Your TA, Section # and Section time:

"SOLUTIONS"

Your name:

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Cornell TAM 2020

Prelim 1

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

Sept 28, 2010

Directions. To ease your TA's grading and to maximize your score, please:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- TIDILY REDUCE and box in your answers (Don't leave simplifiable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct. You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`". Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}$, $\hat{\jmath}$, $\hat{e}_r$, $\hat{e}_\theta$, $\hat{\boldsymbol{\lambda}}$, $\hat{\boldsymbol{n}} \ldots$) and signs ($\pm$) with sketches, equations or words.
- **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** Put your name on each extra sheet, fold it in, and refer to it at the relevant problem. Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 1: ____ /25

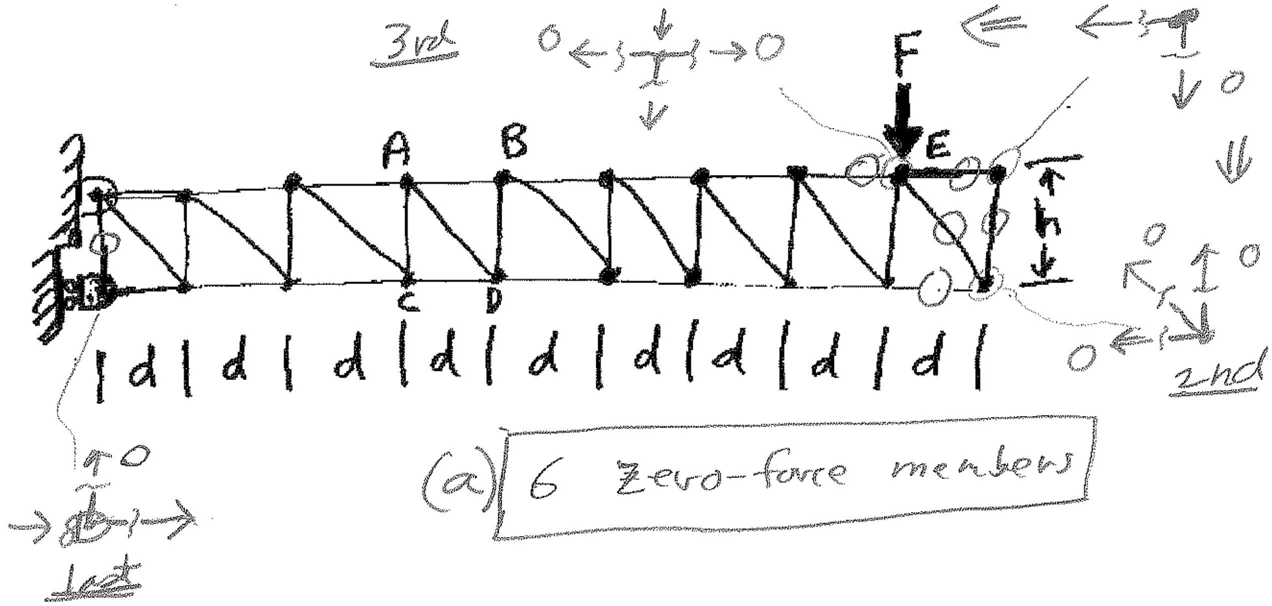
Problem 2: ____ /25

Problem 3: ____ /25

1) The negligible-mass truss shown has height h and all horizontal bars have length d. The force F, with $F > 0$, is given.

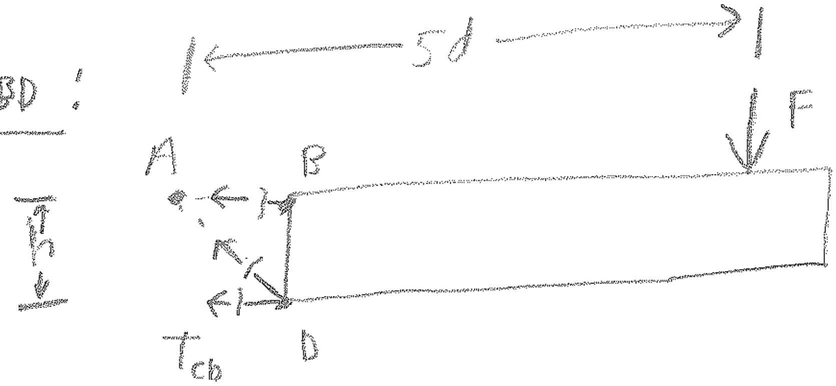
a) Find the 'zero force members' (all the bars that have zero tension). How many are there? Mark them all clearly.

b) Find T_{CD} (the tension in bar CD). Answer in terms of some or all of d, h and F.



(a) $\boxed{6 \text{ zero-force members}}$

Section FBD:



$$\circlearrowleft \sum M_A = 0 \Rightarrow -h T_{CD} - 5d F = 0$$

$$\Rightarrow \boxed{T_{CD} = \frac{-5d}{h} F} \quad \text{(b)}$$

If $F>0$ bar CD has compression of $\boxed{\frac{5d}{h} F}$

2) A door is held open by two opposing taut cables. The top horizontal cable is known to have a tension of 80 N. *What is the tension in the opposing diagonal cable?*

These things apply to the picture at the right:

The door is 2 m high and 50 cm wide.
The door knob is 1.2 m above the floor.
The outer door edge is 30 cm from the back wall &
40 cm from the left wall.
The top horizontal cable is parallel to the back wall.
The right anchor point is
at the intersection of the floor and the back wall,
and 80 cm from the left wall.

Also: $g = 10\,\mathrm{m/s^2}$ and the door mass m = 20 kg.

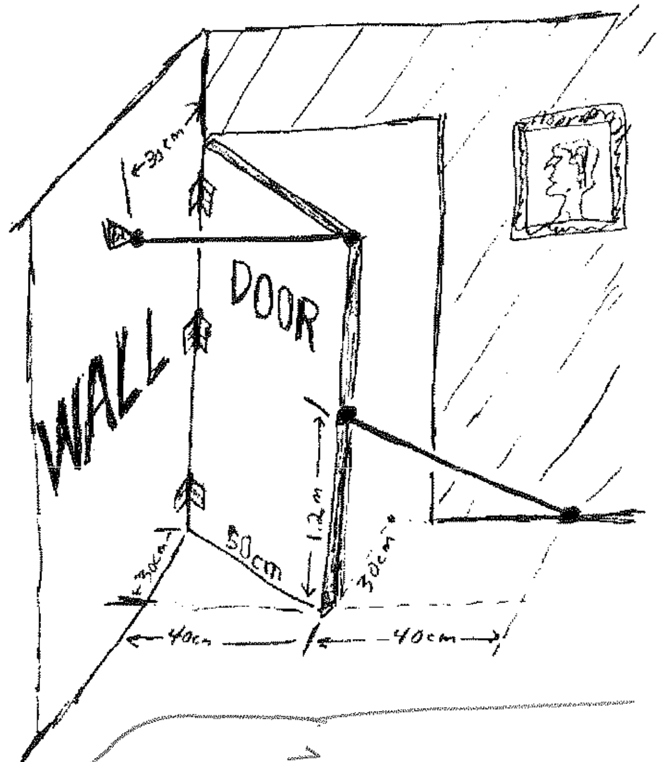
Some perfect squares:

$3^2 = 9$, $4^2 = 16$, $5^2 = 25$

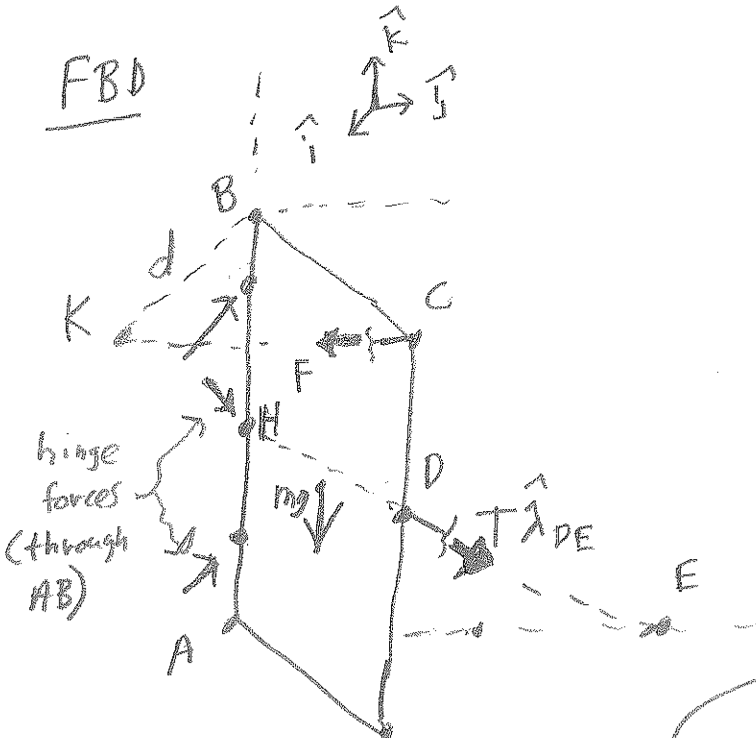
$12^2 = 144$, $13^2 = 169$

Some addition facts:

$9 + 16 = 25$, $25 + 144 = 169$



FBD



$$\hat{\lambda}_{DE} = \vec{r}_{DE} / |\vec{r}_{DE}|$$

$$\hat{\lambda}_{DE} = \frac{40\,\mathrm{cm}\,\hat{j} - 30\,\mathrm{cm}\,\hat{i} - 120\,\mathrm{cm}\,\hat{k}}{\sqrt{(40^2 + 30^2 + 120^2)\,\mathrm{cm}^2}}$$

$$\lambda_{DE} = (-3\hat{i} + 4\hat{j} - 12\hat{k})/13$$

$$\Sigma M_{AB} = 0 \Rightarrow \underbrace{(\vec{r}_{HD} \times T\hat{\lambda}_{DE})}_{(40\hat{j} + 30\hat{i})\,\mathrm{cm}} \cdot \hat{k} - \underbrace{Fd}_{\text{slide } F \text{ to } K} = 0$$

(gravity ∥ to axis)

$$\Rightarrow \hat{k} \cdot \frac{[240\hat{k} + (\ )\hat{i} + (\ )\hat{j}]}{13} T\,\mathrm{cm} - 80\,\mathrm{N} \cdot 30\,\mathrm{cm} = 0$$

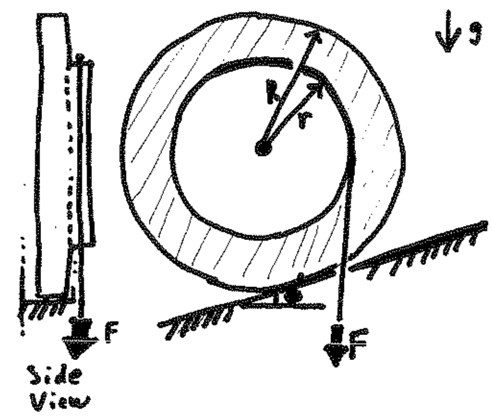
$$\hat{k} \cdot \hat{k} = 1 \Rightarrow T = \frac{2400}{240} \cdot 13\,\mathrm{N} \Rightarrow \boxed{T = 130\,\mathrm{N}}$$

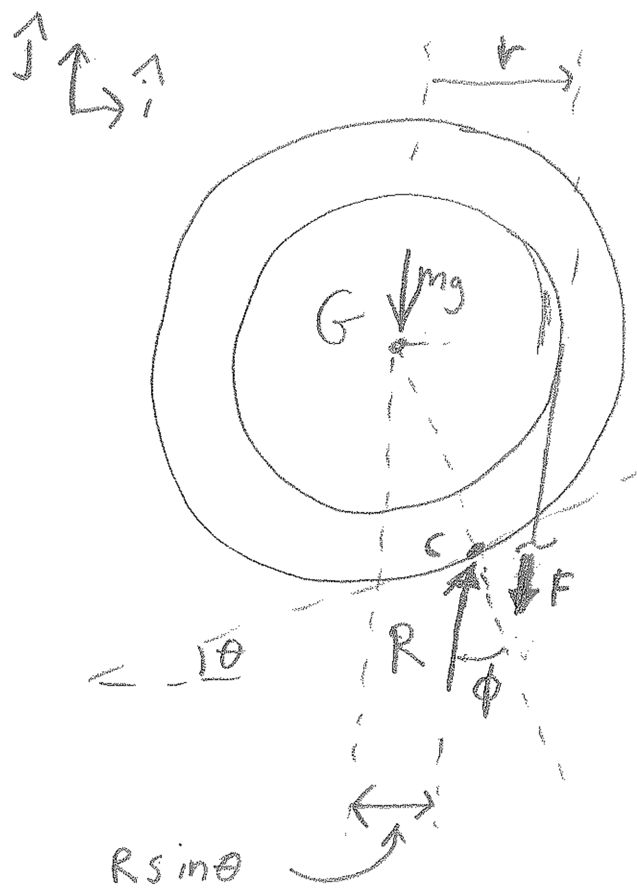
3) A uniform cylinder with mass m and radius R is attached to a spool with radius r. A cable wrapped around the spool is pulled down with force F. The cylinder is free to roll on the slope θ but does not slide on the slope.

a) Find F in order to keep the spool in equilibrium (not rolling up or down). Answer in terms of some or all of m, g, r, R and θ.

b) What is the coefficient of friction μ needed to keep the spool from slipping when it is held in equilibrium. [Alternatively, you can provide the friction angle ϕ, where ϕ is defined by the equation $\tan\phi = \mu$]. Answer in terms of some or all of m, g, r, R and θ.

c) Assume $R = 10\,\text{cm}, r = 5\,\text{cm}, \mu = 1, m = 1\,\text{kg}$ and $g = 10\,\text{m/s}^2$. Assume that you can pull down with arbitrarily large F without damaging the rope, spool or ground. What is the biggest slope θ for which equilibrium is possible?





$$\left\{\sum \vec{F} = \vec{0}\right\}\cdot\hat{\imath}$$

$$\Rightarrow -R\sin(\theta-\phi) = 0$$

$$\Rightarrow \theta = \phi$$

Reaction is vertical

Needed $\phi = \theta \Rightarrow \boxed{\mu \geq \tan\theta}$ (b)

$$\circlearrowleft^{+} \sum M_{/C} = 0 \Rightarrow mg R\sin\theta - (r - R\sin\theta)F = 0$$

$$\Rightarrow \boxed{F = mg\,\frac{R\sin\theta}{r - R\sin\theta}} \quad (a)$$

Bad!

Impossible to get $\sum M_{/C} = 0$!

For $r/R = .5$, $F \to \infty$ as $\theta \to \pi/6 = 30°$.

If $R\sin\theta > r$ then C is to the right of the rope. $\Rightarrow$