Your TA, Section # and Section time:

"Solutions"

Your name:

RUINA

Cornell TAM/ENGRD 2030

Prelim 3

April 16, 2013

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- ☐ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** The last page is blank for your use. Ask for more extra paper if you need it. Put your name on each extra sheet, fold it in, and refer to back pages or extra sheets on the page of the relevant problem.

Problem 7: ____ /25

Problem 8: ____ /25

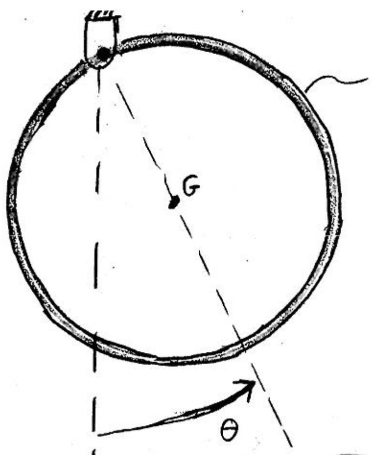
Problem 9: ____ /25

1) A circular hoop swings as a pendulum from a hinge at a point on its edge. Answer in terms of some or all of m, R and g.

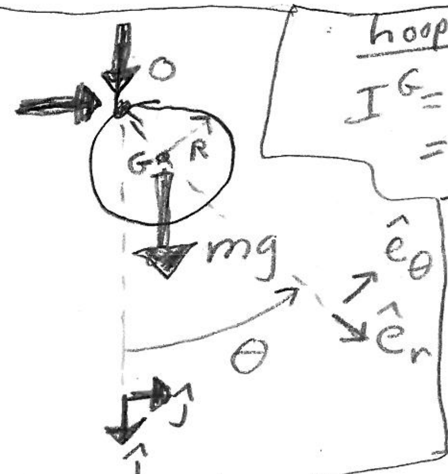
a) What is the period of small oscillation near to hanging straight down?

b) If it was launched from the straight down position, what is the minimum launch speed needed by the center of mass in order to swing the hoop over the top?

c) In terms of some or all of m, R and g, what is the length ℓ of a simple pendulum (ie, point mass m and string with the same gravity g) that has the same period of small oscillation as does this hoop?



FBD



hoop

$I^G = \int r^2\,dm$
$= mR^2$

AMB$_{/O}$: $\sum \vec{M}_{/O} = \dot{\vec{H}}_{/O}$

$$\left\{-mg\sin\theta R\hat{k} = R\hat{e}_r \times m\vec{a}_G + I^G\ddot{\theta}\hat{k}\right\}$$

$$\vec{a}_G = R\ddot{\theta}\hat{e}_\theta - R\dot{\theta}^2\hat{e}_r$$

$$\{\ \}\cdot\hat{k} \Rightarrow -mgR\sin\theta = (I^G + R^2m)\ddot{\theta}$$

$$-\cancel{m}g\cancel{R}\sin\theta = 2\cancel{m}R^{\cancel{2}}\ddot{\theta}$$

$$\ddot{\theta} = \frac{-g}{2R}\theta \quad (*)\ \text{(ODE)}$$

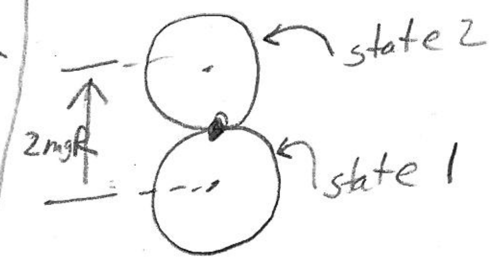
$\theta \ll 1 \Rightarrow$

$\theta =$, e.g. $\quad \theta = \theta_0 \cos\sqrt{\frac{g}{2R}}\,t \quad$ (one soln)

one period $\equiv t^*$: $\quad \sqrt{\frac{g}{2R}}\,t^* = 2\pi$

(a) $\boxed{t^* = 2\pi\sqrt{2R/g}}$

Energy



$$E_{k_1} + E_{p_1} = E_{k_2} + E_{p_2}$$

$$\Delta E_k = -\Delta E_p$$

$$-\frac{mv_1^2}{2} - \frac{I^G\omega_1^2}{2} = -2mgR \qquad \omega_1 = v_1/R$$

$$-\frac{v_1^2}{2}\left(m + \frac{I^G}{R^2}\right) = -2mgR$$

$$\frac{\cancel{m}v_1^2}{\cancel{2}}(\cancel{2}\cancel{m}) = 2\cancel{m}gR$$

$$v_1^2 = 2gR$$

$\boxed{v_1 = +\sqrt{2gR}}$ (b)

Simple pendulum:

$$\ddot{\theta}_p = \frac{-g}{L}\theta_p$$

as $*$ above

to have same soln, $\frac{g}{L} = \frac{g}{2R}$

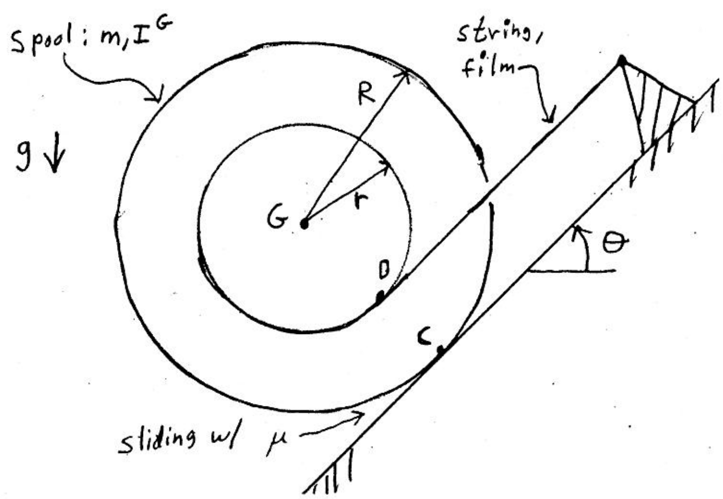
$\Rightarrow \boxed{L = 2R}$ (c)

2) A spool, like the movie spool in lecture, is progressing down a slope. The inextensible film is held firmly at one end and unwinds from the spool. The friction between the spool and the ground is low enough so that the spool slides on the surface. You are given the spool outer radius R, the film radius r, the spool inertia about its COM I^G, the spool mass m, the slope θ, the friction coefficient μ, the gravity constant g and the present speed v_G of the spool down the slope.

Hint: it might help you to picture the motion by first imagining that there is no friction. Then put in the friction.

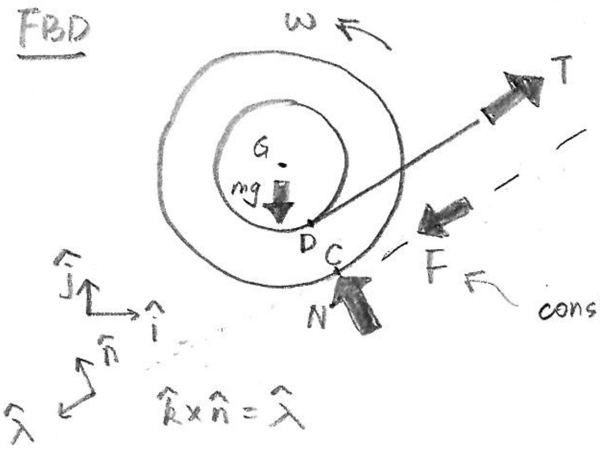
a) Find the normal component of the force of the ground on the spool.

b) Find any one of these quantities: The acceleration a_G of the spool center down the slope, the angular acceleration α of the spool, or the tension in the film/string. If you find more than one quantity, *clearly label the one you want graded.*



2) Solution:

FBD



consistant with $V_G > 0$ & $V_C < 0$

$\uparrow$ bottom of spool is sliding up hill

$\hat{k} \times \hat{n} = \hat{\lambda}$

Geometry

$\vec{V}_G = V_G \hat{\lambda}, \quad \vec{a}_G = a_G \hat{\lambda}$

$\vec{0} = \vec{V}_D = \vec{V}_G + \vec{V}_{D/G} = \vec{V}_G + \omega \hat{k} \times (-r\hat{n})$

$= (V_G - \omega r)\hat{\lambda}$

$\{ \vec{0} = (V_G - \omega r)\hat{\lambda} \}$

$\{ \} \cdot \hat{\lambda} \Rightarrow \boxed{V_G = \omega r}$

$\boxed{a_G = \dot{\omega} r} \quad (*)$

$\vec{V}_C = \vec{V}_D + \vec{V}_{C/D}$

$= \vec{0} + \omega \hat{k} \times [(R-r)(-\hat{n})]$

$\vec{V}_C = -\omega(R-r)\hat{\lambda}$

$\quad \llcorner\ \omega = V_G / r$

if $V_G > 0 \Rightarrow V_C < 0$

$\Rightarrow$ slides uphill

$\Rightarrow$ friction points down

LMB $\cdot \hat{n} \Rightarrow N - mg\cos\theta = 0$

$\Rightarrow \boxed{N = mg\cos\theta} \quad (a)$

LMB $\cdot \hat{\lambda} \Rightarrow F - T + mg\sin\theta = m a_G \quad (**)$

$\quad \llcorner\ F = \mu N = \mu mg\cos\theta$

AMB/D: $\sum \vec{M}_{/D} = \dot{\vec{H}}_{/D}$

$\{ [mgr\sin\theta - (R-r)F]\hat{k} = \vec{r}_{G/D} \times m\vec{a}_G + I^G \dot{\omega}\hat{k}$

$= (mr a_G + I^G \dot{\omega})\hat{k}$

$= (mr a_G + \frac{I^G}{r} a_G)\hat{k} \}$

$\{ \} \cdot \hat{k} \Rightarrow mgr\sin\theta - (R-r)F = (mr + \frac{I^G}{r}) a_G$

$\Rightarrow \boxed{a_G = g \dfrac{\sin\theta - \mu\cos\theta (R-r)/r}{I^G/mr^2 + 1}} \quad (b)$

$* \Rightarrow \boxed{\dot{\omega} = \dfrac{a_G}{r} = \dfrac{g}{r} \dfrac{\sin\theta - \mu\cos\theta (R-r)/r}{I^G/mr^2 + 1}} \quad (b)$

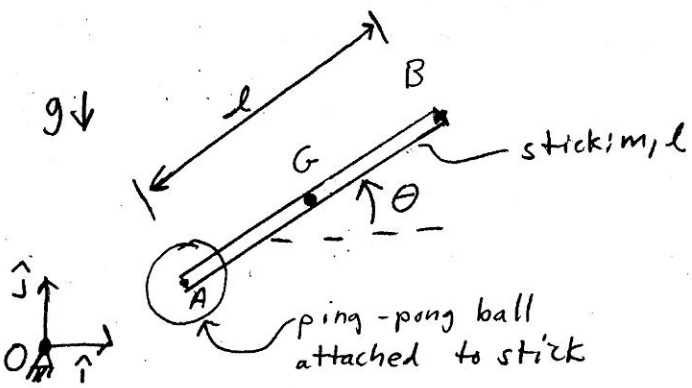
$** \Rightarrow T = F + mg\sin\theta - m a_G$

$\boxed{T = mg\left[\mu\cos\theta + \sin\theta - \dfrac{\sin\theta - \mu\cos\theta (R-r)/r}{I^G/mr^2 + 1}\right]} \quad (b)$

3) Equations of motion for a simple model of an arrow. 2D. A uniform rigid stick (length ℓ, mass m) flies through the air. As it flies, the line segment AB makes an angle $\theta(t)$ with the positive horizontal x axis. Attached to one end, labeled A, is a negligible-mass ping-pong ball that has air friction. The friction is modeled as linear: the ping-pong ball drag-force resists motion of point A with magnitude $F_D = cv_A$. The air friction on the rest of the stick is negligible. The center of mass G is at the center of the stick. You are given $\theta, \dot{\theta}, \vec{r}_G, \vec{v}_G, m, \ell, c, d$ and g.

a) Find $\ddot{\theta}$ and $\vec{a}_G$ (a vector expression for $\vec{a}_G$ without explicit components is fine, but scalar components are also fine). You are given $\theta, \dot{\theta}, \vec{r}_G, \vec{v}_G, m, \ell, c$ and g.

b) Harder. If, instead of a ping-pong ball at A there were feathers. And, more like a real arrow, these feathers had no resistance to the motion of A in the AB direction. But the feathers resisted motion perpendicular to AB with a force $F_L^{\perp} = dv^{\perp}$, where $v^{\perp}$ is the component of $\vec{v}_A$ orthogonal to the line AB[1]. You are given $\theta, \dot{\theta}, \vec{r}_G, \vec{v}_G, m, \ell, d$ and g. Find $\ddot{\theta}$.



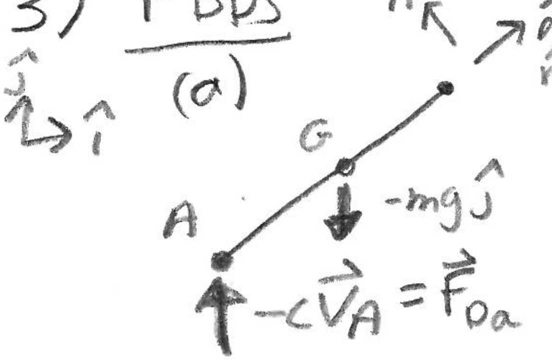
[1]Aside: A more realistic model of an arrow would have that 'lift' force depend quadratically on the forward speed. More realistic still would be to have a lift and drag model for the feathers, this would be a model more like that used for an airplane wing.

3) <u>FBDs</u>

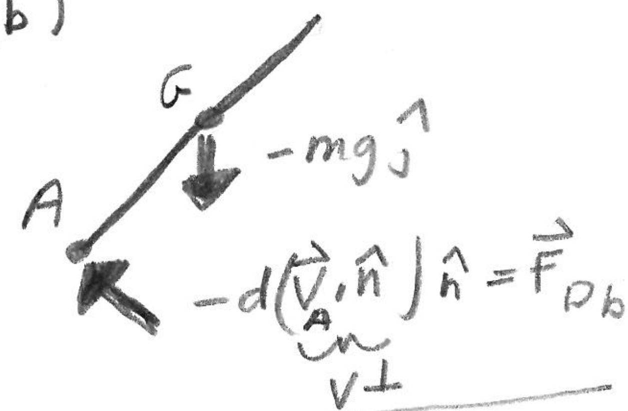
$\hat{\lambda} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\hat{n} = \hat{k} \times \hat{\lambda}$

(a)



(b)



<u>Kinematics</u>

$$\vec{V}_A = \underbrace{\vec{V}_G}_{\text{given}} + \underbrace{\vec{\omega}}_{\dot{\theta}\hat{k}} \times \underbrace{\vec{r}_{A/G}}_{-\hat{\lambda}\,\ell/2}$$

$$I^G = \int r^2\,dm = \int_{-\ell/2}^{\ell/2} s^2 \underbrace{\rho}_{m/\ell}\,ds$$

$$I^G = m\ell^2/12$$

<u>LMB</u>!

$$\sum \vec{F} = m\vec{a}_G$$

$$-mg\hat{\jmath} - c\vec{V}_A = m\vec{a}_G$$

$$\boxed{\vec{a}_G = \frac{-c}{m}\vec{V}_A - g\hat{\jmath}}$$

(b) <u>AMB</u>$_{/G}$

$$\sum \vec{M}_{/G} = \dot{\vec{H}}_{/G}$$

$$\vec{r}_{A/G} \times \vec{F}_{D_b} = I^G \ddot{\theta}\hat{k}$$

$$\boxed{\ddot{\theta} = \frac{\vec{r}_{A/G} \times \vec{F}_{D_b} \cdot \hat{k}}{I^G}} \quad (b)$$

where all quantities are calculated above in terms of given quantities

<u>AMB</u>$_{/G}$ $\quad \sum \vec{M}_{/G} = \dot{\vec{H}}_{/G}$

$$\vec{r}_{A/G} \times \vec{F}_{D_a} = \underline{I}^G \ddot{\theta}\hat{k}$$

see above

$$\boxed{\ddot{\theta} = \frac{\vec{r}_{A/G} \times \vec{F}_{D_a}}{I^G} \cdot \hat{k}}$$

see above

see above