Your TA, Section # and Section time:

"SOLUTIONS"

Your name:

RUINA

Cornell TAM 2020

Final Exam

No calculators, books or notes allowed.

December 10, 2010

5 Problems, 150 minutes (no overtime, as per university policy)

How to get the highest score?

Please, please, please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- □ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifiable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60 –100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.
  Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 10: ____ /25

Problem 11: ____ /25

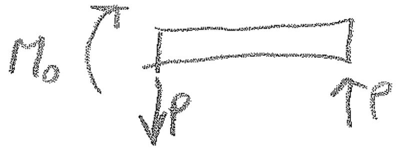
Problem 12: ____ /25

Problem 13: ____ /25

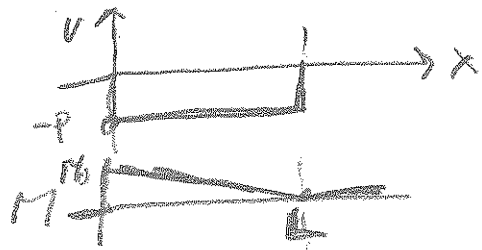
Problem 14: ____ /25

and force

10) A horizontal W12×96 I-beam, has flanges at top and bottom, and has length $L = 100$ in. The complete free body diagram of the beam shows a moment M_0 at one end and a vertical load of $P = 1000$ lbf at the other. Neglect the weight of the beam. Write a clear ordered sequence of formulas that would calculate the quantities in parts (a), (b) and (c) below. The right side of each formula can only have variables which have been on the left side of the formulas above it. For example, the first three formulas in this list are:



$$F = P = 1000\text{ lbf}$$
$$L = 100\text{ in}$$
$$M_0 = LP$$



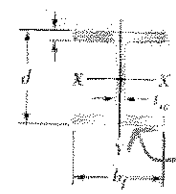
a) The maximum tension stress in the beam?

b) The maximum shear stress in the beam (on a surface whose normal is along the length of the beam)? Clearly explain any estimates you make to do this calculation. You need not repeat formulas used in part (a).

c) What is the weight of this beam? (Use the table below and or numbers you remember and do approximate arithmetic.) Full credit for answers within 50% of correct. Was it reasonable to neglect the weight of this beam?

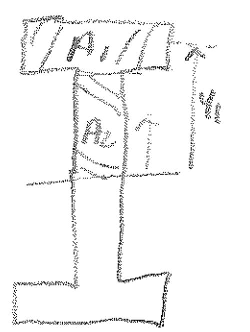
APPENDIX B Properties of Rolled-Steel Shapes
(U.S. Customary Units)
Continued from page 680

W Shapes
(Wide-Flange Shapes)



"96" means 96 lbf/ft.

Designation†	Area A, in²	Depth d, in.	Flange Width b_f, in.	Flange Thickness t_f, in.	Web Thickness t_w, in.	Axis X-X I_x, in⁴	Axis X-X S_x, in³	Axis X-X r_x, in.	Axis Y-Y I_y, in⁴	Axis Y-Y S_y, in³	Axis Y-Y r_y, in.
W12 × 96	28.2	12.7	12.2	0.900	0.550	833	131	5.44	270	44.4	3.09
72	21.1	12.3	12.0	0.670	0.430	597	97.4	5.31	195	32.4	3.04
50	14.6	12.2	8.08	0.640	0.370	391	64.2	5.18	56.3	13.9	1.96
40	11.7	11.9	8.01	0.515	0.295	307	51.5	5.13	44.1	11.0	1.94
35	10.3	12.5	6.56	0.520	0.300	285	45.6	5.25	24.5	7.47	1.54
30	8.79	12.3	6.52	0.440	0.260	238	38.6	5.21	20.3	6.24	1.52
26	7.65	12.2	6.49	0.380	0.230	204	33.4	5.17	17.3	5.34	1.51
22	6.48	12.3	4.03	0.425	0.260	156	25.4	4.91	4.66	2.31	0.848
16	4.71	12.0	3.99	0.265	0.220	103	17.1	4.67	2.82	1.41	0.773
W10 × 112	32.9	11.4	10.4	1.25	0.755	716	126	4.66	236	45.3	2.68
68	20.0	10.4	10.1	0.770	0.470	394	75.7	4.44	134	26.4	2.59
54	15.8	10.1	10.0	0.615	0.370	303	60.0	4.37	103	20.6	2.56
45	13.3	10.1	8.02	0.620	0.350	248	49.1	4.32	53.4	13.3	2.01
39	11.5	9.92	7.99	0.530	0.315	209	42.1	4.27	45.0	11.3	1.98
33	9.71	9.73	7.96	0.435	0.290	171	35.0	4.19	36.6	9.20	1.94
30	8.84	10.5	5.81	0.510	0.300	170	32.4	4.38	16.7	5.75	1.37



$P = 1000$ lbf
$L = 100$ in
$M_0 = LP$
$I = 833\text{ in}^4$
$d = 12.7$ in
$t_w = 0.55$ in
$t_f = 0.9$ in
$b_f = 12.2$ in

$$\sigma = -M_0 y / I$$

$$\boxed{\sigma_{max} = \frac{M_0 (d/2)}{I}}$$

$$\tau = \frac{VQ}{It}$$
$$A_1 = t_f b_f$$
$$A_2 = t_w \left(\frac{d}{2} - t_f\right)$$
$$y_1 = d - t_f/2$$

$$y_2 = (d/2 - t_f)/2$$
$$Q = y_1 A_1 + y_2 A_2$$
$$V = P$$
$$\boxed{\tau_{max} = \frac{VQ}{I t_w}}$$ (b)

Not (c) reasonable to neglect W! 800 lbf ≈ 1000 lbf

$W = 96\ \text{lbf/ft} \cdot 100\text{ in} \cdot \frac{1\text{ ft}}{12\text{ in}} = \boxed{800\text{ lbf}}$

OR steel has 8× density of H_2O ← water

$W = A \cdot L \cdot \gamma = 28\text{ in}^2 \cdot 100\text{ in} \cdot 8 \cdot 64\text{ lbf/ft}^3\ (1\text{ ft}/12\text{ in})^3$

$\approx \frac{2800 \cdot 8 \cdot 64}{12 \cdot 12 \cdot 12}\text{ lbf} \approx \boxed{800\text{ lbf}}$

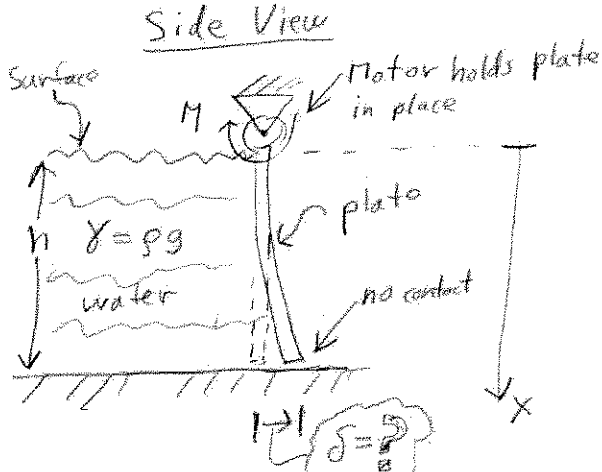
11) Water is held back by the rotary dam shown. The whole dam is a big plate that is held at the top. Assume the following quantities are given:

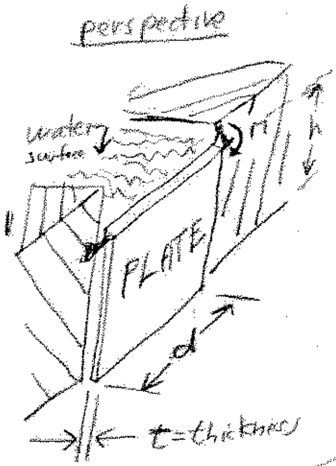
E, G, ν Elastic moduli of plate

$\cancel{w}\, d, t, h$ width, thickness and height of plate (h is also the depth of the water)

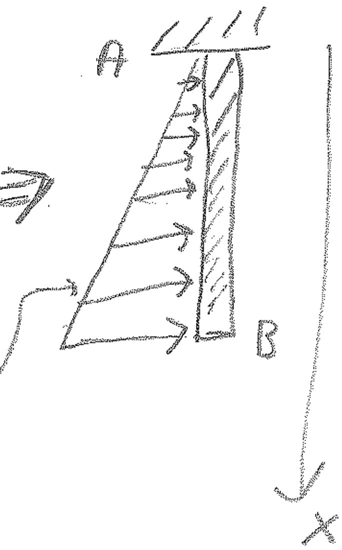
$\gamma = \rho g$ density of water (weight per unit volume)

Find the deflection δ of the bottom of the plate due to the water pressure. Answer in terms of the given quantities (Hint, first find M).



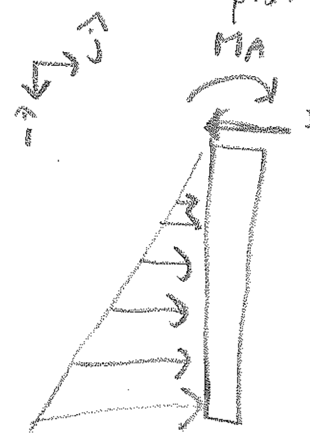


Find $w(x)$:



$w(x) = p \cdot d = \underbrace{\gamma x}_{\rho g h} d$

plate material: E, G, ν



$$\sum F_y = 0 \Rightarrow -F_A + \int_0^h w(x)\,dx = 0 \Rightarrow F_A = \gamma d h^2/2$$

$$-F_A + \int_0^h \gamma d x\, dx = 0$$

$$+\circlearrowleft \sum M_A = 0 \Rightarrow -M_A + \underbrace{\int_0^h (\gamma d x)\, x\, dx}_{\gamma d h^3/3} = 0 \Rightarrow M_A = \gamma d h^3/3$$

$$\boxed{I = dt^3/12}$$

$$w(x) = -d\gamma x$$

$$V = -\int w\,dx = +d\gamma x^2/2 \overbrace{- \gamma d h^2/2}^{V_A} = \frac{\gamma d}{2}(x^2 - h^2)$$

$$M = \int V\,dx = \frac{\gamma d}{2}\left[x^3/3 - h^2 x\right] + \underbrace{\gamma d h^3/3}_{M_A}$$

$$\underbrace{\frac{M}{EI}}_{u''} \int \Rightarrow u' = \frac{1}{EI}\left[\frac{\gamma d}{2}\left[x^4/12 - h^2 x^2/2\right] + \gamma d h^3 x/3\right] + \underbrace{0}_{u'(0)=0}$$

$$\int \Rightarrow u = \frac{1}{EI}\left[\frac{\gamma d}{2}\left[\frac{x^5}{60} - h^2 x^3/6\right] + \gamma d h^3 x^2/6\right] + 0$$

$$\delta = u(h) = \frac{\gamma d}{E\, d t^3/12}\, h^5\left[\frac{1}{2}\left[\frac{1}{60} - \frac{10}{60}\right] + \frac{20}{120}\right] = \boxed{\frac{\gamma h^5}{E t^3}\,\frac{11}{10}}$$

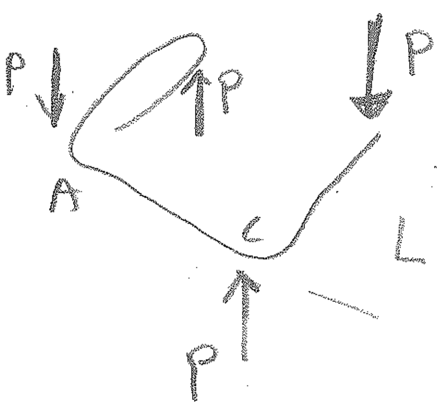
12) A standard paperclip is partially unfolded and held at the edge of a rigid table as shown. The folded part is held firmly on the table at A and B. The part overhanging has force P applied at D. Neglect the bending of segment CD (treat it as rigid). Consider only the torsion of the segment AC. In terms of some or all of

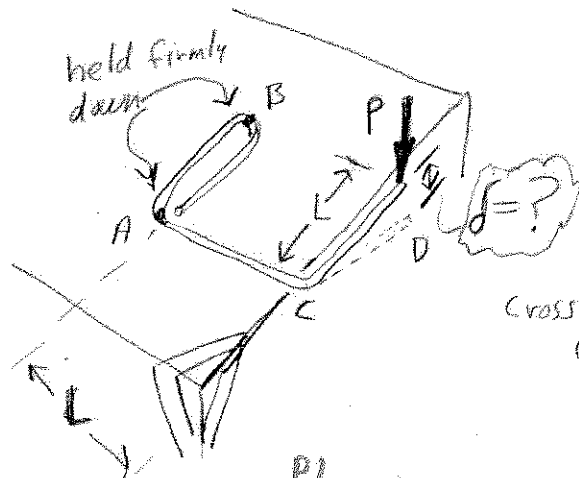
E, G, ν Elastic moduli of paper clip
L length of torsion segment AC and of rigid segment CD
R radius of paperclip
P applied force

a) find the deflection δ.

b) What is the Young's modulus E of the paper clip? A numerical answer is desired. Answer in any units you like. Full credit if answer is within 20% of correct.

FBD

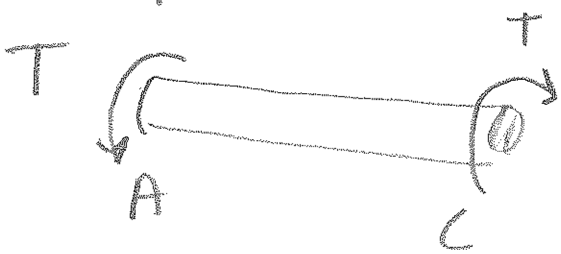




Cross section: ⊘ $2R$

pure torsion

$T = \text{torque} = PL$



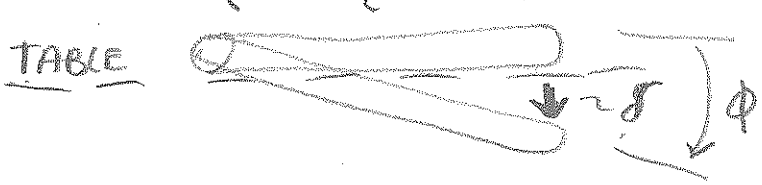
$$\phi = \frac{TL}{JG} = \frac{2PL^2}{\pi R^4 G}$$

$T = PL$, $J = \pi R^4/2$

$$\delta = \phi L = \frac{2PL^2}{\pi R^4 G} L$$

$$\boxed{\delta = \frac{2PL^3}{\pi R^4 G}} \quad (a)$$

END VIEW



$\delta = \phi L$

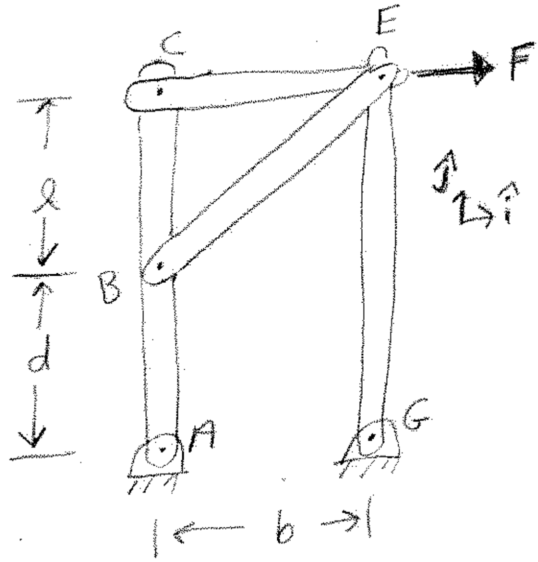
Paper clips usually made of steel:

$E_s = 2 \cdot 10^{11}\ N/m^2$ (b)
$= 2 \cdot 10^{11}\ Pa$
$= 200\ GPa$
$= 200{,}000\ N/mm^2$

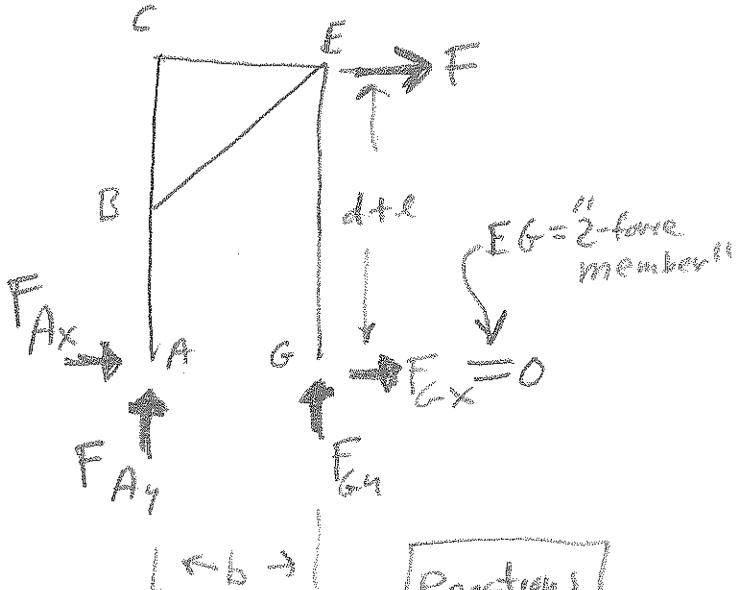
$\boxed{E_s = 20\ \text{tons}/mm^2}$
$\approx 15000\ \text{tons}/in^2$

$E_s = 30 \cdot 10^6\ PSI$
$= 2 \cdot 10^6\ atm$
$= 2 \cdot 10^6\ bars$
$= 2 \cdot 10^6 \cdot 10^5\ N/m^2$

13) Four rigid bars (EG, EC, BE and ABC) are connected with pins as shown with the force F applied to the end of bar CE at E. Find the force of bar CE on bar ABC at C. Answer in terms of some or all of $F, b, d, \ell, \hat{\imath}$ and $\hat{\jmath}$.



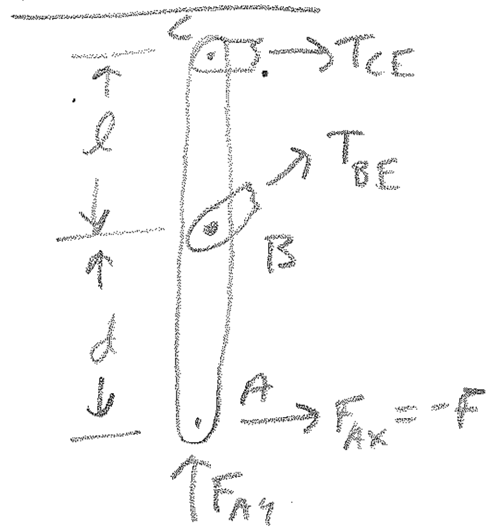
FBD of Structure



Reactions

$$\Rightarrow \boxed{\begin{aligned} F_{Ay} &= \frac{-(d+\ell)}{b} F \\ F_{Gy} &= +\left(\frac{d+\ell}{b}\right) F \\ F_{Ax} &= -F \end{aligned}}$$

FBD of ABC



$$+\circlearrowleft \sum M_{/B} = 0$$

$$\Rightarrow \quad F_{Ax} \cdot d - T_{CE} \cdot \ell = 0$$

$$T_{CE} = \frac{d}{\ell} F_{Ax} = \frac{-d}{\ell} F$$

Force of CE on ABC $= T_{CE}\,\hat{\imath} = \boxed{\frac{-d}{\ell} F \hat{\imath}}$

14) A uniform solid rigid cube with sides $2a$ and mass m has edges parallel to the xyz axis. The cube is held in place by 6 rods each with length ℓ. Each rod is parallel to one of the axes. Find the tensions in the 6 rods in terms of some or all of m, g, a and ℓ. [Hint, at least one tension is zero.]

Answer here:

$T_{KJ} = 0$

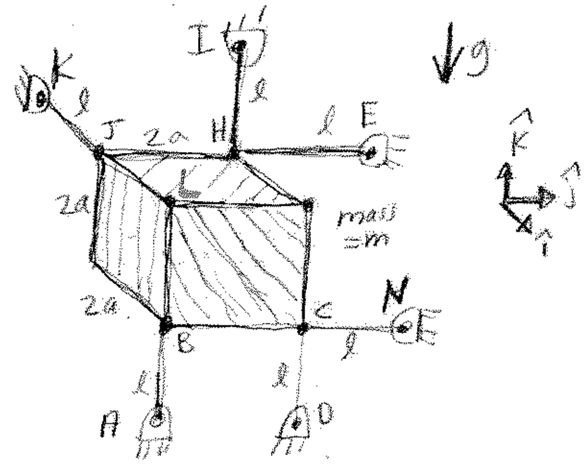
$T_{HI} = mg/2$

$T_{EH} = 0$

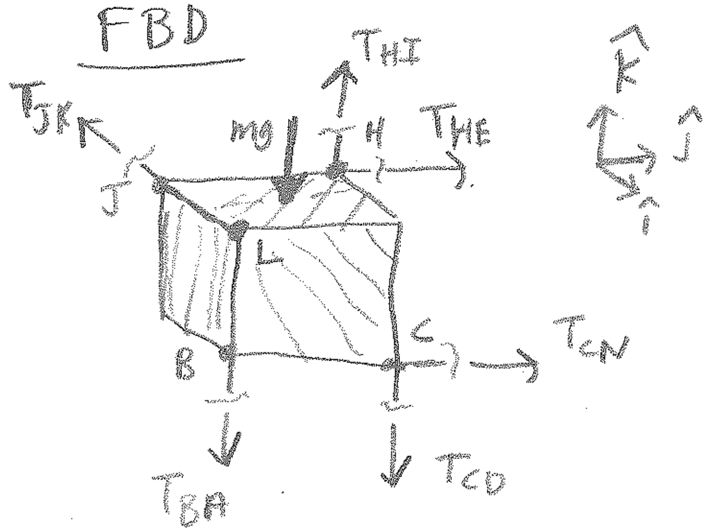
$T_{CN} = 0$

$T_{CD} = 0$

$T_{AB} = -mg/2$



FBD



QUICK SOLN:

$T_{HI} = T_{BA} = mg/2$ would hold up mass

$\Rightarrow$ all others $\boxed{0 = T}$

& prob. is stat. det., unique soln.

SLOW SOLN

$\sum F_x = 0 \Rightarrow \boxed{T_{JK} = 0}$

$\sum M_{HZ} = 0 \Rightarrow \boxed{T_{CN} = 0}$

$\sum M_{Cz} = 0 \Rightarrow \boxed{T_{HE} = 0}$

$\sum M_{HL} = 0 \Rightarrow \boxed{T_{CD} = 0}$

$\sum M_{CB} = 0 \Rightarrow mga = T_{HI} \cdot 2a$

$\Rightarrow \boxed{T_{HI} = mg/2}$

$\sum F_z = 0 \Rightarrow \boxed{T_{BA} = -mg/2}$