2.1.6 a. \[ \text{resultant vector: } \sqrt{3} \hat{N} - 3 \hat{i} + 2 \hat{j} \]

b. \[ -3 \hat{i} + 2 \hat{j} \]

\[ \text{mag} = \sqrt{(-3)^2 + (2^2)} = \sqrt{13} \]

\[ \hat{j} \]

c. \[ \sqrt{13} \hat{N} \]

\[ \text{correct mag.} \]

\[ \text{dir.} \quad -3 \hat{i} + 2 \hat{j} \]

d. \[ \text{correct magnitude but wrong direction} \]

\[ 3 \hat{i} - 2 \hat{j} \]

2.1.7 a. \[ \frac{4N}{2\sqrt{3}} \]

\[ \frac{1}{2} \sqrt{3} \hat{i} + 2 \hat{j} \]

c. \[ 2N \]

\[ \frac{\hat{i}}{2\sqrt{3}N} + \frac{2N}{\sqrt{3}} \]

d. \[ 2N (\hat{i} + \sqrt{3} \hat{j}) \]

\[ = -2 \hat{i} + 2 \sqrt{3} \hat{j} \]

\[ \text{magnitude} = \sqrt{4 + 4(3)} = 4 \text{N} \]

e. \[ 3 \hat{i} + 1 \hat{j} \]

\[ \text{mag.} \quad \sqrt{10} \text{N} \]

\[ f. \quad 3N (\frac{1}{3} \hat{i} + \hat{j}) \]

\[ = \hat{i} + 3 \hat{j} \]

\[ \text{magnitude} = \sqrt{10} \text{N} \]

\[ \therefore \text{same vectors} = a \neq c \]
Components of $W$ along $\hat{e}_R$ and $\hat{e}_\theta$?

$W_{\hat{e}_R} = W \hat{j} \cdot (\sin \theta \hat{i} - \cos \theta \hat{j}) = W \cos \theta$

$W_{\hat{e}_\theta} = -W \hat{j} \cdot (\cos \theta \hat{i} + \sin \theta \hat{j}) = -W \sin \theta$
For parts 'c' & 'd' of 2.3.2, I googled the
properties of cross products & came across the
scalar/vector triple product

a) \( \vec{B} \times \vec{C} = \vec{C} \times \vec{B} \) \text{ NEVER TRUE because cross product of vectors is not commutative, direction of the cross product change depending on whether you do } \vec{B} \times \vec{C} \text{ or } \vec{C} \times \vec{B} \\
\text{ex) } \vec{B} = \hat{i} + \hat{j} + \hat{k} \quad \vec{C} = 2\hat{i} + \hat{j} + 3\hat{k} \\
\vec{B} \times \vec{C} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \end{bmatrix} = 2\hat{i} - \hat{j} - \hat{k} \text{ opposite direction so } \vec{C} \times \vec{B} \\
\hat{c} \times \hat{b} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \end{bmatrix} = -2\hat{i} + \hat{j} + \hat{k} \\
\vec{b} \times \vec{c} \neq \vec{c} \times \vec{b} \\

b) \( \vec{B} \times \vec{C} = \vec{C} \cdot \vec{B} \) \text{ NONSENSE cross products yield vectors while dot products yield scalars. And vectors do not equate scalars (vectors \neq scalars) }

c) \( \vec{C} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{C} \times \vec{A}) \) ALWAYS TRUE \((\vec{C} \cdot (\vec{A} \times \vec{B})) \in (\vec{B} \cdot (\vec{C} \times \vec{A}))\) are identities of the scalar triple product
\text{ex) } \vec{A} = \hat{i} + \hat{j} + \hat{k} \quad \vec{B} = 2\hat{i} + \hat{j} + \hat{k} \quad \vec{C} = \hat{i} + 2\hat{j} + \hat{k} \\
(\vec{A} \times \vec{B}) = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \end{bmatrix} = \hat{i} - \hat{k} \\
\vec{C} \cdot (\vec{A} \times \vec{B}) = (\vec{i} + 2\hat{j} + \hat{k}) \cdot (\hat{i} - \hat{k}) = 2 - 1 = 1 \\
\vec{C} \times \vec{A} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \end{bmatrix} = \hat{i} - \hat{k} \\
\vec{B} \cdot (\vec{C} \times \vec{A}) = (2\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} - \hat{k}) = 2 - 1 = 1
\[ \vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C} \]

**ALWAYS TRUE property of the vector triple product**

\[ \vec{A} = \vec{i} + \vec{j} + \vec{k} \quad \vec{B} = 2\vec{i} + \vec{j} + \vec{k} \quad \vec{C} = \vec{i} + 2\vec{j} + \vec{k} \]

\[ \vec{B} \times \vec{C} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -\vec{i} - \vec{j} + 3\vec{k} \]

\[ \vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ -1 & -1 & 3 \end{vmatrix} = 4\vec{i} - 4\vec{j} \]

(\vec{A} \cdot \vec{C}) = 1 + 2 + 1 = 4

(\vec{A} \cdot \vec{C})\vec{B} = 8\vec{i} + 4\vec{j} + 4\vec{k}

(\vec{A} \cdot \vec{B})\vec{C} = 4\vec{i} + 8\vec{j} + 4\vec{k}

(\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C} = 4\vec{i} - 4\vec{j}

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2. 3. 14

a) **unit normal vector to plane**

\[ \vec{n} = \vec{r}_B \times \vec{r}_{EC} = (-2\vec{i} + 4\vec{k}) \times (-2\vec{j} + 4\vec{k}) \]

\[ = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 4 \\ 0 & -2 & 4 \end{vmatrix} = 8\vec{i} + 8\vec{j} + 8\vec{k} \]

\[ \vec{n} = (2, 2, 1) \quad |\vec{n}| = \sqrt{9} \]

\[ \hat{n} = \frac{3\vec{i} + 2\vec{j} + \vec{k}}{\sqrt{9}} = \frac{1}{3} (2\vec{i} + 2\vec{j} + \vec{k}) \]
\[ \vec{r}_{DE} = \vec{r}_D - \vec{r}_E \]

b) \[ \vec{r}_{DE} = 3\hat{i} + 3\hat{j} + 3\hat{k} \]

(1) Distance from \(D\) to plane = \( \vec{r}_{DE} \cdot \hat{n} \)

\[ d = \vec{r}_{DE} \cdot \hat{n} = (-3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot \frac{1}{3} (2\hat{i} + 2\hat{j} + \hat{k}) \]

\[ d = \frac{1}{3} (-6 + 6 + 3) \]

\[ d = 1 \]

c) Point on the plane closest to point \(D = E\)

\[ \vec{r}_E = \vec{r}_D - \vec{r}_{DE} \] where \( \vec{r}_{DE} \) is equal to \( \hat{n} \) since the closest line would be on a line thru point \(D\) & perpendicular to plane

\[ \vec{r}_{DE} = \frac{1}{3} (2\hat{i} + 2\hat{j} + \hat{k}) \]

\[ \vec{r}_E = (0, 7, 4) - \frac{1}{3} (2, 2, 1) \]

\[ = (-\frac{2}{3}, \frac{1}{3}, \frac{11}{3}) = \frac{1}{3} (-2, 19, 11) \]

d) Point \(E = (-\frac{2}{3}, \frac{19}{3}, \frac{11}{3})\)

The triangular plane given does not contain any points in the negative \(x\) direction since the \(x\)-component of point \(E\) is negative, it cannot lie on the plane.
2.2.11 (a) \( \hat{A} + \hat{B} = \hat{B} + \hat{A} \) This is always true. Example: \( \hat{A} = (0, 17, -2) \) \( \hat{B} = (-1, 3, 2) \)
\( \hat{A} + \hat{B} = (-1, 20, 0) \) \( \hat{B} + \hat{A} = (-1, 20, 0) \) easily verified by using commutative property of real numbers on components.

(b) \( \hat{A} + \hat{b} = \hat{b} + \hat{A} \) Nonsensical. The concept of adding a vector to a scalar does not make sense.

(c) \( \hat{A} \cdot \hat{B} = \hat{B} \cdot \hat{A} \) Always true. Let \( \hat{A} = (a_x, a_y, a_z) \) \( \hat{B} = (b_x, b_y, b_z) \)
\( \hat{A} \cdot \hat{B} = a_x b_x + a_y b_y + a_z b_z = b_x a_x + b_y a_y + b_z a_z = \hat{B} \cdot \hat{A} \)

(d) \( \hat{B} / \hat{C} = \hat{B} / \hat{C} \) Nonsensical. There is no defined concept of division of vectors.

(e) \( \hat{b} / \hat{A} = \hat{b} / \hat{A} \) Nonsensical. You cannot divide a scalar by a vector.

(f) \( \hat{A} = (\hat{A} \cdot \hat{B}) \hat{B} + (\hat{A} \cdot \hat{C}) \hat{C} + (\hat{A} \cdot \hat{D}) \hat{D} \) This is at least sometimes true (for example it is true if \( \hat{A} \) is the zero vector

if \( \hat{A} = \hat{B} = \hat{C} = \hat{D} = (1, 1, 1) \)

then \( (\hat{A} \cdot \hat{B}) \hat{B} + (\hat{A} \cdot \hat{C}) \hat{C} + (\hat{A} \cdot \hat{D}) \hat{D} = (a, a, a) \neq (1, 1, 1) = \hat{A} \) so the statement is sometimes true possibly (f) is only true if \( \hat{B}, \hat{C}, \hat{D} \) are a set of orthonormal basis vectors.
\( \vec{r}_F = \vec{r}_D + \vec{r}_{C/D} + \vec{r}_{F/C} \)

where \( \vec{r}_D = 6k \)

\( \vec{r}_{C/D} \) is a position of \( C \) relative to \( D \)

\( = 6i \)

\( \vec{r}_{F/C} \) is a position of \( F \) relative to \( D \)

\( = 6j \)

Hence, \( \vec{r}_F = 6k + 6i + 6j \)

\( |\vec{r}_F| = |\vec{r}_F|^2 + |\vec{r}_F|^2 + |\vec{r}_F|^2 = \sqrt{6^2 + 6^2 + 6^2} = 6\sqrt{3} \)

3. We can find \( \vec{r}_G \) as a sum of \( \vec{r}_F \) and \( \vec{r}_{G/F} \).

Hence, \( \vec{r}_G = 6k + 6i + 6j + (-6k) = 6i + 6j \)

Note that \( \vec{r}_{G/F} \) represents a position of \( G \) relative to \( F \). Graphically, this makes sense because as we "move" from \( F \) to \( G \), we are only sliding down on the z-axis (6 units down).
\[ F = 1.027 \text{ ft} \]

\[ \text{Vertical:} \quad y = \sqrt{y_0^2 + (0.232)^2} \]

\[ r_{\text{ef}} = \frac{r_a - r_{\text{ef}}}{1.0} = 1.7327 + 0.232 \text{ ft} \]

\[ r_a = 1.7327 + 1.5 \text{ ft} \]

\[ \cos(\theta) = \frac{z}{y} \]

\[ \sin(\theta) = \frac{z}{y} \]

A (0, 1.5)

B (1, 1.7327)

\[ \frac{z}{y} = \frac{2}{y} \]

\[ \frac{2}{y} = \frac{2}{y} \]

\[ y = 2 \text{ ft} \]

\[ \text{Horizontal:} \quad x = 2 \text{ ft} \]
**SOLUTION 2.1.23**

**Known:** A structure containing a spring is given.

**Schematic & Given Data:**

- \( h = 1.5 \text{ ft} \)
- \( l = 2 \text{ ft} \)
- \( \theta_1 = 90^\circ \)
- \( \theta_2 = 60^\circ \)

**Force in the spring:**

\[
F = k\vec{r}_{AB}
\]

\[
k = 100 \text{ lb/ft}
\]

**Find:**

(a) A unit vector \( \hat{\lambda}_{AB} \) along \( AB \)

(b) calculate the spring force \( F = F\hat{\lambda}_{AB} \)

**Analysis:**

(a) \[
\vec{r}_A = 1.5 \cos(90^\circ)\hat{i} + 1.5 \sin(90^\circ)\hat{j} = 1.5\hat{j}
\]

(b) \[
\vec{r}_B = 2 \cos(60^\circ)\hat{i} + 2 \sin(60^\circ)\hat{j} = 1\hat{i} + \sqrt{3}\hat{j}
\]

\[
\vec{r}_A + \vec{r}_{AB} = \vec{r}_B \Rightarrow \vec{r}_{AB} = \vec{r}_B - \vec{r}_A
\]

\[
\vec{r}_{AB} = 1\hat{i} + \left( \frac{\sqrt{3} - 3}{2} \right)\hat{j}
\]

\[
|\vec{r}_{AB}| = \sqrt{1^2 + 0.23^2} = \sqrt{1.054} = 1.027
\]

\[
\hat{\lambda}_{AB} = \frac{1.027 (1 + 0.232\hat{j})}{|\vec{r}_{AB}|} = 0.974\hat{i} + 0.220\hat{j}
\]

(b) \[
F = k\vec{r}_{AB} = k|\vec{r}_{AB}|\hat{\lambda}_{AB} = F\hat{\lambda}_{AB}
\]

\[
F = (100 \text{ lb/ft})(1.027) = 102.7 \text{ lb/ft}
\]