2.5.9

\[ \vec{A} + \vec{B} = \vec{C} \]

Component Breakdown:

\[ A \hat{\lambda}_x + B \hat{\lambda}_y = C_x = 20 \text{ mi} \]
\[ A \hat{\lambda}_x + B \hat{\lambda}_y = C_y = 60 \text{ mi} \]

where

\[ \hat{\lambda}_x = \frac{\sqrt{2}}{2} \mathbf{i} + \frac{\sqrt{2}}{2} \mathbf{j} \]
\[ \hat{\lambda}_y = \mathbf{j} \]

\[ A \left( \frac{\sqrt{2}}{2} \right) + B (0) = 30 \text{ mi} \quad \rightarrow \quad A = \frac{30}{\sqrt{2}} \text{ mi} = 30\sqrt{2} \text{ mi} \]

\[ A \left( \frac{\sqrt{2}}{2} \right) + B (1) = 100 \text{ mi} \]

2.6.8

\[ \sum F_x = (3/2 \vec{F} - \vec{F}) \hspace{1cm} \sum F_y = (-\vec{F})N \]

\[ \vec{F}_{net} = \frac{1}{2} \vec{F}_{x} - \vec{F}_{y} \]

\[ M_c = \vec{r}_{A/c} \times \frac{3}{2} \vec{F} + \vec{r}_{B/c} \times \vec{F} + \vec{r}_{D/c} \times \vec{F} \]
\[ = R \cdot \frac{3}{2} \vec{F} \cdot \sin 135 \cdot \hat{k} + R \cdot \vec{F} \cdot \sin 135 \cdot \hat{k} \]
\[ \hspace{1cm} + R \cdot \vec{F} \cdot \sin 135 \cdot \hat{k} \]
\[ = \left( \frac{3RF}{2} \cdot \frac{\sqrt{2}}{2} + \frac{RF \cdot \sqrt{2}}{2} + \frac{RF \cdot \sqrt{2}}{2} \right) \hat{k} \]
\[ = -\frac{RF \sqrt{2}}{2} \left( \frac{3}{2} + 2 \right) \hat{k} = -\frac{7RF \sqrt{2}}{4} \hat{k} = M_c \]
Let \( m_1 \) = mass of plate with cutout
Let \( m_2 \) = mass of cutout
Let \( m \) = mass of the whole plate = 1 kg
Let \( x_{cm} \) = center of mass with cutout
Let \( x_A \) = center of mass of cutout = 100 mm
Let \( x_0 \) = center of mass of whole plate

\[
M_1 x_{cm} + M_2 x_A = m x_0 \quad x_{cm} = -\frac{M_2 x_A}{M_1}
\]

\[
M_2 = \frac{(0.2 \text{ m})^2}{\pi (0.25 \text{ m})^2 - (0.2 \text{ m})^2} = 0.04 \text{ m}^2 \\
= 0.04 \text{ m}^2
\]

\[
x_{cm} = -0.254 (100 \text{ mm}) = -25.58 \text{ mm}
\]

Center of mass at \((-25.58 \text{ mm}, 0 \text{ mm})\)

Let \( m_1 \) = mass of plate with cutout
Let \( m_2 \) = mass of cutout
Let \( m \) = mass of the whole plate = 1 kg
Let \( x_{cm} \) = center of mass with cutout
Let \( x_A \) = center of mass of cutout
Let \( x_0 \) = center of mass of whole plate

\[
x_{cm} = -\frac{m_2 x_A}{m}
\]

\[
m_2 = \frac{\pi (0.1 \text{ m})^2}{\pi (0.25 \text{ m})^2 - \pi (0.1 \text{ m})^2} = 0.0314 \text{ m}^2 \\
= 0.0314 \text{ m}^2
\]

\[
x_{cm} = -0.1964 (0.05 \text{ m}) = -9.82 \text{ mm}
\]

Center of mass at \((-9.82 \text{ mm}, 0 \text{ mm})\)
4.1.23 a.  

\[ mg = T_{NB} = 3 \text{ kg} \left( 10 \text{ m/s}^2 \right) = 30 \text{ N} \]

\[ T_{AC} \hat{\lambda}_{AC} = mg \hat{k} = \vec{0} \]

\[ \hat{\lambda}_{AC} = \frac{\hat{r}_{AC}}{1 \sqrt{13} m} = -2 \hat{m} + 3 \hat{k} \]

\[ \hat{\rho}_{AB} = \frac{\hat{r}_{AB}}{5 \text{ m}} = 3 \hat{m} + 4 \hat{k} \]

\[ \left( \frac{-2 T_{AC} + 3 T_{AB}}{\sqrt{13}} \right) \hat{k} + \left( \frac{3 T_{AC} + 4 T_{AB} - mg}{\sqrt{15}} \right) \hat{k} = \vec{0} \]

\[ T_{AC} = 2 \]

\[ \hat{r}_{AC} \cdot \left( T_{AC} \hat{r}_{AC} + T_{AB} \hat{r}_{AB} - mg \hat{k} \right) = \vec{0} \cdot \hat{r}_{AC} \]

\[ \left( \hat{r}_{AC} \cdot \hat{r}_{AB} \right) T_{AB} = \left( \hat{r}_{AC} \cdot mg \hat{k} \right) = 0 \]

\[ T_{AB} = \frac{\hat{r}_{AC} \cdot mg \hat{k}}{\hat{r}_{AC} \cdot \hat{r}_{AB}} \]

\[ \text{Where } \hat{r}_{AB} = \frac{\hat{r}_{A}}{3} + \frac{\hat{r}_{C}}{2} \]

\[ T_{AB} = \left( \frac{3 \sqrt{13} \hat{k} + \frac{2}{\sqrt{13}} \hat{k}}{17 \sqrt{13}} \right) \cdot (30 \hat{k}) \]

\[ = \frac{60 \sqrt{13}}{17 \sqrt{13}} \]

\[ \left( \frac{3 \sqrt{13} \hat{k} + \frac{2}{\sqrt{13}} \hat{k}}{17 \sqrt{13}} \right) \cdot \left( \frac{3 \hat{k} + 4 \hat{k}}{17 \sqrt{13}} \right) \]

\[ T_{AB} = 300 \text{ N} \]

\[ \frac{17}{1} \]
\[ T_{AD} \hat{a}_{AD} + T_{AB} \hat{a}_{AB} + T_{AC} \hat{a}_{AC} - mg \hat{k} = 0 \]

\[ \hat{a}_{AD} = \frac{\vec{r}_{AD}}{\| \vec{r}_{AD} \|} = -4m \hat{j} + 3m \hat{k} \]

\[ \hat{a}_{AB} = \frac{\vec{r}_{AB}}{\| \vec{r}_{AB} \|} = -m \hat{i} + 4m \hat{j} + 3m \hat{k} \]

\[ \hat{a}_{AC} = \frac{\vec{r}_{AC}}{\| \vec{r}_{AC} \|} = m \hat{i} + 4m \hat{j} + 3m \hat{k} \]

\[ X : \frac{1}{\sqrt{2}b} T_{AB} + \frac{1}{\sqrt{2}b} T_{AC} = 0 \]

\[ Y : \frac{4}{5} T_{AD} + \frac{4}{\sqrt{2}b} T_{AB} + \frac{4}{\sqrt{2}b} T_{AC} = 0 \]

\[ Z : \frac{3}{5} T_{AD} + \frac{3}{\sqrt{2}b} T_{AB} + \frac{3}{\sqrt{2}b} T_{AC} = 30 \]

\[
\begin{bmatrix}
0 & -1/\sqrt{2}b & 1/\sqrt{2}b \\
-4/5 & 4/\sqrt{2}b & 4/\sqrt{2}b \\
3/5 & 3/\sqrt{2}b & 3/\sqrt{2}b
\end{bmatrix}
\begin{bmatrix}
T_{AD} \\
T_{AB} \\
T_{AC}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
mg = 30
\end{bmatrix}
\]

Used a calculator to find

\[ T_{AB} = 12.78 \text{ N} = \frac{5\sqrt{2}b}{2} \text{ N} \]
\[ \sum \vec{M}_{A} = 0 = (30 \text{N})(0.5 \text{m}) + (65 \text{N})(1 \text{m}) - T \cos 37^\circ (0.25 \text{m}) - T \sin 37^\circ (1 \text{m}) \]

\[ 80 = T (0.25 \cos 37^\circ + \sin 37^\circ) \]

\[ T = 99.8 \text{ N} \]

b. \[ A_x = T_x = T \cos 37^\circ = 99.8 \cos 37^\circ \]

\[ A_y = 30 \text{N} + 65 \text{N} - T \sin 37^\circ \]

\[ A_x = 79.8 \text{ N} \]

\[ A_y = 34.94 \text{ N} \]

\[ \theta = \tan^{-1} \left( \frac{A_y}{A_x} \right) = 23.6^\circ \]

\[ A = \sqrt{A_x^2 + A_y^2} = 87.1 \text{ N} \]
Free body diagram:

*since surface air frictionless, 

\( f_i = 0 \)

Forces: Weight = \(-mg\uparrow\)

at B: \(-F_{Bx}\uparrow + F_{By}\downarrow\)

friction at B: \(M_F\uparrow = F_{By}\)

at A: \(F_{A, x}\downarrow + F_{A, y}\downarrow\)

\[ \Sigma F_x = 0 \Rightarrow A \cos \theta - F_{Bx} = 0 \]

\[ \Sigma F_y = 0 \Rightarrow A \sin \theta + F_{By} - mg = 0 \]

use moments:

\[ A = \frac{mg(1 - \frac{r}{l})}{\sin \theta} \]

\[ F_{Bx} = \frac{mg(1 - \frac{r}{l}) \cos \theta}{\sin \theta} \]

\[ M_{bc} = Rf = R_{F_{By}}F \]

\[ M_C + M_{bc} = 0 \]

\[ F_{By} = \frac{mgr}{R} \]

\[ F = \left[ mg \left(1 - \frac{r}{l} \right) \cot \theta \right] \uparrow + \frac{mgr}{R} \uparrow \]