Your TA, Section # and Section time:

"Solutions"

Your name:

Matthew Kelly

Cornell TAM/ENGRD 2030

Prelim 2

March 26, 2013

No calculators, books or notes allowed. This version slightly improves the given version.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- □ TIDILY REDUCE and box in your answers (Don't leave simplifyable algebraic expressions).
- \>> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions $(\ell, h, d, \ldots)$, coordinates $(x, y, r, \theta \ldots)$, variables $(v, m, t, \ldots)$, base vectors $(\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots)$ and signs $(\pm)$ with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60 – 100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** The last page is blank for your use. Ask for more extra paper if you need it. Put your name on each extra sheet, fold it in, and refer to back pages or extra sheets on the page of the relevant problem.

Problem 4: ____ /25

Problem 5: ____ /25

Problem 6: ____ /25

1) Pulleys. One dimensional mechanics. Draw three pulley systems. Each one has only one mass m and only one applied force F. For each system you can use any number of ideal massless pulleys and any number of pieces of inextensible massless string. Neglect gravity.

You can label any number of points on one drawing. On your drawings find and label a point

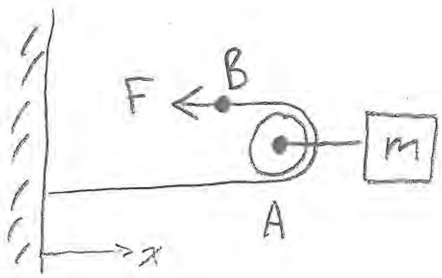
a) A with acceleration whose magnitude is $2F/m$.

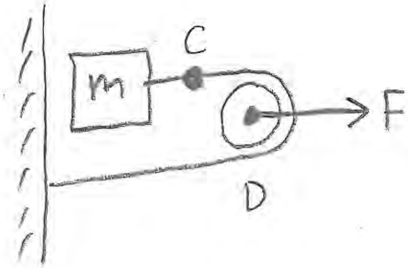
b) B with acceleration whose magnitude is $4F/m$.

c) C with acceleration whose magnitude is $F/(2m)$.

d) D with acceleration whose magnitude is $F/(4m)$.

e) E with acceleration whose magnitude is $9F/m$.





$L = x_A + (x_A - x_B)$

$2\ddot{x}_A = \ddot{x}_B$

FBD:

LMB: $\Sigma F = ma$

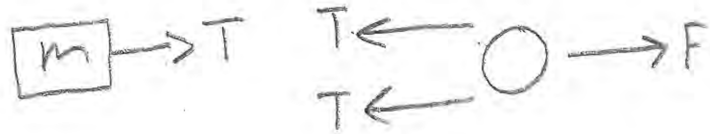
$-2F = m\ddot{x}_a$

$$\boxed{\begin{aligned} \dot{x}_a &= \frac{-2F}{m} \\ \ddot{x}_b = 2\ddot{x}_a &= \frac{-4F}{m} \end{aligned}}$$

$L = x_D + (x_D - x_C)$

$2\ddot{x}_D = \ddot{x}_C$

FBD



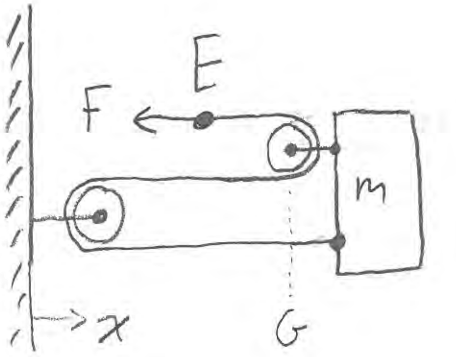
LMB: $\Sigma F = ma$

massless pulley

$T = m\ddot{x}_c$ $\qquad -2T + F = 0$

$\ddot{x}_c = \frac{T}{m}$ $\qquad T = \frac{-F}{2}$

$$\boxed{\begin{aligned} \ddot{x}_c &= \frac{-F}{2m} \\ \ddot{x}_D = \frac{1}{2}\ddot{x}_c &= \frac{-F}{4m} \end{aligned}}$$

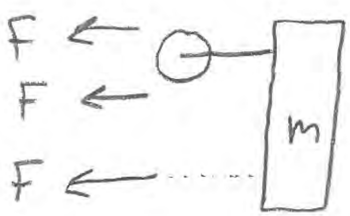


Bonus Option/Bonus project.
Find a single pulley system on which you can mark all 5 points.
Is it even possible?

$$L = 2x_G + (x_G - x_E)$$

$$3\ddot{x}_G = \ddot{x}_E$$

FBD



LMB

$$-3F = m\ddot{x}_G$$

$$\ddot{x}_G = \frac{-3F}{m}$$

$$\ddot{x}_E = 3\ddot{x}_G = \frac{-9F}{m}$$

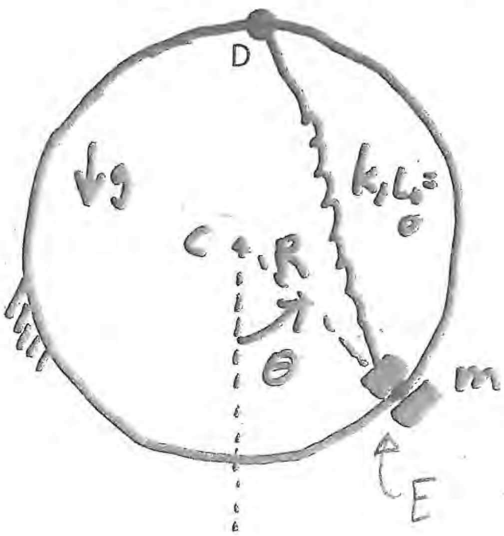
$$\boxed{\ddot{x}_E = \frac{-9F}{m}}$$

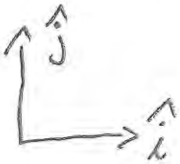
2) A small collar m slides on a rigid stationary hoop with radius R. There is gravity g but no friction. A spring with constant k and rest length $L_0 = 0$ pulls on the mass. One end of the spring is at the fixed point D directly above C. Answer in terms of some or all of m, R, g, θ and $\dot\theta$ find

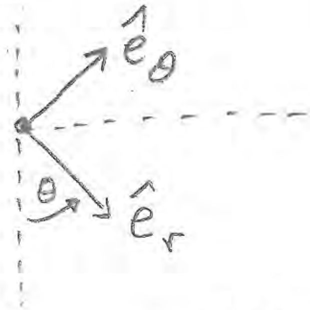
a) Find $\ddot\theta$

a) Find the net force (a vector) on the block from the hoop (Please read the 6th line in the directions on the front cover: "Clearly define . . . ").

a) Are there any special values of k (in terms of m, g and R) for which you can find the general exact solution to the equations of motion? If so, name the k and give the solution. This problem part depends on correct solution of (a)). No partial credit for Matlab on this problem.



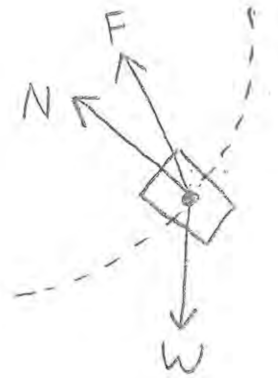




$$\left.\begin{aligned}\hat e_r &= \sin\theta\,\hat\imath - \cos\theta\,\hat\jmath \\ \hat e_\theta &= \cos\theta\,\hat\imath + \sin\theta\,\hat\jmath\end{aligned}\right]$$

Note: not the conventional def. in terms of $\hat\imath$ and $\hat\jmath$

FBD



$\vec W = -mg\,\hat\jmath$

$\vec N = -N\,\hat e_r$

Assumes $L_0 = 0$

$\vec F = K(\vec r_{D/E}) = K(R\hat\jmath - R\hat e_r)$

LMB $\quad \Sigma\vec F = \dot{\vec L}$

$\vec W + \vec N + \vec F = m(R\ddot{\hat e}_r)$

$\{(-mg\hat\jmath) + (-N\hat e_r) + (KR\hat\jmath - KR\hat e_r) = mR(\ddot\theta\,\hat e_\theta - \dot\theta^2\,\hat e_r)\}$

a) $\{\}\cdot\hat{e}_\theta$: $-mg(\hat{j}\cdot\hat{e}_\theta) + 0 + KR(\hat{j}\cdot\hat{e}_\theta) = mR\ddot{\theta}$

$$-mg\sin\theta + KR\sin\theta = mR\ddot{\theta}$$

$$\boxed{\ddot{\theta} = \left(\frac{K}{m} - \frac{g}{R}\right)\sin\theta} \quad (a)$$

b) $\{\}\cdot\hat{e}_r$: $-mg(\hat{j}\cdot\hat{e}_r) - N + \left(KR(\hat{j}\cdot\hat{e}_r) - KR\right) = -mR\dot{\theta}^2$

$$mg\cos\theta - N - KR\cos\theta - KR = -mR\dot{\theta}^2$$

$$\boxed{\begin{aligned} &N = mg\cos\theta - KR(\cos\theta + 1) + mR\dot{\theta}^2 \\ &\vec{N} = -N\hat{e}_r \end{aligned}} \quad (b)$$

c) Sol'n when $\left(\frac{K}{m} - \frac{g}{R}\right) = 0$

(c) $\boxed{K = \frac{gm}{R}}$ $\Longrightarrow$ $\ddot{\theta} = 0$

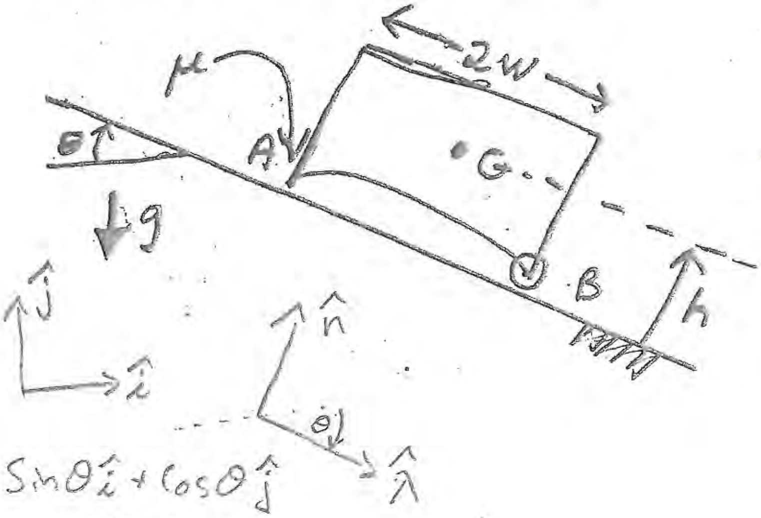
(c) $\boxed{\theta(t) = \omega_0 t + \theta_0}$ $\omega_0 \equiv \dot{\theta}(0)$; $\theta_0 \equiv \theta(0)$

3) A uniformly dense suitcase with mass m slides down (with $v > 0$) a straight ramp with slope θ with $0 < \theta < \pi/2$. The suitcase has height $2h$ and width $2w$. The front (downhill) end is supported by well lubricated and negligible-mass wheels at B. The uphill end drags with friction coefficient $\mu > 0$ at A.

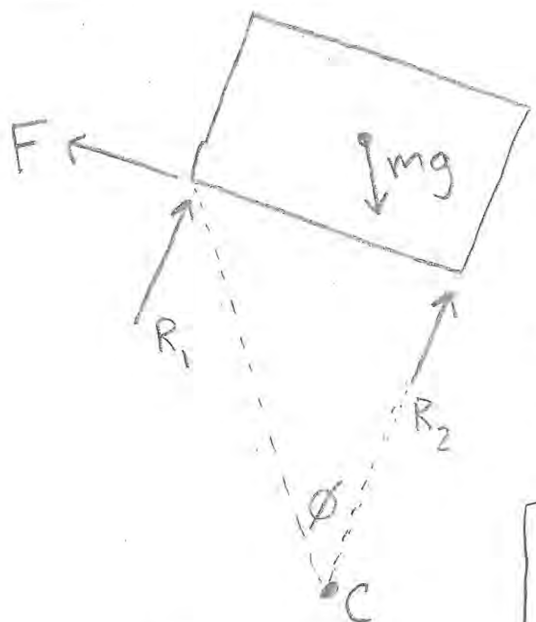
a) What is the acceleration of the suitcase? Answer in terms of some or all of m, g, h, w, θ, v and μ.

b) For what values of the parameters is the solution not applicable because the suitcase would tip over forwards? Answer in terms of some or all of m, g, h, w, θ, v and μ. [Hint: some people may find the answer surprising]

c) Given the other parameters, for some slopes θ the suitcase is slowing and for some slopes it is speeding. What is the minimum θ for which it is assured that the suitcase will speed up as it goes along no matter how big is the friction μ? Answer in terms of some or all of m, g, h and w. (It is possible to answer this without use of the answer to (a) above. No partial credit for correct algebra based on an incorrect answer to (a) above.)

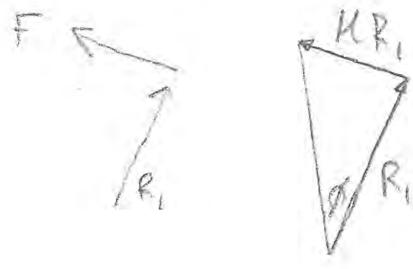


FBD



$$\hat{n} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$
$$\hat{\lambda} = \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$$

Aside: $F = \mu R_1$



$$\tan\phi = \frac{\mu R_1}{R_1}$$
$$\tan\phi = \mu$$

$$\tan\phi = \frac{2w}{L} = \mu \longrightarrow L = \frac{2w}{\mu}$$

AMB$_{/C}$ $\quad \sum \vec{M}_{/C} = \dot{\vec{H}}_{/C}$

$$\vec{r}_{G/C} \times (-mg\,\hat{j}) = \vec{r}_{G/C} \times (ma\,\hat{\lambda})$$

$$\vec{r}_{G/C} = \left(\frac{2w}{\mu} + h\right)\hat{n} + (-w)\hat{\lambda}$$

$$\left(\frac{2w}{\mu}+h\right)(-mg)(\hat{n}\times\hat{j})+(mgw)(\hat{\lambda}\times\hat{j})$$

$$=\left(\frac{2w}{\mu}+h\right)(ma)(\hat{n}\times\hat{\lambda})+(-maw)(\hat{\lambda}\times\hat{\lambda})$$

$$\left(\begin{array}{ll} \hat{n}\times\hat{j}=\sin\theta & \hat{\lambda}\times\hat{j}=\cos\theta \\ \hat{n}\times\hat{\lambda}=-1 & \hat{\lambda}\times\hat{\lambda}=0 \end{array}\right)$$

$$-mg\left(\frac{2w}{\mu}+h\right)\sin\theta+mgw\overbrace{\vphantom{w}}^{\cos\theta}=-ma\left(\frac{2w}{\mu}+h\right)+0$$

$$\boxed{a=g\left(\sin\theta-\frac{w\mu\cos\theta}{2w+\mu h}\right)}\quad\boxed{\vec{a}_G=a\hat{\lambda}}\ \text{(a)}$$

CHECKS

1. Units: accel = accel ✓
2. $g=0 \Rightarrow a=0$ ✓
3. $\mu=0 \Rightarrow a=g\sin\theta$ ✓
4. $h=0,\ \theta=0 \Rightarrow a=\mu/2$ ✓
5. $w\to 0 \Rightarrow a=g\sin\theta$ ✓

<u>LMB</u> $\quad \Sigma\vec{F}=\dot{\vec{L}}$

$$\left\{(-\mu R_1\hat{\lambda})+(R_1\hat{n})+(R_2\hat{n})+(-mg\hat{j})=(ma\hat{\lambda})\right\}$$

$$\{\}\cdot\hat{\lambda}:\ -\mu R_1+0+0-mg(-\sin\theta)=ma$$

$$R_1=\frac{m}{\mu}(g\sin\theta-a)$$ *Now use Sol'n

$$R_1=\frac{m}{\mu}\left(\frac{wg\mu\cos\theta}{2w+\mu h}\right)=\frac{mgw\cos\theta}{2w+\mu h}$$

Assume $\theta<\pi/2 \Rightarrow \cos\theta>0$

Tip Forwards $\longleftrightarrow (R_1<0)$ — NEEDS TO BE TRUE

$$\frac{mgw}{2w+\mu h}>0 \quad\therefore\quad R_1>0\quad\therefore$$ Always True

For all h,μ,θ, ect...

$\boxed{\text{Cannot Tip Forward}}$

Slowing Down $\longleftrightarrow (a<0)$

Speeding Up $\longleftrightarrow (a>0)$

$$a = g\left(\sin\theta - \frac{w\mu\cos\theta}{2w+\mu h}\right)$$

$$a(\theta) \equiv 0 \longrightarrow \sin\theta - \frac{w\mu\cos\theta}{2w+\mu h} = 0$$

$$\boxed{\begin{array}{l} \theta_{crit} = \tan^{-1}\left(\dfrac{w\mu}{2w+\mu h}\right) \\ \\ \text{if } \theta > \theta_{crit} \text{ then } a>0 \end{array}}$$

For all $\mu \Rightarrow$ worst case is $\mu \to \infty$

$$\Rightarrow \boxed{\boxed{\underset{\text{for } a>0 \text{ guarantee}}{\theta > \tan^{-1}\left(\frac{w}{h}\right)}}} \text{(c)}$$

Alt. derivation of (c) w/out using (a). Worst case is $\mu = \infty$
$\Rightarrow R_1 = 0$. Critical case is $a=0 \Rightarrow$ statics. Solve statics problem:

$\sum \vec{M}_{/B} = \vec{0} \Rightarrow G$ directly above B

FBD

$\Rightarrow \tan\theta_c = w/h$

$$\Rightarrow \boxed{\begin{array}{l} \theta_c = \tan^{-1}(w/h) \\ \text{accel} > 0 \text{ if } \theta > \theta_c \end{array}}$$