Your TA, Section # and Section time: "Solutions"

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Cornell TAM/ENGRD 2030

No calculators, books or notes allowed. This version slightly improves the given version.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

Prelim 1

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How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation.**
- A+ Be (I) neat, (II) clear and (III) well organized.
- ☐ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{n}} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60 – 100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** The last page is blank for your use. Ask for more extra paper if you need it. Put your name on each extra sheet, fold it in, and refer to back pages or extra sheets on the page of the relevant problem.

Problem 1: ____ /25

Problem 2: ____ /25

Problem 3: ____ /25

1) A mass m bounces back and forth between two springs each with constant k. The space between the springs is L. Neglect the width of the mass. Neglect the mass of the springs and their supports. In this motion energy is conserved. For some initial condition, assume that the peak speed of the mass in a cycle of oscillation is v_p. Answer the questions below in terms of some or all of m, k, L and v_p.

a) What is the total energy? Assume the potential energy is zero when the springs are unstretched.

$E_T = E_K + \cancel{E_P}$

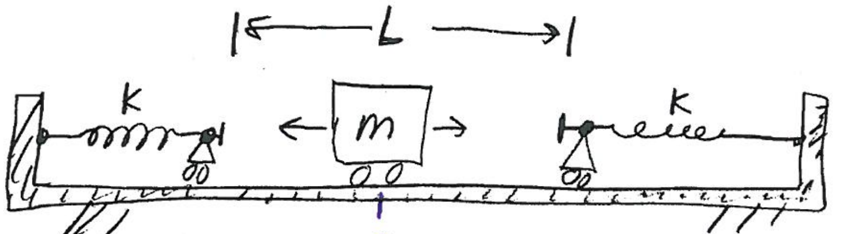
$\boxed{E_T = \frac{1}{2} m V_p^2}$ (a)

b) In a cycle of oscillation what is the maximum deflection of the right spring?

c) What is the period T of oscillation?

d) Plot the cyclic frequency f (defined as $f = 1/T$) *vs* peak speed v_p. Clearly mark and label any key slopes, intercepts, intersections or asymptotes on this plot.

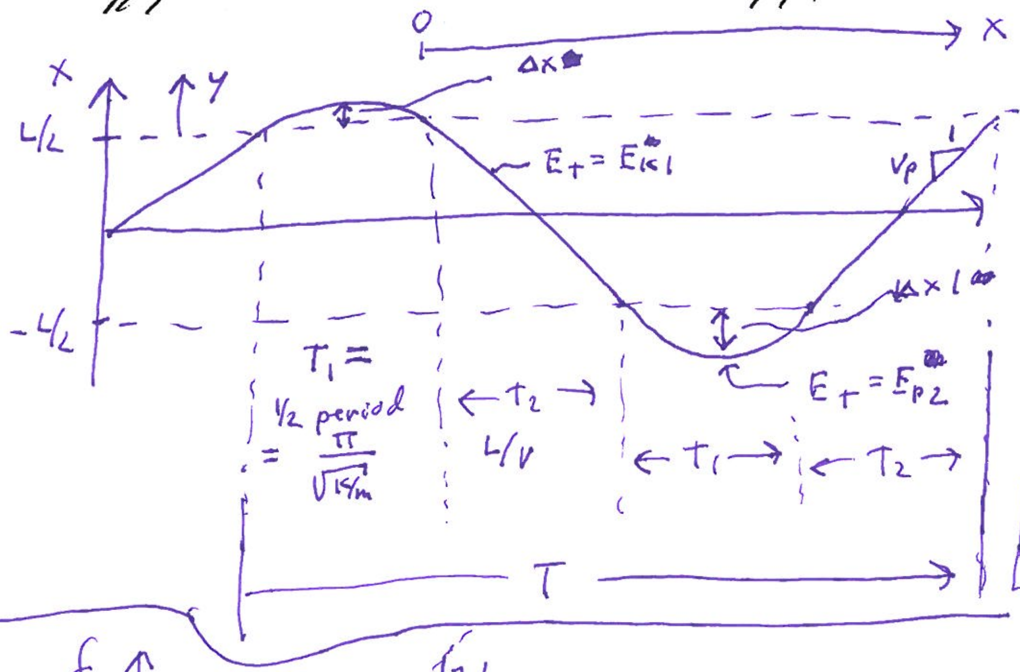
e) EXTRA CREDIT. Hard. *Only think about this if you have nothing more to add to anything else on the exam.* Assume that the ends of the springs also each have mass $m = m/3$. Assume that all collisions are with restitution coefficient $e = 0$. Assume there is no other friction. Can you find a situation where oscillations persist with no decay? This means finding the right combination of v_p, k, m and L, as well as the right initial conditions. Hint: How, with $e = 0$, can there be collisions with no energy loss?



$E_{K1} = E_{P2}$

$\frac{1}{2} m V_p^2 = \frac{1}{2} k (\Delta x)^2$

$\boxed{|\Delta x| = \sqrt{m/k}\, V_p}$ (b)



during contact

$y = A\sin(\omega t) + B\sin(\omega t)$

$\omega = \sqrt{k/m}$

$2\omega T_1 = 2\pi$

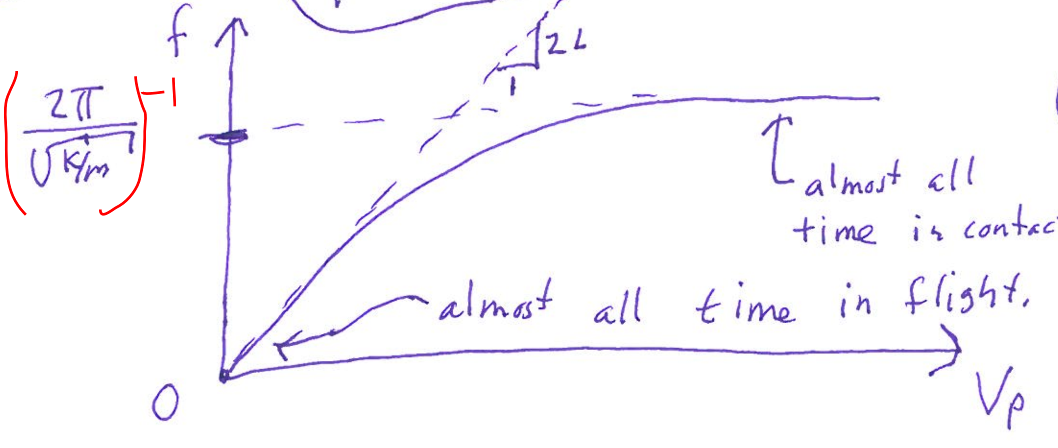
$T_1 = \frac{\pi}{\sqrt{k/m}}$

$T = 2(t_1 + t_2)$ (c)

$\boxed{T = \frac{2L}{V_p} + \frac{2\pi}{\sqrt{k/m}}}$

$f = 1/T$

(d) $\boxed{f = \frac{1}{2L/V_p + 2\pi/\sqrt{k/m}}}$

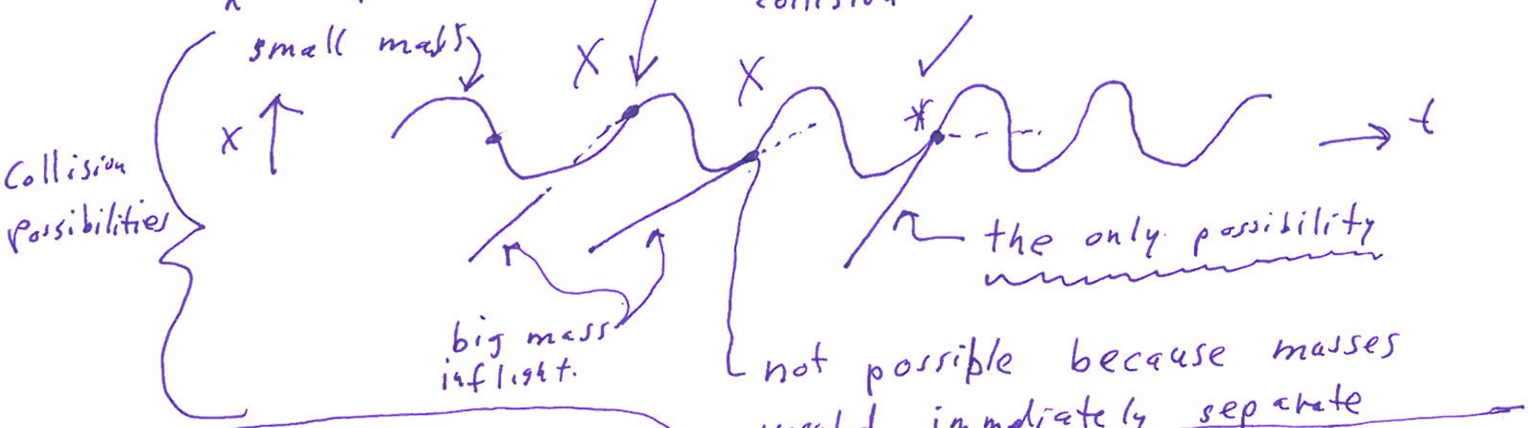


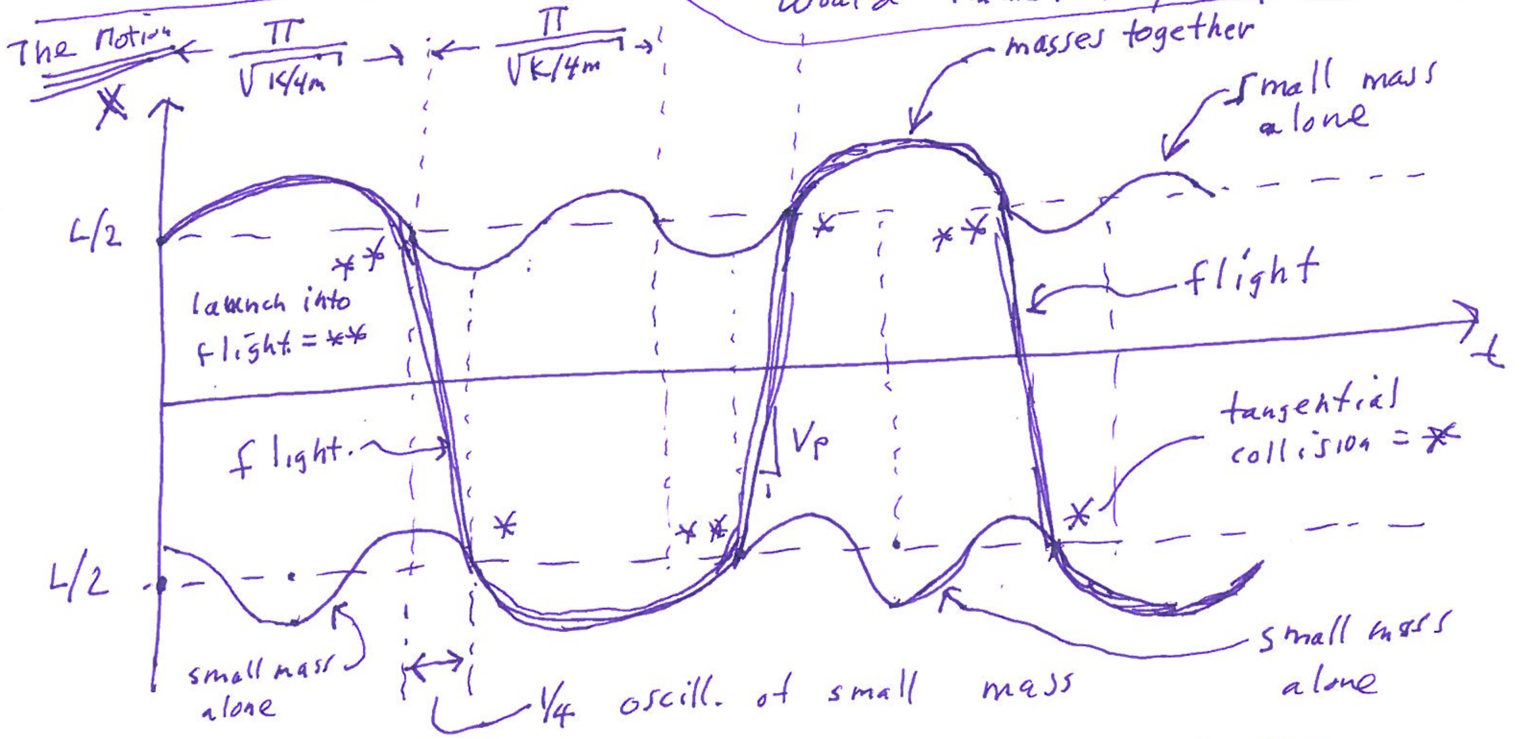
e) No loss ⇒ rel. vel. of collision = 0.

⇒ velocities of masses match

⇒ flight vel. of big mass matches osc. vel of small mass. } *

When is collision in cycle?





When masses all in contact $M_{tot} = 3m + m = 4m$

⇒ oscillations are ~~twice~~ half as fast.

One full cycle = 2 half oscillations of 2 masses together + 2 flights

$$= \cancel{4}\left[\frac{2\pi}{\sqrt{K/4m}} + \frac{\pi}{\sqrt{K/m}}\right] = \boxed{\frac{5\pi}{\sqrt{K/m}} = T}$$ Period of one full oscillation.

flight time

$$\frac{L}{V_p} = \frac{1}{4}\text{ oscill. of small mass} = \frac{1}{4}\frac{2\pi}{\sqrt{K/m}} \Rightarrow \boxed{V_p = \frac{2L\sqrt{K/m}}{\pi}}$$ Flight speed.

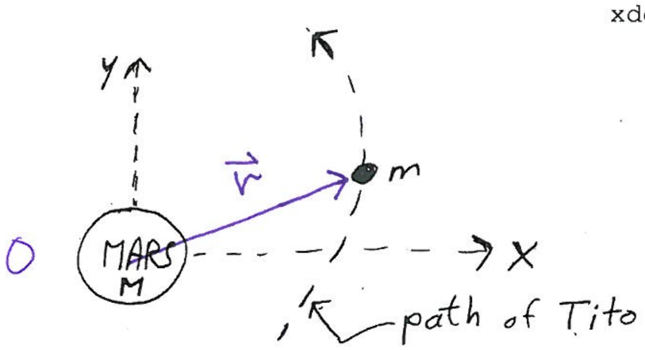
2) The year is 2018 and Dennis Tito's space ship (mass = m) is flying around Mars (mass = $M \gg m$)[1]. The forces acting on the ship include

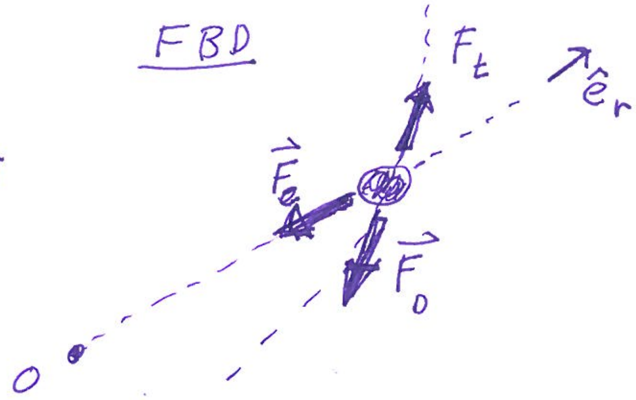
- a gravity force from mars due to the universal law of gravitation with constant G;
- a drag force with magnitude cv^2, due to motion through the thin martian atmosphere;
- a thrust force F_0, from the ion generator, along the motion.

a) (5 points for your signature) *I have read all the directions on the front cover.* ____________________.
(sign above)

b) (20 points) In terms of some or all of $m, M, G, c, F_0, t, x, y, \dot{x}$ and $\dot{y}$ given at some time t, write Matlab commands to find $\ddot{x}$ at that time ~~(No need to solve the ODEs by hand or numerically.)~~ That is, assume non-zero values have already been assigned for the given variables, and you write Matlab commands the last of which should start:

`xddot =`





$$\vec{F}_G = -\frac{GMm}{r^2}\hat{e}_r \qquad \vec{F}_D = -cV^2\frac{\vec{V}}{|V|} = -c|\vec{V}|\vec{V}$$

$$\vec{F}_t = F_0\frac{\vec{V}}{|\vec{V}|}, \qquad \vec{F}_{TOT} = \vec{F}_g + \vec{F}_t + \vec{F}_D$$

LMB: $\vec{F}_{TOT} = m\vec{a} \Rightarrow \vec{a} = \vec{F}/m \Rightarrow a_x = \ddot{x} = [\vec{F}/m]\cdot\hat{\imath}$

r = [x y];

v = ~~[x y]~~ [xdot ydot];

er = r/norm(r);

F_G = -G*M*m*er/r^2;

F_t = F_0 * ~~er~~ ~~V~~ v/norm(v);

F_D = -c* V* norm(V);

a=(FG + Ft + FD)/m

$\boxed{\text{Xddot} = a(1)}$

[1]No kidding. Dennis Tito is a real person and this, according to the New York Times this past weekend, is his plan. The ship is to be designed by Space X, many of whose top engineers, so the rumor goes, learned dynamics in TAM 2030 at Cornell. And by 2018, one of them could be you.

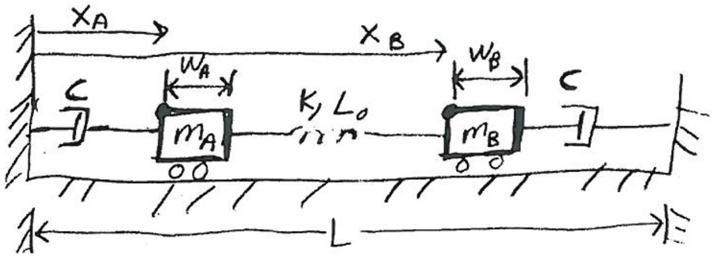
3) Two masses are connected to two identical dashpots and a spring, as shown. The positions of the left edges of the masses x_A and x_B, as well as all other variables, are defined as shown.

a) Draw free body diagrams for both masses, and write the equations of linear momentum balance ($F = ma$) for both masses clearly enough so that a differential equations expert would know what to solve.

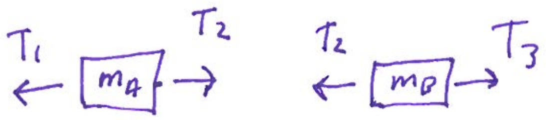
b) That expert wrote some code to generate an approximate numerical solutions, but left some parts blank. Wherever there is a ____ , you fill in the missing code. The meanings of variables are implicitly defined by the physical problem and by other parts of the code.

c) Based on your understanding of both Matlab and mechanics, draw the plot Matlab would make, as accurately as you can.

Symmetric Prob. has symmetric Soln., Like one mass oscillating w/ spring w/ stiffness 2K.



FBDs



$T_1 = c\dot{x}_A$ $\quad$ $T_2 = k(x_B - x_A - L_0 - w)$ $\quad$ $T_3 = -c\dot{x}_B$

LMB

A) $\Sigma F = m_A \ddot{x}_A$

$\ddot{x}_A = (T_2 - T_1)/m_A$

above

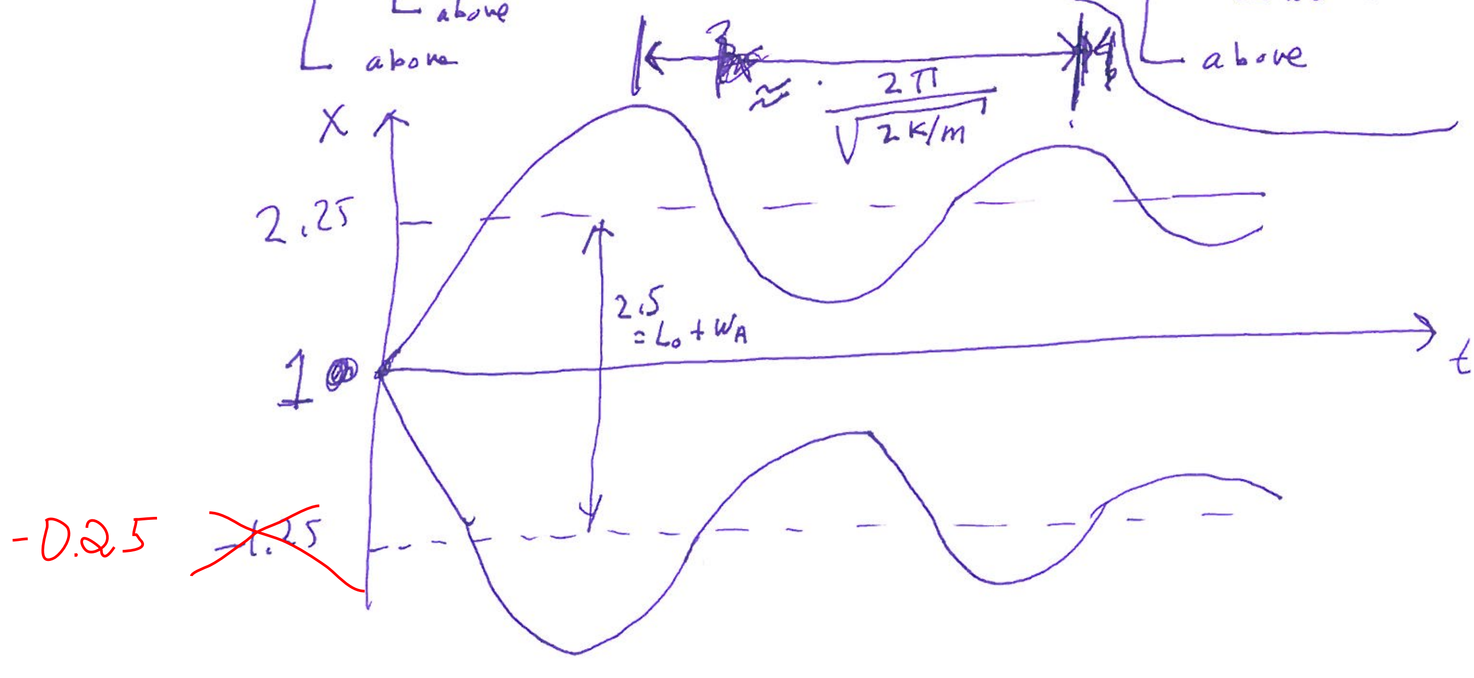
above

B) $\Sigma F = m_B \ddot{x}_B$

$\ddot{x}_B = (T_3 - T_2)/m_B$

above

above



masses on top of each other but computer doesn't care

This code is poorly commented. It is an exam problem, not an example to emulate.

```
function   Prel1Q1
  tarray =  0:.01:20 ;  n = length(tarray);
  z0     = [1 4 0 0]';  %in the original exam this was [1 1 0 0]';
  p.mA = 1;   p.mB = 1;   p.k  = 1;   p.c = .5;
  p.wA = .5;  p.wB = 2;   p.L0 = 2;   p.L = 10;

  [tarray zarray] = mysolver(tarray, z0, p);

  x1array = zarray(:,1);x2array = zarray(:,2);
  plot(tarray, x1array,  'b--',tarray, x2array,  'r-' )
end
```

Spring relaxed when $x_B - x_A = 2.5$

initial stretch is $\Delta L = -2.5$

```
function  [tarray zarray] = mysolver(tarray, z0, p)
  n       = length(tarray);
  zarray  = zeros(n,length(z0));
  z       = z0;              zarray(1,:) = z;

  for i = 2:n
    t    = tarray(i-1);  h = tarray(i) - tarray(i-1);
    zdot = rhs(t,z,p) ;
    z    = z + zdot*h ;
    zarray(i,:) = z';
  end
end


function  zdot=rhs(t,z,p)

  x=z(1:2);  v= z(3:4); ;
  L1dot    = v(1);
  L2       = x(2) - x(1) - p.wA ;     % length of spring
  L3dot    = -v(2) ;

  T1  =  p.c  * L1dot;
  T2  =  p.k * (L2 - p.L0) ;
  T3  =  p.c * (L3dot);

  xdot  = v ;
  v1dot = (T2 - T1)/ p.mA ;
  v2dot = (T3 - T2) / p.mB ;
  vdot   = [v1dot; v2dot] ;

  zdot  = [xdot; vdot];
end
```