TAM 7960
Legged Locomotion of Robots and Animals
(Terrestrial Locomotion)
Spring 2010

Updated March 5, 2010

Homework

Each homework assignment will start on a fresh page. This document will grow in length as the semester progresses.

HW due Tuesday Feb 2

1. A natural measure of energy use is ”The specific energetic cost of transport”,

$$c_t \equiv \frac{\text{Energy used}}{\text{weight} * \text{distance}}.$$

   Note that weight is a force (mass times g) so c_t is a dimensionless number. Estimate, using whatever numbers you know or look up, the c_t for at least 3 of these things.

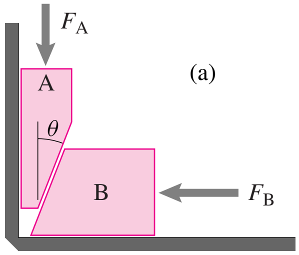
   - a person walking
   - a person biking
   - a car (full weight)
   - a car (only counting weight of passengers)
   - a freight truck
   - a freight train
   - a passenger train (only counting weight of passengers)
   - some other animal of your choice, walking, running or flying.

   Make your assumptions clear and clearly identify (so I can check) any sources you use. Best if you have two different estimates, based on different types of data.

2. What would be a good measure of locomotion speed for comparing different animals or robots? Justify your answer as best you can. If you have more than one candidate, compare them. I am not looking for a quote from some paper or other, but your own thoughts.

HW due Thursday Feb 11

1. An ideal passive transmission conserves energy. One example of a transmission is a screw-drive or ‘worm drive’ (read about such in, say, wikipedia). A model for the basic mechanics of a worm drive (left) is the wedge (right).



   (a) For simplicity, assume that all sliding is frictionless and that the parts have no mass or weight. Think of A as the input and B as the output.

      i. Find F_B in terms of θ and F_A.

      ii. From geometry (kinematics), find the sideways motion of B in terms of θ and the motion of A.

      iii. Note that the results above are consistent with energy conservation.

      iv. Simplify all of the expressions above using a small angle approximation ($\theta \ll 1, \sin\theta \approx \theta$, *etc.*

   (b) Now include friction, but just on the surface AB. Assume a friction coefficient μ or friction angle ϕ with $\tan\phi = \mu$. Assume that the friction against the walls is still zero. For the worm drive this is like assuming that the shafts have good bearings but that all the friction is on the screw surface. Assume the wedge is going down.

      i. Find F_B in terms of θ, F_A and μ (or ϕ).

      ii. Note that the kinematics is unchanged by the friction.

      iii. What fraction of the power is lost?

      iv. Now assume the drive is running backwards, with the wedge A being pushed up. Now what fraction of the power is lost?

      v. Assume small angles. Show that if μ (or ϕ) and θ are such that 50% of the power is lost when A is pushed down that 100% is lost when B is pushed in.

2. The most common idealization of a motor includes two dissipative terms, $c\omega$ and IR:

$$T_m = kI_m - c\omega \qquad \text{Torque equation} \qquad (1)$$

$$V_m = k\omega_m + I_m R + \dot{I}_m L \qquad \text{Voltage equation} \qquad (2)$$

In these equations

T_m is the torque the motor causes on what its connected to,
ω is the motor angular velocity,
I_m is the electric current through the motor,
V_m is the voltage across the motor. The motor is characterized by two numbers:
k is the 'motor constant' or 'torque constant',
R is the internal electrical resistance of the motor, and
c is the viscous damping term, which also includes electrodynamic drag.
L is the motor inductance which is not important for steady steady state operation with a DC power supply.

From these we calculate all manner of quantities, for example the electric power in and mechanical power out are

$$P_i = V_m I_m \quad \text{and} \quad P_o = T_m \omega_m$$

Now imagine that we also have access to a frictionless gear box that changes the torque and angular velocity out to

$$T_o = GT_m \quad \text{and} \quad \omega_o = \omega_m / G$$

and also a lossless transformer that can 'transform' the battery voltage.

$$V_m = DV_{in} \quad \text{and} \quad I_m = I_{in}/D.$$

Assume you are stuck with a motor with given k, c, and R. You are also stuck with a given battery (V_{in}) and output angular velocity (ω_o). Assume that the motor must deliver a specified power P_o.

(a) Find G and D to maximize the efficiency of the system.

(b) What is the efficiency?

(c) Do the calculations above for a real motor. The Faulhaber 4490-024B has a nominal voltage of 24V. This means you should use 24 V to infer the other motor constants. The motor resistance is $R = 0.237\Omega$. The 'no load speed is 9550 rpm'; this means that at 24V that $\omega = 9550 * 2\pi/(60\,\text{s})$ when $T_m = 0$. The stall torque is $2.406\,\text{N·m}$. The no-load current is 0.554 Amps (this is the current when the 24V is applied but no torque). The torque constant is $k = 0.02383\,\text{N·m/Amp}$.

   i. Find the damping constant c.

   ii. Assume that a mechanical power of $P_o = 20\,\text{W}$ is desired at $\omega_o = 1\,\text{rad/s}$ using a 24 Volt battery. Find the best gear ratio and transformer ratio to minimize the power in. What is that power?

HW due Tuesday Feb 16, 2010

1. **Reading.** Get a copy of "The collisional cost ..." from the course web page. Read it. By Feb 23 you should understand the whole paper well. You can also look at the Rashevsky paper.

2. **The cost of frog jumping.** A frog makes a sequence of parabolic jumps, stopping and jumping again each time. Neglect the waiting time between jumps. Here are some possibly relevant variables.

$$\begin{aligned}
v_{xo} &= \text{ the } x \text{ component of jump velocity} = v_o \cos\phi \\
v_{yo} &= \text{ the } y \text{ component of jump velocity} = v_o \sin\phi \\
v_o &= \text{ the launch speed } = \sqrt{v_{xo}^2 + v_{xo}^2} \\
\phi &= \text{ the jump angle, measured from the positive } x \text{ axis} \\
v &= \text{ the average forward speed} = v_x = d/t_f \\
d &= \text{ the step length (flight distance)} \\
h &= \text{ the height of flight} \\
t_f &= \text{ the time of flight} \\
g &= \text{ the downwards gravity constant} \\
m &= \text{ the mass of the frog} \\
b &= \text{ the metabolic cost multiplyer: } E_m = b|W_{neg}|
\end{aligned}$$

   (a) In terms of some of the variables above determine the specific cost of transport.

$$c_t = \frac{\text{energy cost}}{\text{weight} \times \text{distance}}$$

   Express this c_t at least two different ways, using two different minimal sets of variables.

   (b) For a given m, g, d find other variables to minimize c_t. Find the resulting v and c_t in terms of m, g, d. Plug in numbers for $g = 10\,\text{m/s}^2$ and $d = 1$ m.

   (c) For a given m, g, v find other variables to minimize c_t.

   (d) Make any interesting observations you can about the calculations above.

HW due Thursday Feb 25, 2010

1. Equations 15 and 16 in the paper "A collisional model..." (available on course www site) are based on a single metabolic inefficiency b. Rederive equations 15 and 16 using two cost coefficients b_1 and b_2 for negative and positive work. Thus your final formulas will be more complicated.

2. Use two cost coefficients b_1 and b_2 to replace the formulas for curves i and ii in Figure 5 of the "Collisional model" paper. You need not use the formulas from problem (1) above, but can rederive them as needed for this special case.

3. On the course motor page there is a design sheet by David Palombo of Aveox (seller of motors). He says "The motor is at it's peak efficiency when the iron loss equals the copper loss." The 'copper loss' is $I^2R/2$. The iron loss is, presumably, $c\omega^2/2$.

   (a) Using specific simple motor parameters (say $k = 10, R = 1, c = 1$ in consistent units) check this result. Part of your solution should be a set of multiple-line or vertically stacked plots that show, for a fixed V (say 10), all of the following as functions of ω: P_{in}, P_{out}, $I^2R/2$, $c\omega^2/2$. A vertical line should mark the point of maximum efficiency. You can use Matlab code, computer algebra or whatever.

   (b) Inspired by the numerical example above, see if you can definitively answer these questions. Is the result exactly true for our standard motor model? Is it approximately true? Is it true in some limiting cases?

HW due Thursday March 3, 2010

This whole assignment is about the rimless wheel. The basic parameters are here. Where numerical values are needed, assume consistent units and use the numbers in parentheses.

$m =$ Mass of rimless wheel (1).

$I = I^G =$ Moment of inertia about the center of the wheel, G (1/2).

$N =$ Number of spokes (8).

$\phi =$ Angle between spokes = $2\pi/N$.

$\ell =$ Length of spokes (1).

$\gamma =$ Slope of ground, down is to the left (.1 radians).

$g =$ Gravity constant (1).

The basic variables are:

$\theta_n =$ Angle of stance leg n relative to a line normal to the ground, measured CCW.

$\omega = \dot{\theta} =$ Angular velocity of wheel, measured CCW.

$(...)_n^+ = (...)$ just after collision n.

$(...)_n^- = (...)$ just before collision n.

$f(\omega_n^+) = \omega_{n+1}^+$ = Poincare map/Return map/Stride function.

$(...)_*^- =$ Steady state value of (...) if the motion is periodic.

Please answer these questions.

1. Use energy conservation to find ω_{n+1}^- from ω_n^+
2. Use numerical integration of the non-linear ODEs, with the appropriate end condition, to verify your answer to the previous problem for a range of initial conditions.
3. Plot f along with the line $y = x$ using the same scale for both axes. Use your numerical evaluation for the motion between collisions, not the energy conservation result above.
4. Find ω^{+*} to at least 10 digits accuracy by root finding using your numerical evaluation of f. Find it again analytically using the energy conservation result. Compare the solutions.
5. Start with the initial condition that $\omega_0^+ = 1.5\omega^{+*}$. Calculate ω_n^+ for $n = 1...20$.
6. Define $r_n = \frac{\omega_n^+ - \omega^{+*}}{\omega_{n-1}^+ - \omega^{+*}}$. Plot r_n vs n and show it tends to a constant. Find that constant as best you can.
7. Find an analytic formula for the limit $n \to \infty$ of r_n, evaluate the result with the given numbers, and compare it to your numerical result above.

HW due Thursday March 10, 2010

This whole assignment is about Spring mass running, as described in lecture. Here are the model parameters. In parentheses are values you should use, in consistent units, for numerical work.

m = Mass of running human (100).

θ_c = Touchdown angle (variable, you choose).

ℓ_0 = Uncompressed leg length (1).

k = Spring constant (10,000, compression under gravity is 1/10 of leg length).

g = Gravity constant (10).

The basic dynamic variables, and things you can measure, are:

$x, y, \dot{x}, \dot{y}$ = Position and velocity of COM

d = Step length

$\bar{v}$ Average forward speed

h = Height at top of periodic trajectory ($= y_{max}$)

c_{duty} Fraction of time the foot is on the ground

So everyone is on the same page, everyone should use section 3, the peak of the flight. Thus the section variables are h and $\dot{x}$ with $\dot{y} = 0$.

1. Find a periodic motion using the computer recipe. Prize for first one to find, send it out to the class (that is, send h and $\dot{x}$ on the section).
2. Find a stable periodic motion. Prize for first one to find, send it out to the class. Send key information.
3. Graph your trajectories and animate your trajectories, showing the stance leg during stance. You might find that graphing and animation are useful debugging tools, even before you have solutions to the problems above.