = maximum
τ Shear stress
ϕ, θ Rotation of a shaft, slope of a beam