Your TA, Section # and Section time:

"Solutions"

Your name:

ANDY RUINA

Cornell TAM/ENGRD 2030

Prelim 1

February 26, 2013

No calculators, books or notes allowed. This version slightly improves the given version.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation.**
- A+ Be (I) neat, (II) clear and (III) well organized.
- □ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems ***poorly defined***, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** The last page is blank for your use. Ask for more extra paper if you need it. Put your name on each extra sheet, fold it in, and refer to back pages or extra sheets on the page of the relevant problem.

Problem 1: ____ /25

Problem 2: ____ /25

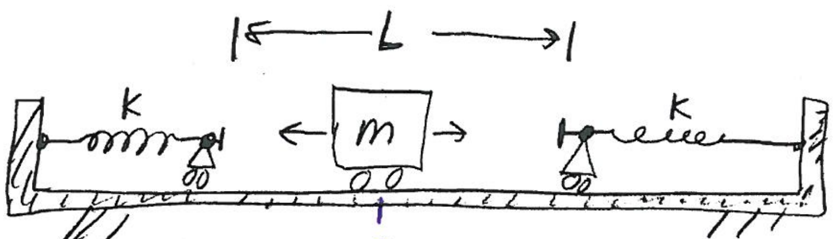
Problem 3: ____ /25

1) A mass m bounces back and forth between two springs each with constant k. The space between the springs is L. Neglect the width of the mass. Neglect the mass of the springs and their supports. In this motion energy is conserved. For some initial condition, assume that the peak speed of the mass in a cycle of oscillation is v_p. Answer the questions below in terms of some or all of m, k, L and v_p.

- **a)** What is the total energy? Assume the potential energy is zero when the springs are unstretched.
- **b)** In a cycle of oscillation what is the maximum deflection of the right spring?
- **c)** What is the period T of oscillation?
- **d)** Plot the cyclic frequency f (defined as $f = 1/T$) *vs* peak speed v_p. Clearly mark and label any key slopes, intercepts, intersections or asymptotes on this plot.
- **e)** EXTRA CREDIT. Hard. *Only think about this if you have nothing more to add to anything else on the exam.* Assume that the ends of the springs also each have mass $m = m/3$. Assume that all collisions are with restitution coefficient $e = 0$. Assume there is no other friction. Can you find a situation where oscillations persist with no decay? This means finding the right combination of v_p, k, m and L, as well as the right initial conditions. Hint: How, with $e = 0$, can there be collisions with no energy loss?

$E_T = E_K + \cancel{E_P}$

$\boxed{E_T = \frac{1}{2} m V_p^2}$ (a)



$E_{K1} = E_{P2}$

$\frac{1}{2} m V_p^2 = \frac{1}{2} k (\Delta x)^2$

$\boxed{|\Delta x| = \sqrt{m/k}\, V_p}$ (b)

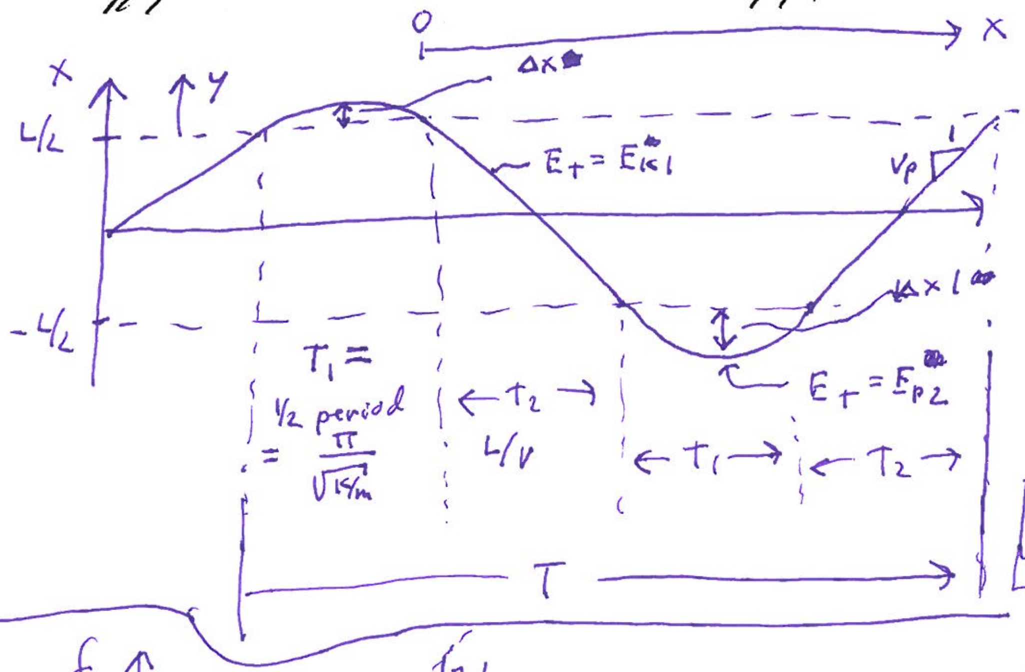
during contact

$y = A\sin(\omega t) + B\sin(\omega t)$

$\omega = \sqrt{k/m}$

$2\omega T_1 = 2\pi$

$T_1 = \frac{\pi}{\sqrt{k/m}}$



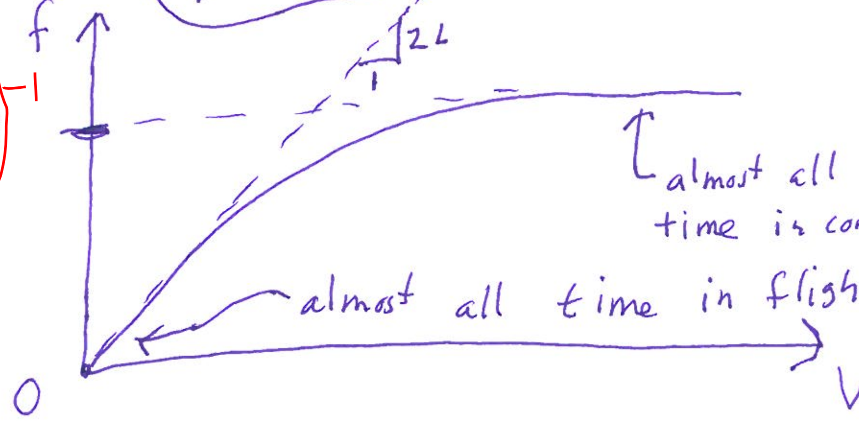
$T = 2(t_1 + t_2)$ (c)

$\boxed{T = \frac{2L}{V_p} + \frac{2\pi}{\sqrt{k/m}}}$

$f = 1/T$

(d) $\boxed{f = \frac{1}{2L/V_p + 2\pi/\sqrt{k/m}}}$

$\left(\frac{2\pi}{\sqrt{k/m}}\right)^{-1}$

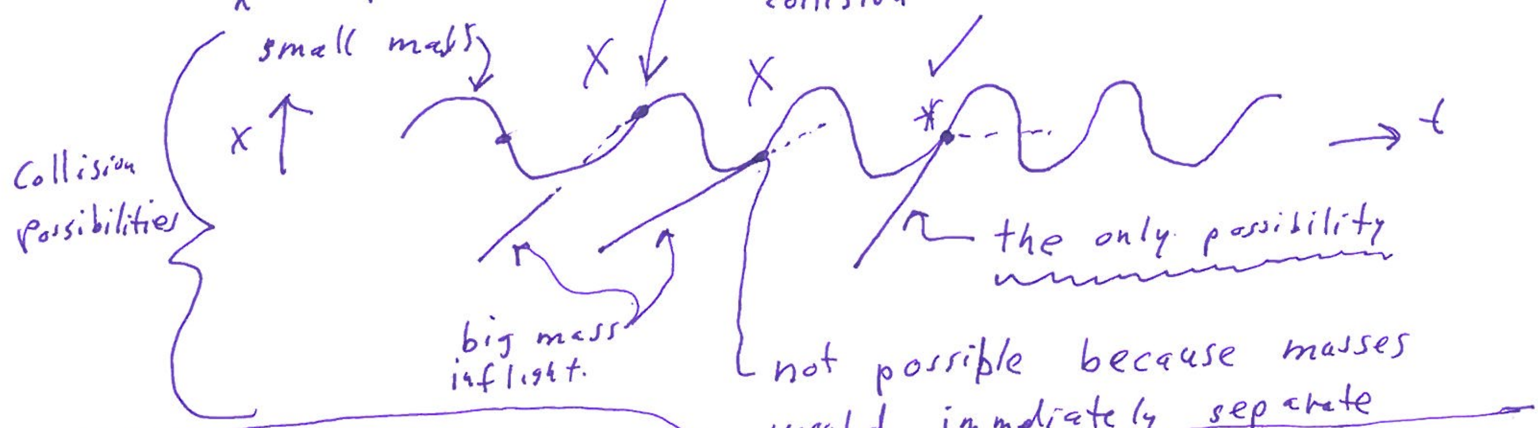


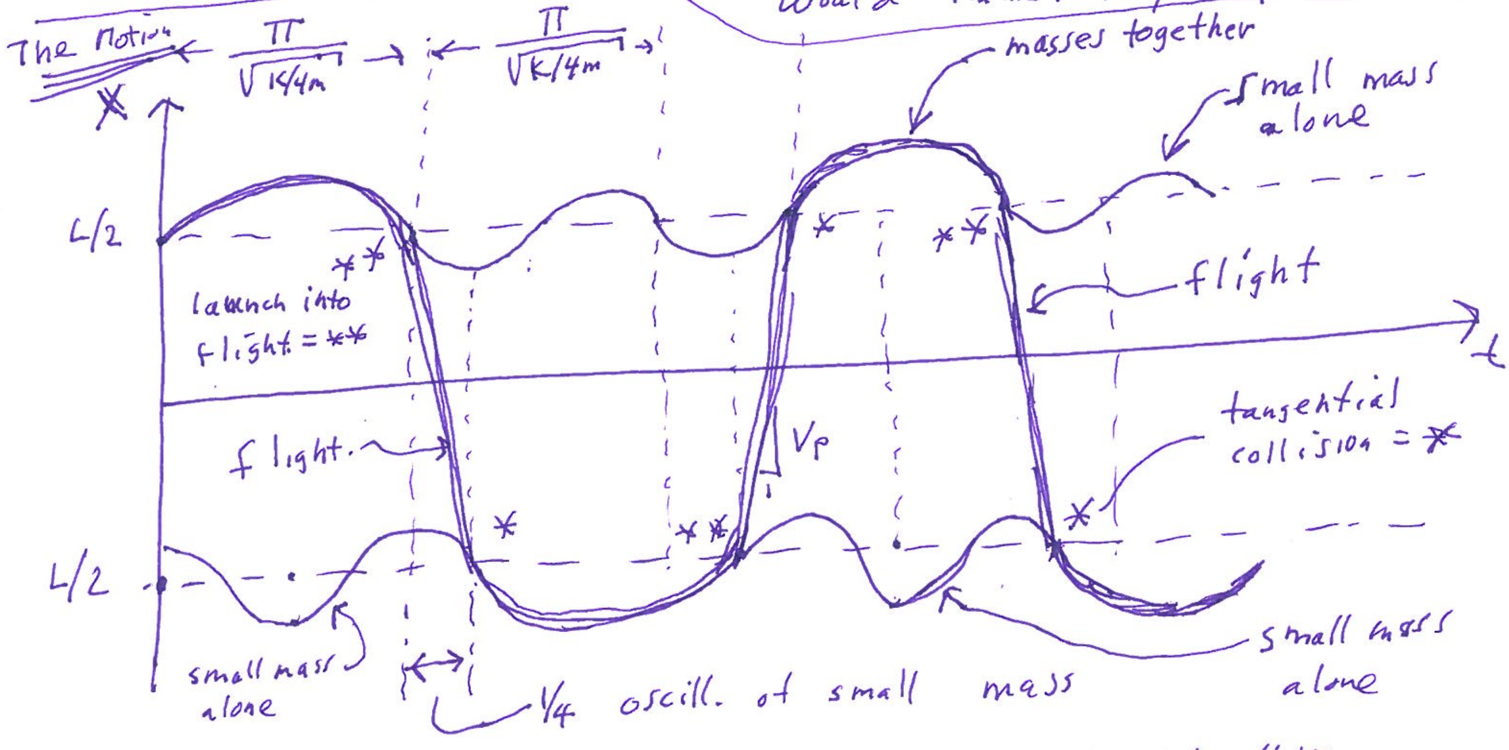
almost all time in contact w/ spring (b/c oscillations)

almost all time in flight. (long slow flight)

e) No loss ⇒ rel. vel. of collision = 0.
⇒ velocities of masses match
⇒ flight vel. of big mass matches osc. vel of small mass. } ✱

When is collision in cycle?





When masses all in contact $M_{tot} = 3m + m = 4m$

⇒ oscillations are ~~twice~~ half as fast.

One full cycle = 2 half oscillations of 2 masses together + 2 flights

$$= \cancel{4}\left[\frac{2\pi}{\sqrt{K/4m}} + \frac{\pi}{\sqrt{K/m}}\right] = \boxed{\frac{5\pi}{\sqrt{K/m}} = T}$$ Period of one full oscillation.

flight time

$\frac{L}{V_p} = \frac{1}{4}$ oscill. of small mass $= \frac{1}{4}\frac{2\pi}{\sqrt{K/m}} \Rightarrow \boxed{V_p = \frac{2L\sqrt{K/m}}{\pi}}$ Flight speed.

2) The year is 2018 and Dennis Tito's space ship (mass = m) is flying around Mars (mass = $M \gg m$)[1]. The forces acting on the ship include

- a gravity force from mars due to the universal law of gravitation with constant G;
- a drag force with magnitude cv^2, due to motion through the thin martian atmosphere;
- a thrust force F_0, from the ion generator, along the motion.

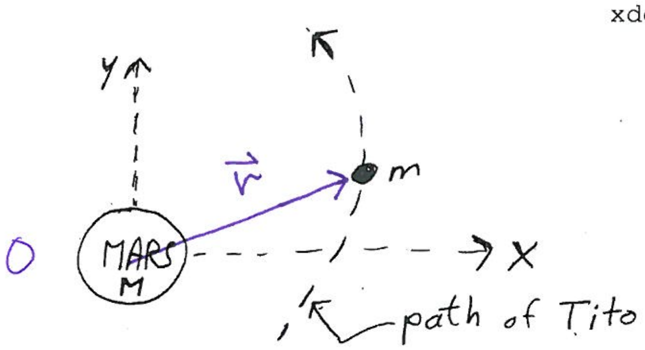
a) (5 points for your signature) *I have read all the directions on the front cover.* ____________________.

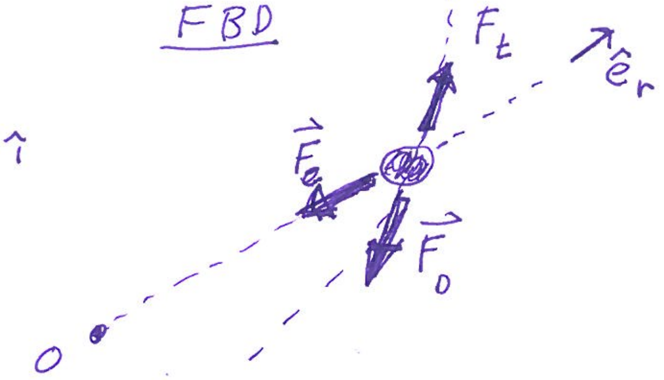
(sign above)

b) (20 points) In terms of some or all of $m, M, G, c, F_0, t, x, y, \dot{x}$ and $\dot{y}$ given at some time t, write Matlab commands to find $\ddot{x}$ at that time ~~(No need to solve the ODEs by hand or numerically.)~~ That is, assume non-zero values have already been assigned for the given variables, and you write Matlab commands the last of which should start:

FBD

`xddot =`





$$\vec{F}_G = -\frac{GMm}{r^2}\hat{e}_r \qquad \vec{F}_D = -cV^2\frac{\vec{V}}{|V|} = -c|\vec{V}|\vec{V}$$

$$\vec{F}_t = F_0\frac{\vec{V}}{|\vec{V}|}, \qquad \vec{F}_{TOT} = \vec{F}_g + \vec{F}_t + \vec{F}_0$$

LMB! $\vec{F}_{TOT} = m\vec{a} \Rightarrow \vec{a} = \vec{F}/m \Rightarrow a_x = \ddot{x} = [\vec{F}/m]\cdot\hat{\imath}$

$$a = (F_G + F_t + F_D)/m$$

```
r = [x y];
V = [xdot ydot];
er = r/norm(r);
FG = -G*M*m*er/r^2;
Ft = F0 * V/norm(V);
FD = -c* V* norm(V);
```

$\boxed{\text{Xddot} = a(1)}$

[1]No kidding. Dennis Tito is a real person and this, according to the New York Times this past weekend, is his plan. The ship is to be designed by Space X, many of whose top engineers, so the rumor goes, learned dynamics in TAM 2030 at Cornell. And by 2018, one of them could be you.

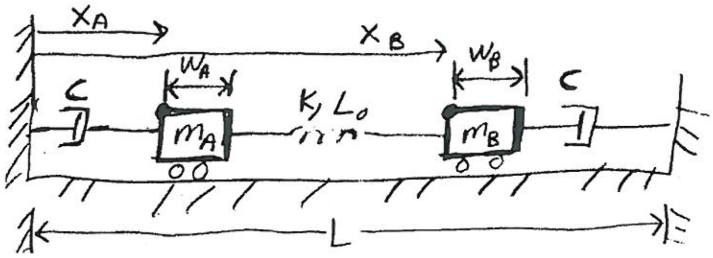
3) Two masses are connected to two identical dashpots and a spring, as shown. The positions of the left edges of the masses x_A and x_B, as well as all other variables, are defined as shown.

a) Draw free body diagrams for both masses, and write the equations of linear momentum balance ($F = ma$) for both masses clearly enough so that a differential equations expert would know what to solve.

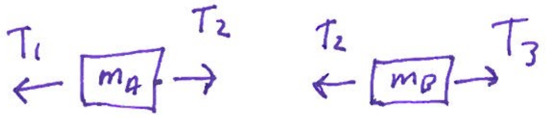
b) That expert wrote some code to generate an approximate numerical solutions, but left some parts blank. Wherever there is a ____ , you fill in the missing code. The meanings of variables are implicitly defined by the physical problem and by other parts of the code.

c) Based on your understanding of both Matlab and mechanics, draw the plot Matlab would make, as accurately as you can.

Symmetric Prob. has symmetric Soln.. Like one mass oscillating w/ spring w/ stiffness 2K.



FBDs



$T_1 = c\dot{x}_A$ $\quad$ $T_2 = k(x_B - x_A - L_0 - w)$ $\quad$ $T_3 = -c\dot{x}_B$

LMB

A) $\Sigma F = m_A \ddot{x}_A$

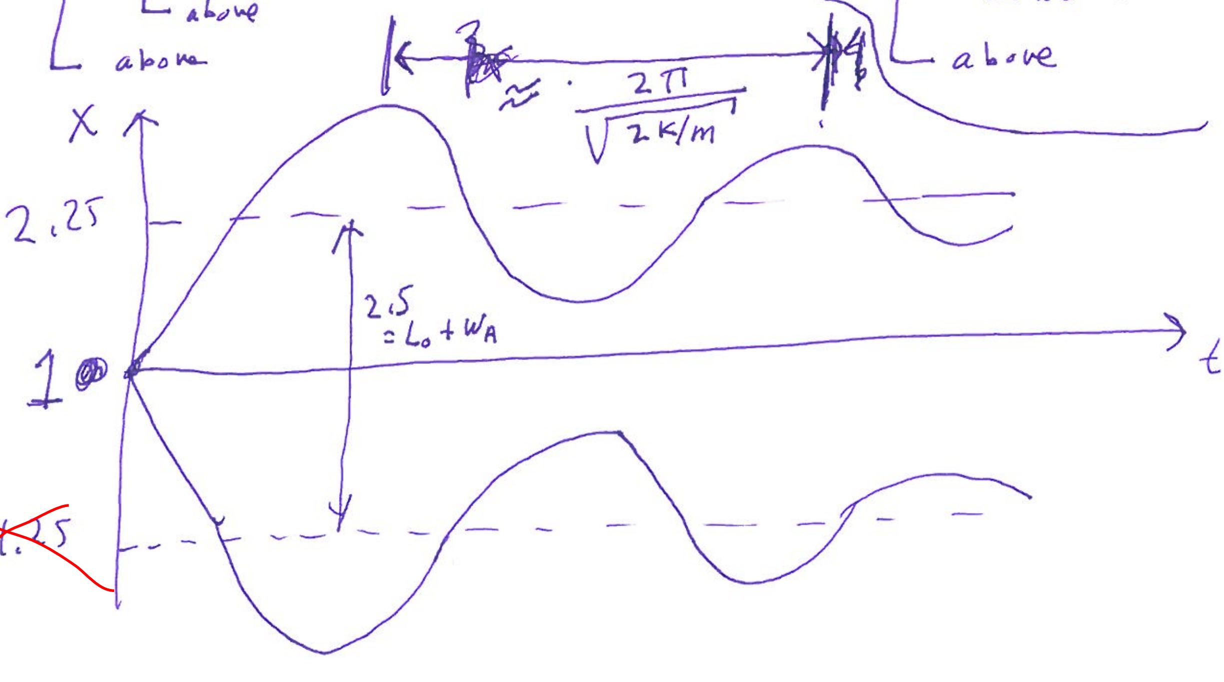
$\ddot{x}_A = (T_2 - T_1)/m_A$

above $\quad$ above

B) $\Sigma F = m_B \ddot{x}_B$

$\ddot{x}_B = (T_3 - T_2)/m_B$

above $\quad$ above



masses on top of each other but computer doesn't care

This code is poorly commented. It is an exam problem, not an example to emulate.

```
function   Prel1Q1
  tarray =  0:.01:20 ;  n = length(tarray);
  z0      = [1 1 0 0]';  %in the original exam this was [1 1 0 0]';
  p.mA = 1;   p.mB = 1;   p.k  = 1;   p.c = .5;
  p.wA = .5;  p.wB = 2;   p.L0 = 2;   p.L = 10;

  [tarray zarray] = mysolver(tarray, z0, p);

  x1array = zarray(:,1);x2array = zarray(:,2);
  plot(tarray, x1array,  'b--',tarray, x2array,  'r-' )
end
```

Spring relaxed when $x_B - x_A = 2.5$

initial stretch is $\Delta L = -2.5$

```
function  [tarray zarray] = mysolver(tarray, z0, p)
  n          = length(tarray);
  zarray   = zeros(n,length(z0));
  z          = z0;                 zarray(1,:) = z;

  for i = 2:n
    t      = tarray(i-1);   h = tarray(i) - tarray(i-1);

    zdot = rhs(t,z,p);

    z    = z + zdot*h;
    zarray(i,:) = z';
  end
end
```

```
function  zdot=rhs(t,z,p)

  x=z(1:2);  v=z(3:4);
  L1dot    = v(1);

  L2       = x(2) - x(1) - p.wA;        % length of spring

  L3dot    = -v(2);

  T1  =  p.c  * L1dot;

  T2  =  p.k * (L2 - p.L0);
  T3  =  p.c * (L3dot);

  xdot  =  v;

  v1dot = (T2 - T1)/p.mA;

  v2dot = (T3 - T2)/p.mB;
  vdot    = [v1dot; v2dot] ;

  zdot   = [xdot; vdot];
end
```

Your TA, Section # and Section time:

"Solutions"

Your name:

Matthew Kelly

Cornell TAM/ENGRD 2030

Prelim 2

March 26, 2013

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Problem 4: ____ /25

Problem 5: ____ /25

Problem 6: ____ /25

1) Pulleys. One dimensional mechanics. Draw three pulley systems. Each one has only one mass m and only one applied force F. For each system you can use any number of ideal massless pulleys and any number of pieces of inextensible massless string. Neglect gravity.

You can label any number of points on one drawing. On your drawings find and label a point

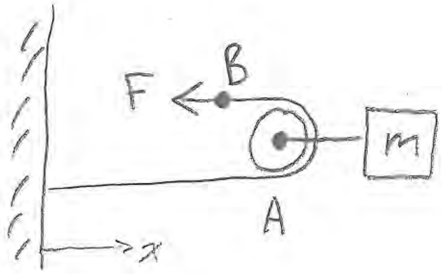
a) A with acceleration whose magnitude is $2F/m$.

b) B with acceleration whose magnitude is $4F/m$.

c) C with acceleration whose magnitude is $F/(2m)$.

d) D with acceleration whose magnitude is $F/(4m)$.

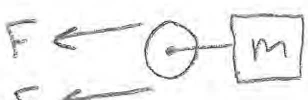
e) E with acceleration whose magnitude is $9F/m$.



$$L = x_A + (x_A - x_B)$$

$$2\ddot{x}_A = \ddot{x}_B$$

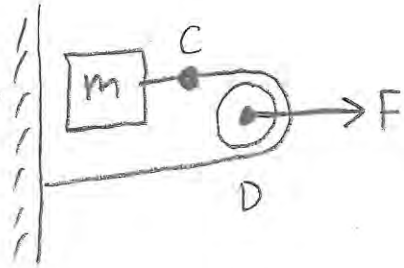
FBD:



LMB: $\Sigma F = ma$

$$-2F = m\ddot{x}_a$$

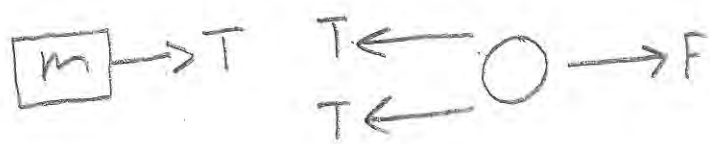
$$\boxed{\ddot{x}_a = \frac{-2F}{m} \qquad \ddot{x}_b = 2\ddot{x}_a = \frac{-4F}{m}}$$



$$L = x_D + (x_D - x_C)$$

$$2\ddot{x}_D = \ddot{x}_C$$

FBD



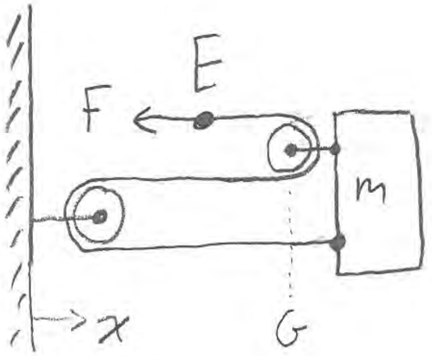
LMB: $\Sigma F = ma$

massless pulley

$$T = m\ddot{x}_C \qquad -2T + F = 0$$

$$\ddot{x}_C = \frac{T}{m} \qquad T = \frac{-F}{2}$$

$$\boxed{\ddot{x}_C = \frac{-F}{2m} \qquad \ddot{x}_D = \frac{1}{2}\ddot{x}_C = \frac{-F}{4m}}$$

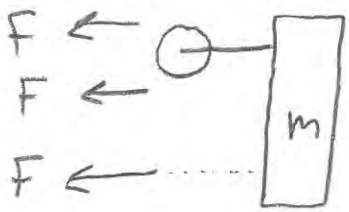


Bonus Option/Bonus project.
Find a single pulley system on which you can mark all 5 points.
Is it even possible?

$$L = 2x_G + (x_G - x_E)$$

$$3\ddot{x}_G = \ddot{x}_E$$

FBD



LMB

$$-3F = m\ddot{x}_G$$

$$\ddot{x}_G = \frac{-3F}{m}$$

$$\ddot{x}_E = 3\ddot{x}_G = \frac{-9F}{m}$$

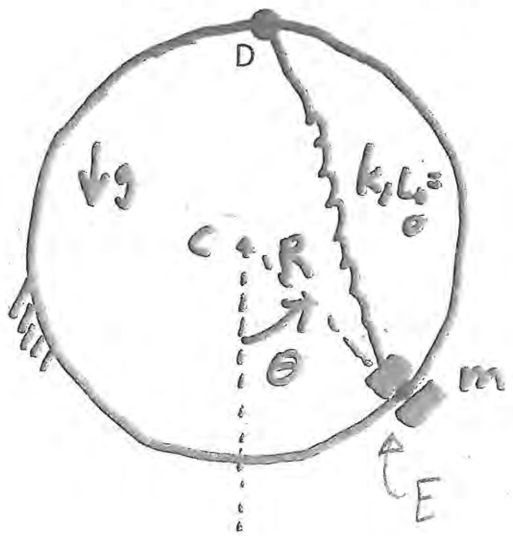
$$\boxed{\ddot{x}_E = \frac{-9F}{m}}$$

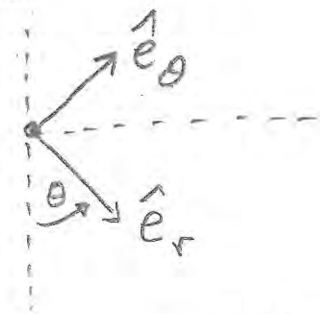
2) A small collar m slides on a rigid stationary hoop with radius R. There is gravity g but no friction. A spring with constant k and rest length $L_0 = 0$ pulls on the mass. One end of the spring is at the fixed point D directly above C. Answer in terms of some or all of m, R, g, θ and $\dot{\theta}$ find

a) Find $\ddot{\theta}$

a) Find the net force (a vector) on the block from the hoop (Please read the 6th line in the directions on the front cover: "Clearly define . . . ").

a) Are there any special values of k (in terms of m, g and R) for which you can find the general exact solution to the equations of motion? If so, name the k and give the solution. This problem part depends on correct solution of (a)). No partial credit for Matlab on this problem.

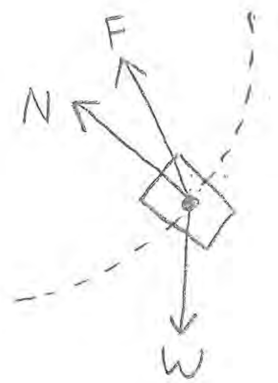




$$\hat{e}_r = \sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath}$$
$$\hat{e}_\theta = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

Note: not the conventional def. in terms of $\hat{\imath}$ and $\hat{\jmath}$

FBD



$$\vec{W} = -mg\,\hat{\jmath}$$
$$\vec{N} = -N\,\hat{e}_r$$
$$\vec{F} = K(\vec{r}_{D/E}) = K(R\hat{\jmath} - R\hat{e}_r)$$

Assumes $L_o = 0$

LMB $\quad \Sigma\vec{F} = \dot{\vec{L}}$

$$\vec{W} + \vec{N} + \vec{F} = m(R\ddot{\hat{e}}_r)$$
$$\left\{(-mg\hat{\jmath}) + (-N\hat{e}_r) + (KR\hat{\jmath} - KR\hat{e}_r) = mR(\ddot{\theta}\hat{e}_\theta - \dot{\theta}^2\hat{e}_r)\right\}$$

a) $\{\}\cdot\hat{e}_\theta$: $-mg(\hat{j}\cdot\hat{e}_\theta) + 0 + KR(\hat{j}\cdot\hat{e}_\theta) = mR\ddot{\theta}$

$$-mg\sin\theta + KR\sin\theta = mR\ddot{\theta}$$

$$\boxed{\ddot{\theta} = \left(\frac{K}{m} - \frac{g}{R}\right)\sin\theta} \quad (a)$$

b) $\{\}\cdot\hat{e}_r$: $-mg(\hat{j}\cdot\hat{e}_r) - N + \left(KR(\hat{j}\cdot\hat{e}_r) - KR\right) = -mR\dot{\theta}^2$

$$mg\cos\theta - N - KR\cos\theta - KR = -mR\dot{\theta}^2$$

$$\boxed{\begin{array}{l} N = mg\cos\theta - KR(\cos\theta + 1) + mR\dot{\theta}^2 \\ \vec{N} = -N\hat{e}_r \end{array}} \quad (b)$$

c) Sol'n when $\left(\frac{K}{m} - \frac{g}{R}\right) = 0$

$$(c)\ \boxed{K = \frac{gm}{R}} \implies \ddot{\theta} = 0$$

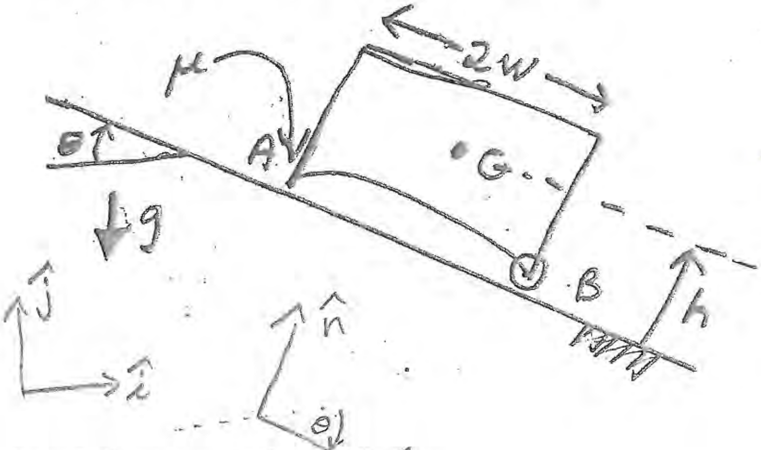
$$(c)\ \boxed{\theta(t) = \omega_0 t + \theta_0} \quad \omega_0 \equiv \dot{\theta}(0)\ ;\ \theta_0 \equiv \theta(0)$$

3) A uniformly dense suitcase with mass m slides down (with $v > 0$) a straight ramp with slope θ with $0 < \theta < \pi/2$. The suitcase has height $2h$ and width $2w$. The front (downhill) end is supported by well lubricated and negligible-mass wheels at B. The uphill end drags with friction coefficient $\mu > 0$ at A.

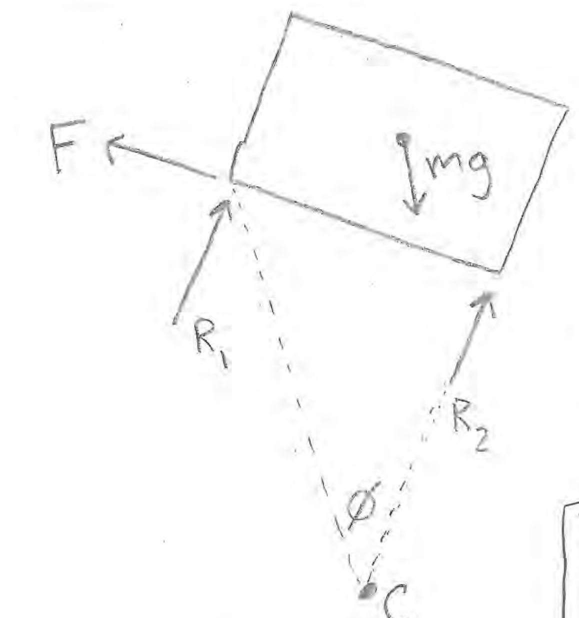
a) What is the acceleration of the suitcase? Answer in terms of some or all of m, g, h, w, θ, v and μ.

b) For what values of the parameters is the solution not applicable because the suitcase would tip over forwards? Answer in terms of some or all of m, g, h, w, θ, v and μ. [Hint: some people may find the answer surprising]

c) Given the other parameters, for some slopes θ the suitcase is slowing and for some slopes it is speeding. What is the minimum θ for which it is assured that the suitcase will speed up as it goes along no matter how big is the friction μ? Answer in terms of some or all of m, g, h and w. (It is possible to answer this without use of the answer to (a) above. No partial credit for correct algebra based on an incorrect answer to (a) above.)



FBD



$\hat{n} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

$\hat{\lambda} = \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

Aside: $F = \mu R_1$

$\tan\phi = \dfrac{\mu R_1}{R_1}$

$\tan\phi = \mu$

$\tan\phi = \dfrac{2w}{L} = \mu \longrightarrow L = \dfrac{2w}{\mu}$

AMB$_{/C}$ $\quad \sum \vec{M}_{/C} = \dot{\vec{H}}_{/C}$

$\vec{r}_{G/C} \times (-mg\hat{j}) = \vec{r}_{G/C} \times (ma\hat{\lambda})$

$\vec{r}_{G/C} = \left(\dfrac{2w}{\mu} + h\right)\hat{n} + (-w)\hat{\lambda}$

$$\left(\frac{2w}{\mu}+h\right)(-mg)(\hat{n}\times\hat{j})+(mgw)(\hat{\lambda}\times\hat{j})$$

$$=\left(\frac{2w}{\mu}+h\right)(ma)(\hat{n}\times\hat{\lambda})+(-maw)(\hat{\lambda}\times\hat{\lambda})$$

$$\left(\begin{array}{ll}\hat{n}\times\hat{j}=\sin\theta & \hat{\lambda}\times\hat{j}=\cos\theta \\ \hat{n}\times\hat{\lambda}=-1 & \hat{\lambda}\times\hat{\lambda}=0\end{array}\right)$$

$$-mg\left(\frac{2w}{\mu}+h\right)\sin\theta+mgw\overset{\cos\theta}{}=-ma\left(\frac{2w}{\mu}+h\right)+0$$

$$\boxed{a=g\left(\sin\theta-\frac{w\mu\cos\theta}{2w+\mu h}\right)} \quad \boxed{\vec{a}_G=a\hat{\lambda}}\ (a)$$

CHECKS

1. Units: accel = accel ✓
2. $g=0 \Rightarrow a=0$ ✓
3. $\mu=0 \Rightarrow a=g\sin\theta$ ✓
4. $h=0,\ \theta=0 \Rightarrow a=\mu/2$ ✓
5. $w\to 0 \Rightarrow a=g\sin\theta$ ✓

LMB $\quad \Sigma\vec{F}=\dot{\vec{L}}$

$$\left\{(-\mu R_1\hat{\lambda})+(R_1\hat{n})+(R_2\hat{n})+(-mg\hat{j})=(ma\hat{\lambda})\right\}$$

$$\{\}\cdot\hat{\lambda}:\ -\mu R_1+0+0-mg(-\sin\theta)=ma$$

$$R_1=\frac{m}{\mu}(g\sin\theta-a)$$ *Now use Sol'n

$$R_1=\frac{m}{\mu}\left(\frac{wg\mu\cos\theta}{2w+\mu h}\right)=\frac{mgw\cos\theta}{2w+\mu h}$$

Assume $\theta<\pi/2$
$\Rightarrow \cos\theta>0$

Tip Forwards $\longleftrightarrow (R_1<0)$

NEEDS TO BE TRUE

$$\frac{mgw}{2w+\mu h}>0 \quad \therefore \quad R_1>0 \quad \therefore$$

Always True

For all h,μ,θ, ect...

$\boxed{\text{Cannot Tip Forward}}$

Slowing Down $\longleftrightarrow (a<0)$

Speeding Up $\longleftrightarrow (a>0)$

$$a = g\left(\sin\theta - \frac{w\mu\cos\theta}{2w+\mu h}\right)$$

$$a(\theta) \equiv 0 \longrightarrow \sin\theta - \frac{w\mu\cos\theta}{2w+\mu h} = 0$$

$$\boxed{\theta_{crit} = \tan^{-1}\left(\frac{w\mu}{2w+\mu h}\right) \quad \text{if } \theta > \theta_{crit} \text{ then } a>0}$$

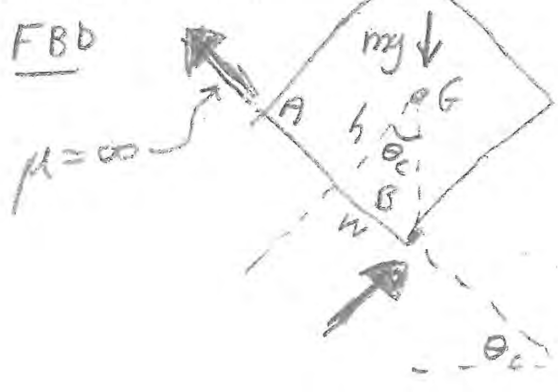
For all $\mu \Rightarrow$ worst case is $\mu \to \infty$

$$\Rightarrow \boxed{\boxed{\theta > \tan^{-1}\left(\frac{w}{h}\right)}} \text{(c)}$$

for $a>0$ guarantee

Alt. derivation of (c) w/out using (a). Worst case is $\mu = \infty$
$\Rightarrow R_1 = 0$. Critical case is $a=0 \Rightarrow$ statics. Solve statics problem:

FBD



$\sum \vec{M}_{/B} = \vec{0} \Rightarrow G$ directly above B

$\Rightarrow \tan\theta_c = w/h$

$$\Rightarrow \boxed{\theta_c = \tan^{-1}(w/h) \quad \text{accel} > 0 \text{ if } \theta > \theta_c}$$

Your TA, Section # and Section time:

"Solutions"

Your name:

RUINA

Cornell
TAM/ENGRD 2030

Prelim 3

April 16, 2013

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{n}} \ldots$) and signs ($\pm$) with sketches, equations or words.
- **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** The last page is blank for your use. Ask for more extra paper if you need it. Put your name on each extra sheet, fold it in, and refer to back pages or extra sheets on the page of the relevant problem.

Problem 7: ____ /25

Problem 8: ____ /25

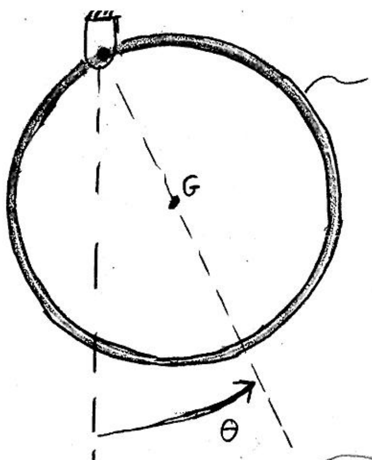
Problem 9: ____ /25

1) A circular hoop swings as a pendulum from a hinge at a point on its edge. Answer in terms of some or all of m, R and g.

a) What is the period of small oscillation near to hanging straight down?

b) If it was launched from the straight down position, what is the minimum launch speed needed by the center of mass in order to swing the hoop over the top?

c) In terms of some or all of m, R and g, what is the length ℓ of a simple pendulum (ie, point mass m and string with the same gravity g) that has the same period of small oscillation as does this hoop?



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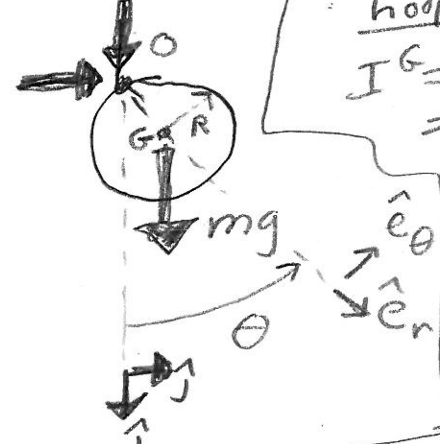


hoop

$I^G = \int r^2\,dm$
$= mR^2$

AMB$_{/O}$: $\sum \vec{M}_{/O} = \dot{\vec{H}}_{/O}$

$$\{-mg\sin\theta R\hat{k} = R\hat{e}_r \times m\vec{a}_G + I^G\ddot{\theta}\hat{k}\}$$

$$\vec{a}_G = R\ddot{\theta}\hat{e}_\theta - R\dot{\theta}^2\hat{e}_r$$

$$\{\}\cdot\hat{k} \Rightarrow -mgR\sin\theta = (I^G + R^2m)\ddot{\theta}$$

$$-\cancel{m}g\cancel{R}\sin\theta = 2\cancel{m}R^{\cancel{2}}\ddot{\theta}$$

$\theta \ll 1 \Rightarrow$

$$\ddot{\theta} = \frac{-g}{2R}\theta \quad (*)\ \text{(ODE)}$$

$\theta =$, e.g.

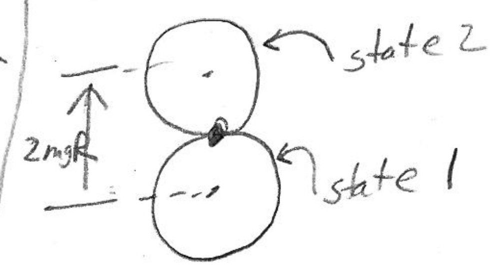
$$\theta = \theta_0\cos\sqrt{\frac{g}{2R}}\,t \quad \text{(one soln)}$$

one period $\equiv t^*$:

$$\sqrt{\frac{g}{2R}}\,t^* = 2\pi$$

$$\text{(a)} \quad \boxed{t^* = 2\pi\sqrt{2R/g}}$$

Energy



$$E_{k_1} + E_{p_1} = E_{k_2} + E_{p_2}$$

$$\Delta E_k = -\Delta E_p$$

$$-\frac{mv_1^2}{2} - \frac{I^G\omega_1^2}{2} = -2mgR \qquad \omega_1 = v_1/R$$

$$-\frac{v_1^2}{2}\left(m + \frac{I^G}{R^2}\right) = -2mgR$$

$$\frac{\cancel{m}v_1^2}{\cancel{2}}(\cancel{2}\cancel{m}) = 2\cancel{m}gR$$

$$v_1^2 = 2gR$$

$$\boxed{v_1 = +\sqrt{2gR}} \text{ (b)}$$

Simple pendulum:

$$\ddot{\theta}_p = \frac{-g}{L}\theta_p$$

as * above

to have same soln $\frac{g}{L} = \frac{g}{2R}$

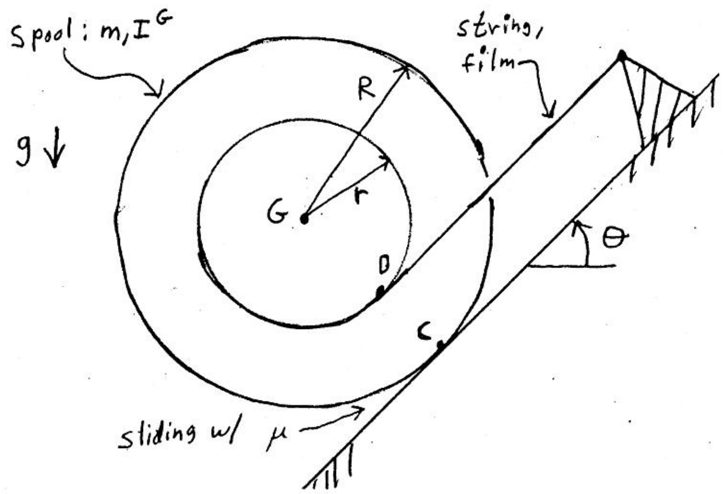
$$\Rightarrow \boxed{L = 2R} \text{ (c)}$$

2) A spool, like the movie spool in lecture, is progressing down a slope. The inextensible film is held firmly at one end and unwinds from the spool. The friction between the spool and the ground is low enough so that the spool slides on the surface. You are given the spool outer radius R, the film radius r, the spool inertia about its COM I^G, the spool mass m, the slope θ, the friction coefficient μ, the gravity constant g and the present speed v_G of the spool down the slope.

Hint: it might help you to picture the motion by first imagining that there is no friction. Then put in the friction.

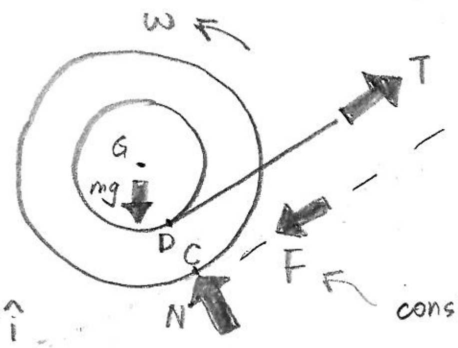
a) Find the normal component of the force of the ground on the spool.

b) Find any one of these quantities: The acceleration a_G of the spool center down the slope, the angular acceleration α of the spool, or the tension in the film/string. If you find more than one quantity, *clearly label the one you want graded.*



2) Solution:

FBD



consistant with $V_G > 0$ & $V_C < 0$

$\underbrace{V_C < 0}$ bottom of spool is sliding up hill

$\hat{j}$, $\hat{i}$, $\hat{n}$, $\hat{\lambda}$ $\qquad \hat{k} \times \hat{n} = \hat{\lambda}$

Geometry

$\vec{V}_G = V_G \hat{\lambda}, \quad \vec{a}_G = a_G \hat{\lambda}$

$\vec{0} = \vec{V}_D = \vec{V}_G + \vec{V}_{D/G} = \vec{V}_G + \omega \hat{k} \times (-r\hat{n})$

$= (V_G - \omega r)\hat{\lambda}$

$\{ \vec{0} = (V_G - \omega r)\hat{\lambda} \}$

$\{ \} \cdot \hat{\lambda} \Rightarrow \boxed{V_G = \omega r}$

$\boxed{a_G = \dot{\omega} r}$ (*)

$\vec{V}_C = \vec{V}_D + \vec{V}_{C/D}$

$= \vec{0} + \omega \hat{k} \times [(R-r)(-\hat{n})]$

$\vec{V}_C = -\omega(R-r)\hat{\lambda}$

$\omega = V_G / r$

if $V_G > 0 \Rightarrow V_C < 0$

$\Rightarrow$ slides uphill

$\Rightarrow$ friction points down

LMB $\cdot \hat{n} \Rightarrow N - mg\cos\theta = 0$

$\Rightarrow \boxed{N = mg\cos\theta}$ (a)

LMB $\cdot \hat{\lambda} \Rightarrow F - T + mg\sin\theta = m a_G$ (**)

$F = \mu N = \mu mg\cos\theta$

AMB/D: $\sum \vec{M}_{/D} = \dot{\vec{H}}_{/D}$

$\{ [mgr\sin\theta - (R-r)F]\hat{k} = \vec{r}_{G/D} \times m\vec{a}_G + I^G \dot{\omega} \hat{k}$

$= (mr a_G + I^G \dot{\omega})\hat{k}$

$= (mr a_G + \frac{I^G}{r} a_G)\hat{k} \}$

$\{ \} \cdot \hat{k} \Rightarrow mgr\sin\theta - (R-r)F = (mr + \frac{I^G}{r}) a_G$

$\Rightarrow \boxed{a_G = g \dfrac{\sin\theta - \mu\cos\theta (R-r)/r}{I^G/mr^2 + 1}}$ (b)

$* \Rightarrow \boxed{\dot{\omega} = \dfrac{a_G}{r} = \dfrac{g}{r} \dfrac{\sin\theta - \mu\cos\theta (R-r)/r}{I^G/mr^2 + 1}}$ (b)

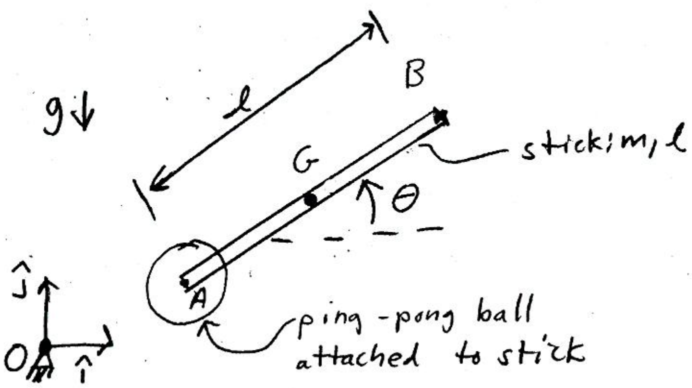
$** \Rightarrow T = F + mg\sin\theta - m a_G$

$\boxed{T = mg\left[\mu\cos\theta + \sin\theta - \dfrac{\sin\theta - \mu\cos\theta (R-r)/r}{I^G/mr^2 + 1}\right]}$ (b)

3) Equations of motion for a simple model of an arrow. 2D. A uniform rigid stick (length ℓ, mass m) flies through the air. As it flies, the line segment AB makes an angle $\theta(t)$ with the positive horizontal x axis. Attached to one end, labeled A, is a negligible-mass ping-pong ball that has air friction. The friction is modeled as linear: the ping-pong ball drag-force resists motion of point A with magnitude $F_D = cv_A$. The air friction on the rest of the stick is negligible. The center of mass G is at the center of the stick. You are given $\theta, \dot{\theta}, \vec{r}_G, \vec{v}_G, m, \ell, c, d$ and g.

a) Find $\ddot{\theta}$ and $\vec{a}_G$ (a vector expression for $\vec{a}_G$ without explicit components is fine, but scalar components are also fine). You are given $\theta, \dot{\theta}, \vec{r}_G, \vec{v}_G, m, \ell, c$ and g.

b) Harder. If, instead of a ping-pong ball at A there were feathers. And, more like a real arrow, these feathers had no resistance to the motion of A in the AB direction. But the feathers resisted motion perpendicular to AB with a force $F_L^{\perp} = dv^{\perp}$, where $v^{\perp}$ is the component of $\vec{v}_A$ orthogonal to the line AB[1]. You are given $\theta, \dot{\theta}, \vec{r}_G, \vec{v}_G, m, \ell, d$ and g. Find $\ddot{\theta}$.



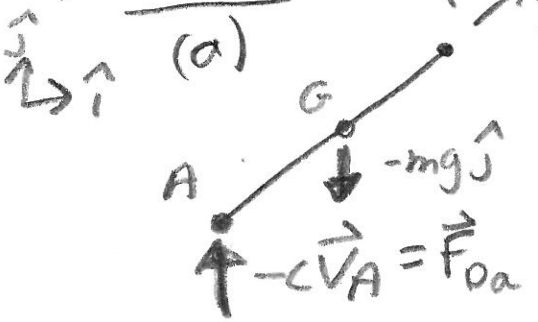
[1]Aside: A more realistic model of an arrow would have that 'lift' force depend quadratically on the forward speed. More realistic still would be to have a lift and drag model for the feathers, this would be a model more like that used for an airplane wing.

3) FBDs

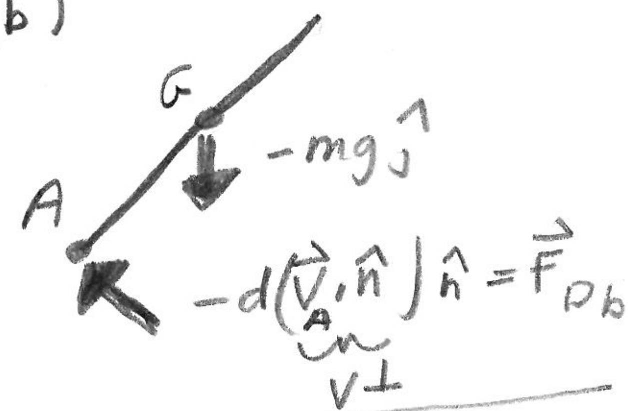
(a)

$\hat{n}$ $\hat{\lambda} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\hat{n} = \hat{k} \times \hat{\lambda}$



(b)



Kinematics

$$\vec{V}_A = \underbrace{\vec{V}_G}_{\text{given}} + \underbrace{\vec{\omega}}_{\dot{\theta}\hat{k}} \times \overbrace{\vec{r}_{A/G}}^{-\hat{\lambda}\,\ell/2}$$

$$I^G = \int r^2 dm = \int_{-\ell/2}^{\ell/2} s^2 \underbrace{\rho}_{m/\ell} ds$$

$$I^G = m\ell^2/12$$

LMB !

$$\sum \vec{F} = m\vec{a}_G$$

$$-mg\hat{j} - c\vec{V}_A = m\vec{a}_G$$

$$\boxed{\vec{a}_G = \frac{-c}{m}\vec{V}_A - g\hat{j}}$$

(b) AMB$/_G$

$$\sum \vec{M}_{/G} = \dot{\vec{H}}_{/G}$$

$$\vec{r}_{A/G} \times \vec{F}_{D_b} = I^G \ddot{\theta}\hat{k}$$

$$\boxed{\ddot{\theta} = \frac{\vec{r}_{A/G} \times \vec{F}_{D_b} \cdot \hat{k}}{I^G}} \quad (b)$$

where all quantities are calculated above in terms of given quantities

AMB$/_G$ $\quad \sum \vec{M}_{/G} = \dot{\vec{H}}_{/G}$

$$\vec{r}_{A/G} \times \vec{F}_{D_a} = \underline{I}^G \ddot{\theta}\hat{k}$$

see above

$$\boxed{\ddot{\theta} = \frac{\vec{r}_{A/G} \times \vec{F}_{D_a}}{I^G} \cdot \hat{k}}$$

see above

see above

Your TA, Section # and Section time:

Your name:

Cornell TAM/ENGRD 2030

Makeup Prelim

May 4, 2013

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

How to get the highest score?

Please do these things:

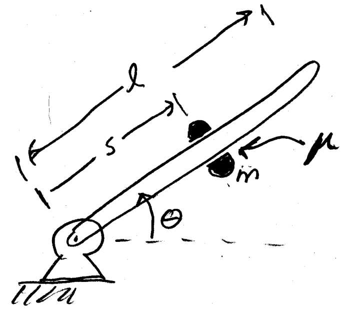
- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- ☐ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** Ask for more extra paper if you need it. Put your name on each extra sheet.

Problem 10: ____ /25

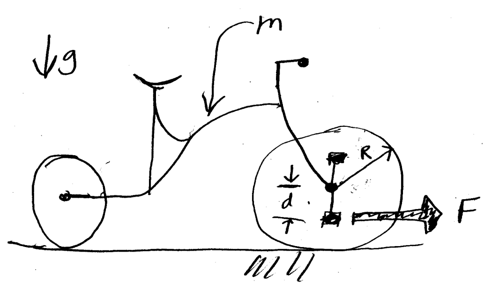
Problem 11: ____ /25

Problem 12: ____ /25

10) 2D. No gravity. A bead m slides with friction coefficient μ on a rigid straight rod with length ℓ that is rotated by a motor. At the instant of interest the angle of the rod is θ, the rotation rate is $\dot{\theta}$ and the angular acceleration is $\ddot{\theta}$. The bead is a distance s from the motor axle and has rate of sliding $\dot{s} > 0$. In terms of some or all of $\mu, \theta\dot{\theta}, \ddot{\theta}, \ell, s$ and $\dot{s}$, find $\ddot{s}$.



11) 2D. A tricycle has weight mg and wheels with negligible mass. The steering is locked straight forwards. Assume the friction μ is big enough so that the wheels roll without slip. The front wheel has radius R and the front crank has length $d < R$. A forwards force $F > 0$ is applied to the bottom pedal from a person standing at the side. In terms of some or all of m, g, R, d, F, μ and g, which direction does the tricycle accelerate (right or left) and with what acceleration?



12) Write all of the Matlab commands to solve the following problem using ODE23 or ODE45. The result should be printed by Matlab in the command window.

The equation of a damped simple pendulum is $\ddot{\theta} = -\frac{g}{\ell}\sin\theta - c\dot{\theta}$.

Find the angle θ at $t = t_f$.

Use any non-zero values you like for g, ℓ, c and t_f and for the initial conditions.

Your TA, Section # and Section time:

Your name:

Cornell
TAM/ENGRD 2030

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

2nd Makeup Prelim

May 17, 2013

How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- ◻ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \dots$), coordinates ($x, y, r, \theta \dots$), variables ($v, m, t, \dots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \dots$) and signs ($\pm$) with sketches, equations or words.
- **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** Ask for more extra paper if you need it. Put your name on each extra sheet.

Problem 10: ______ /25

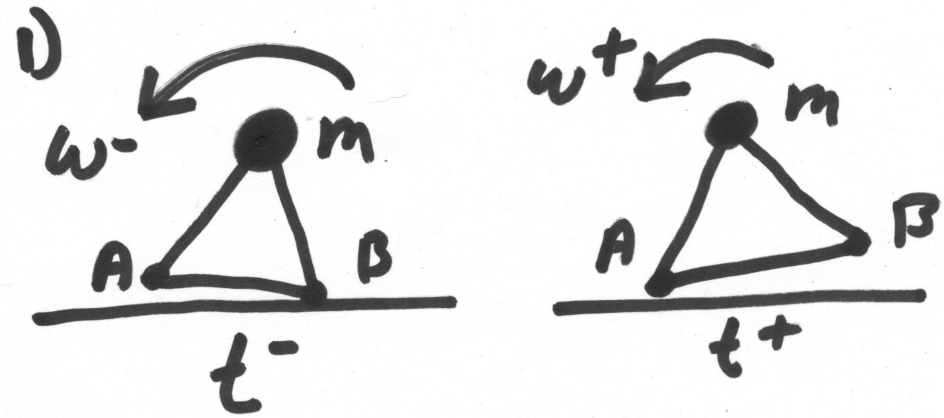
Problem 11: ______ /25

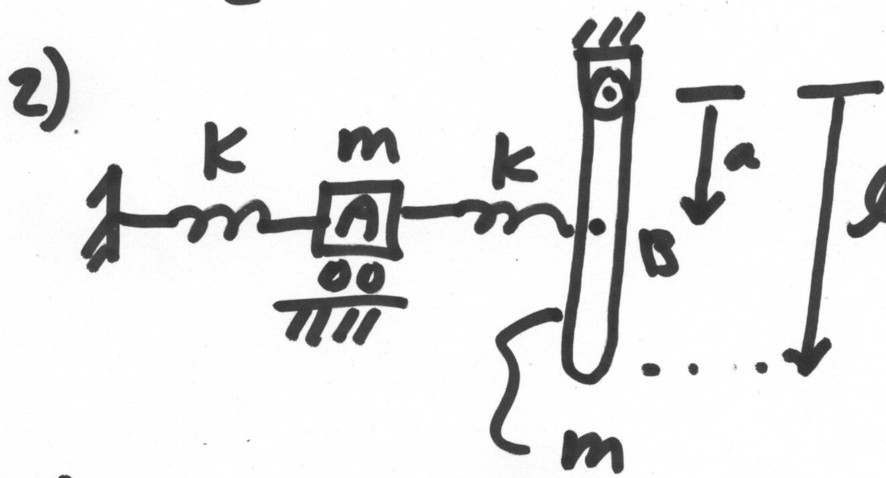
Problem 12: ______ /25

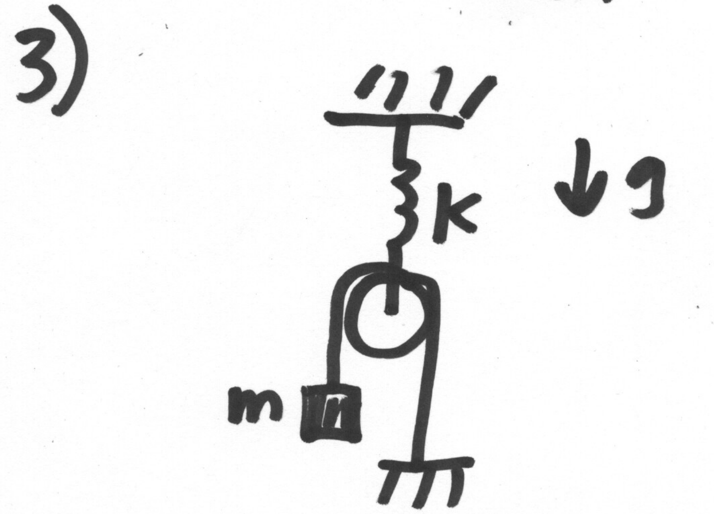
1=10) 2D. A rigid massless equilateral triangle (all lengths ℓ and angles are the same) has a point mass m at it's top vertex. It pivots on corner B and falls to the left. Just before corner A collides it has known angular velocity $\vec{\omega} = \omega^{-}\hat{\boldsymbol{k}}$. Then it has a sticking plastic no-slip collision at A and starts to rock back up pivoting about A. What is the angular velocity just after the collision? Answer in terms of some or all of m, ℓ, g and ω^{-}.

2=11) 2D. Do not neglect gravity. Find the equations of motion for the system shown. Assume the angle of swing of the rigid uniform-mass pendulum is small enough so that the tip of right spring is negligible and so that $\sin\theta \ll 1$, where θ is the angle of swing of the pendulum. Do not try to solve the equations.

3=12) 1D motion. Find the frequency of small vibration of the system shown. Assume the pulley has negligible mass and is otherwise ideal.







Your TA, Section # and Section time:

"SOLUTIONS"

Your name:

ANDY RUINA

Cornell TAM/ENGRD 2030

Final exam

May 10, 2013

No calculators, books or notes allowed.

5 Problems, 150 minutes (+no extra time: University policy $\Rightarrow$ *budget your time!*)

How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- □ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- $\rightarrow\!|$ **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- $\approx$ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** The last page is blank for your use. Ask for more extra paper if you need it. Put your name on each extra sheet, fold it in, and refer to back pages or extra sheets on the page of the relevant problem.

Problem 13: ____ /25

Problem 14: ____ /25

Problem 15: ____ /25

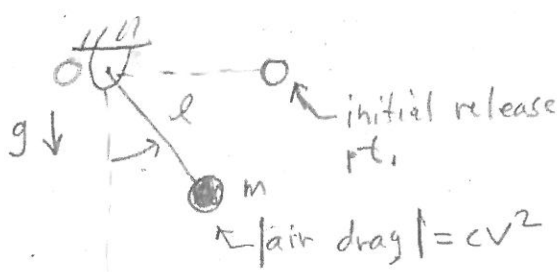
Problem 16: ____ /25

Problem 17: ____ /25

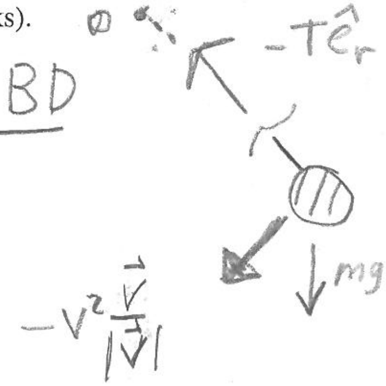
13) 2D, with gravity g. A mass m is attached to the end of a negligible-mass rigid rod with a length ℓ. The other end of the rod is attached to a hinge with negligible friction. The mass is slowed by air friction which resists motion with a force with magnitude $|F| = cv^2$ where $v = |\vec{v}|$ is the speed of the mass. The pendulum is released from rest at time $t = 0$ with the rod horizontal and to the right of the hinge. Assume any non-zero positive values that please you for all parameters (*e.g.*, g, m, ℓ, c, and t_1). Do *not* attempt an analytic solution.

a) Using ODE23 or ODE45 write all the Matlab commands needed to find the speed of the mass at time t_1. Some partial credit if you never learned ODE23 or ODE45 and can write your own ODE solver.

b) The output to the command window should be 'At time ____ the speed is ____.' (with numbers, calculated by the computer, instead of blanks).



FBD



Drag Force

$$\vec{F}_D = -V^2 \frac{\vec{V}}{|\vec{V}|} c$$

$$\vec{F}_D = -c(\ell\dot{\theta})^2 \frac{\ell\dot{\theta}\hat{e}_\theta}{+\sqrt{(\ell\dot{\theta})^2}}$$

$$\boxed{\vec{F}_D = -c\ell^2\dot{\theta}|\dot{\theta}|\hat{e}_\theta}$$

AMB$_{/O}$:

$$\sum \vec{M}_{/O} = \dot{\vec{H}}_{/O}$$

$$\Rightarrow \ell\hat{e}_r \times (-mg\hat{j} - \ell^2\dot{\theta}|\dot{\theta}|\hat{e}_\theta)$$

$$= \ell\hat{e}_r \times [-\ell\dot{\theta}^2\hat{e}_r + \ell\ddot{\theta}\hat{e}_\theta]m$$

$$\Rightarrow -\ell mg\sin\theta\hat{k} - c\ell^3\dot{\theta}|\dot{\theta}|\hat{k} = \ell^2 m\ddot{\theta}\hat{k}$$

$$\{\}\cdot\hat{k} \Rightarrow \boxed{\ddot{\theta} = \underbrace{\frac{-g}{\ell}\sin\theta - \frac{c\ell}{m}\dot{\theta}|\dot{\theta}|}_{*}}$$

$$\boxed{|\vec{V}| = |\dot{\theta}|\,\ell}$$

$$\omega \equiv \dot{\theta} \Rightarrow \boxed{\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= *\end{aligned}}$$

```
function   givemeahighgrade( )
p.L=1;  p.c=1; p.m=1; p.g=1;  t1=10;
tspan= [0 t1];
theta0 = pi/2; omega0 = 0;
z0= [theta0   omega0]';

[t zarray]= ode45(@myrhs, tspan, z0,[], p);

vend= abs(p.L * zarray(end, 2));

disp(['At time ' num2str(t1) ...
     ' the speed is '  num2str(vend)])

end


function  zdot = myrhs(t, z, p)
theta= z(1);  omega= z(2)

thetadot= omega;

omegadot= -p.c*(p.L/p.m) * omega * abs(omega) ...
           - (p.g/p.L) sin(theta);

zdot = [thetadot ; omegadot];

end
```

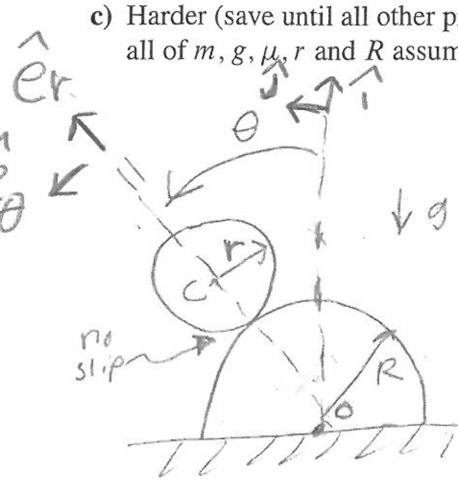
the heart of the matter

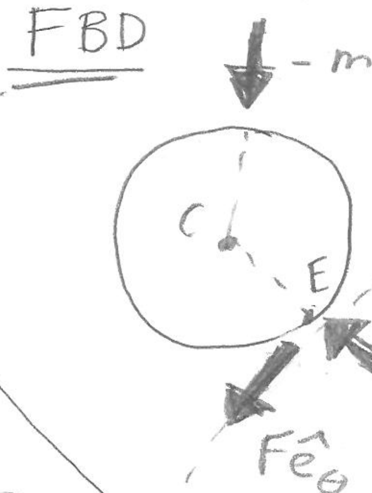
14) 2D, with gravity g. A solid uniform disk with radius r and mass m rolls on the top of a rigid unmoving hollow pipe with radius R. Line OC, between the center of the pipe and the center of the disk makes an angle of θ CCW (counterclockwise) from straight up. Assume θ and $\dot{\theta}$ are small enough, and μ big enough, so there is no separation or slip.

a) Find the equations of motion (That is, find $\ddot{\theta}$ in terms of some or all of $\theta, \dot{\theta}, m, g, r, \mu$ and R).

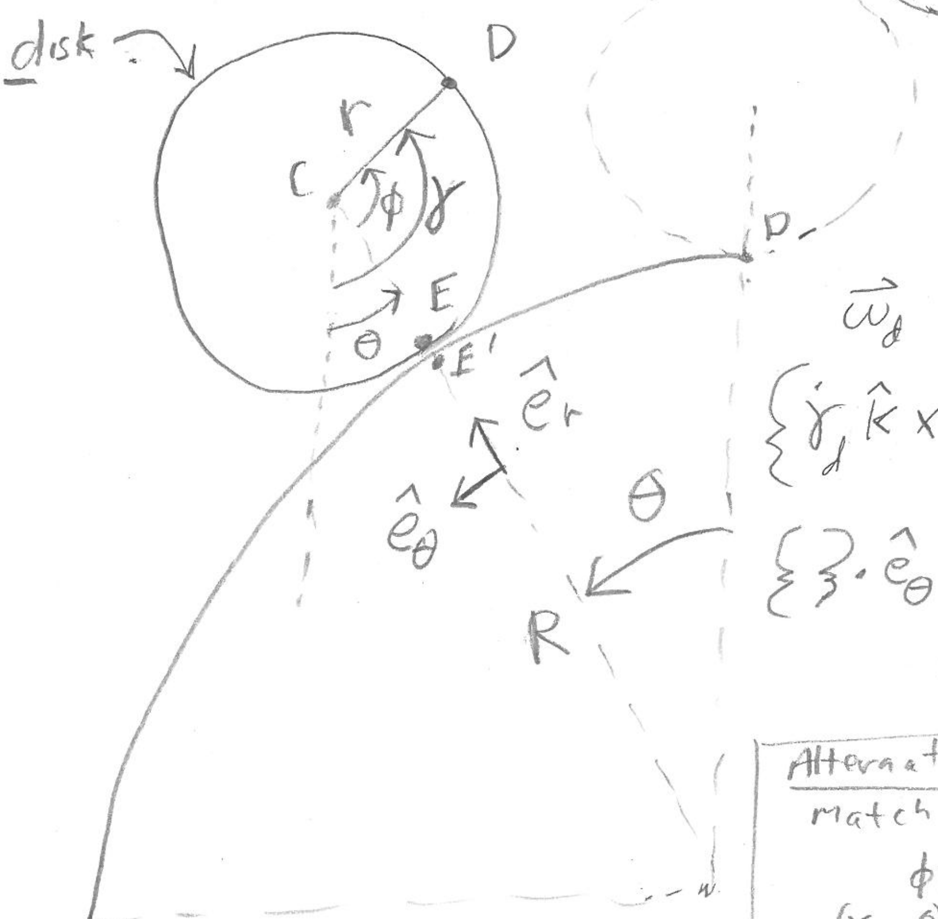
b) Find a function f so that the equation $0 = f(\theta, \dot{\theta}, m, g, \mu, r, R)$ describes the condition when the wheel would first lose contact.

c) Harder (save until all other problems are done). Find the angle θ when contact is first lost in terms of some or all of m, g, μ, r and R assuming that the rolling starts from rest at $\theta_0 = 0^+$ and all rolling is without slip.





Kinematics



$$\boxed{\vec{\omega}_d = \dot{\gamma}\hat{k}}$$

$$\vec{V}_E = \vec{V}_{E'}$$

$$\vec{\omega}_d \times \vec{r}_{E/C} + \vec{V}_C = \vec{0}$$

$$\left\{\dot{\gamma}_d \hat{k} \times (-r\hat{e}_r) + (R+r)\dot{\theta}\hat{e}_\theta = \vec{0}\right\}$$

$$\{\ \}\cdot\hat{e}_\theta \Rightarrow -\dot{\gamma}r + (R+r)\dot{\theta} = 0$$

$$\Rightarrow \boxed{\dot{\gamma} = \frac{R+r}{r}\dot{\theta}} \ *$$

Alternative Kinematics

match arc lengths:

$$\phi r = \theta R$$
$$(\gamma - \theta)r = \theta R$$
$$\gamma r - \theta r = \theta R$$
$$\rightarrow \gamma r = (r+R)\theta$$
$$\boxed{\gamma = \frac{R+r}{r}\theta} \ *$$

(again)

(14 cont'd)

AMB$_{/E}$:

$$\sum \vec{M}_{/E} = \dot{\vec{H}}_{/E}$$

$$\vec{r}_{c/E} \times (-mg\hat{\imath}) = \underbrace{\vec{r}_{c/E}}_{r\hat{e}_r} \times m\vec{a}_c + I\underbrace{\dot{\omega}_d}_{=\frac{R+r}{r}\ddot{\theta}} \hat{k}$$

from *

$$\vec{a}_c = -\dot{\theta}^2(R+r)\hat{e}_r + \ddot{\theta}(R+r)\hat{e}_\theta$$

$$\left\{ mgr\sin\theta\,\hat{k} = \left[mr(R+r)\ddot{\theta} + I\left(\frac{R+r}{r}\right)\ddot{\theta}\right]\hat{k} \right\}$$

$$\{\ \}\cdot\hat{k} \Rightarrow \quad \ddot{\theta} = \frac{mgr\sin\theta}{mr(R+r) + \underbrace{I}_{I = mr^2/2}\frac{R+r}{r}}$$

uniform disk

$$= \frac{g\sin\theta}{(R+r) + (R+r)/2}$$

$$\boxed{\ddot{\theta} = \frac{2g}{3(R+r)}\sin\theta} \quad \boxed{a}$$

LMB $\left\{\sum \vec{F} = \dot{\vec{L}}\right\}\cdot\hat{e}_r$

$$\Rightarrow \quad N = -(R+r)\dot{\theta}^2 m + mg\cos\theta$$

Liftoff $\Rightarrow$ $N=0$

$$\Rightarrow \boxed{b} \quad \boxed{\cos\theta = \frac{(R+r)\dot{\theta}^2}{g}}$$

c) Cons. of Energy

$$E_0 = E_1$$

$$(R+r)\cancel{m}g = (R+r)\cancel{m}g\cos\theta + m(R+r)^2\dot{\theta}^2/2 + \underbrace{I}_{\frac{1}{2}\cancel{m}r^2}\left(\frac{R+r}{r}\dot{\theta}\right)^2/2$$

$$\Rightarrow (R+r)g(1-\cos\theta) = \left[\frac{(R+r)^2}{2} + \frac{(R+r)^2}{4}\right]\dot{\theta}^2$$

$$\Rightarrow \dot{\theta}^2 = \frac{4}{3}\,\frac{(1-\cos\theta)g}{R+r} \quad (**)$$

apply ** to b $\Rightarrow$

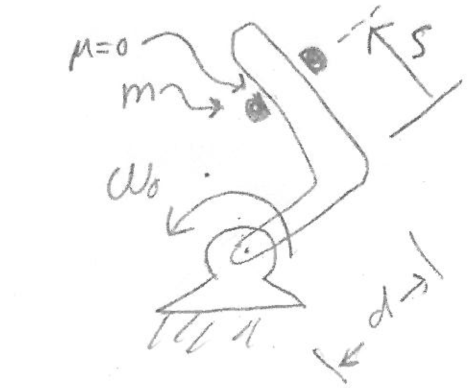
$$\cos\theta = \frac{4}{3}(1-\cos\theta)$$

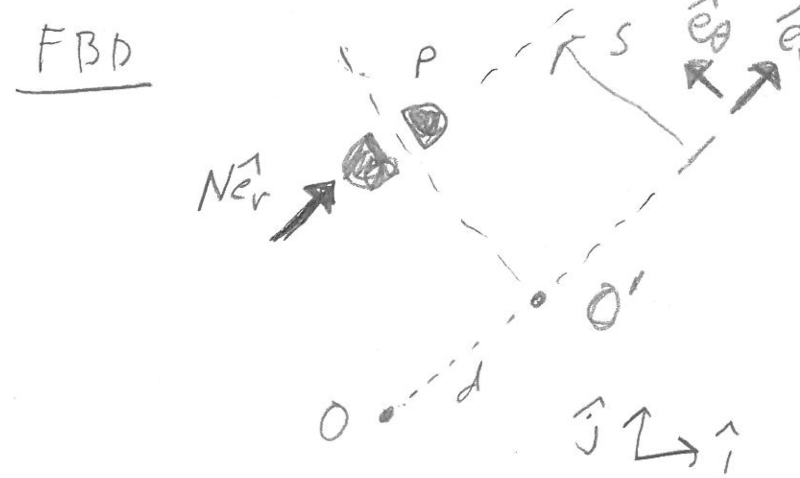
$$\Rightarrow \frac{7}{3}\cos\theta = \frac{4}{3} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{4}{7}\right)} \quad \boxed{c}$$

15) 2D, no gravity. A bead with mass m slides on an L-shaped (right angle at bend) frictionless rigid rod which is turned by a motor at constant angular velocity $\vec{\omega}_0 = \omega_0 \hat{k}$. The bead only moves on the straight part of the bar marked by s. If s passes through zero in your solution, only consider until $s = 0$.

[redacted] **a)** Find the equations of motion of the bead. That is, find $\ddot{s}$ in terms of some or all of $\omega_0, s, \dot{s}, m$ and d.

[redacted] **b)** Given that $s(0) = s_0 > 0$ and $\dot{s}(0) = 0$, find s in terms of some or all of t, ω_0, m, s_0 and d. (No Matlab).





Kinematics: $\vec{a}_P = \vec{a}_{O'} + \vec{a}_{P/O'}$

$$= \underbrace{-\omega_0^2 d\hat{e}_r + (\ddot{s} - s\omega_0^2)\hat{e}_\theta + (0 + 2\dot{s}\omega_0)\hat{e}_r}$$

LMB: $\sum \vec{F} = m\vec{a}$

$$\{N\hat{e}_r = m[\quad]\}$$

$$\{\}\cdot\hat{e}_\theta \Rightarrow \quad 0 = \ddot{s} - s\omega_0^2 \Rightarrow \boxed{\ddot{s} = s\omega_0^2} \text{ (a)}$$

ODE soln is: $s = A\cosh(\omega_0 t) + B\sinh(\omega_0 t)$

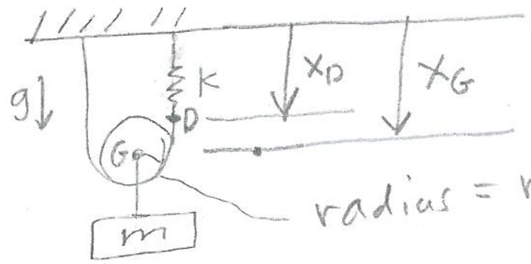
$$\dot{s}(0) = 0, \quad s(0) = s_0 \Rightarrow \boxed{s = s_0\cosh(\omega_0 t)} \text{ (b)}$$

o

16) 1D with gravity g. A mass m hangs from an ideal round negligible-mass frictionless pulley, an inextinsible strings, and spring k as shown. Give all answers in terms of some or all of m, g and k. As for all problems, clearly define any other variables you may use in your solution.

a) At equilibrium how much lower is the pulley than when there is no mass (but the string and pulley are not slack)?

b) What is the frequency of small oscillation (so small that the strings do not go slack)? You can find ω or f, as you please.



String length = L_S
Spring rest length = L_0
Spring stretched length = X_D
Spring stretch = $X_D - L_0 = \Delta\ell$

Before stretch: $2X_{G0} + \pi r = L_S + L_0$ (1)

After stretch: $2X_G + \pi r = L_S + X_D$ (2)

Subtract (2) − (1) ⇒ $\boxed{2\Delta X_G = \underbrace{X_D - L_0}_{\Delta\ell}}$

FBD

$T \uparrow \ \uparrow T = k(\Delta\ell)$

LMB $\quad \sum \vec{F} = m\vec{a}$

$mg - 2T = m\ddot{X}_G$

$T = k\Delta\ell = 2k\,\Delta X_G$

$\ddot{X}_G = g - \dfrac{4k(\Delta X_G)}{m}$ (1)

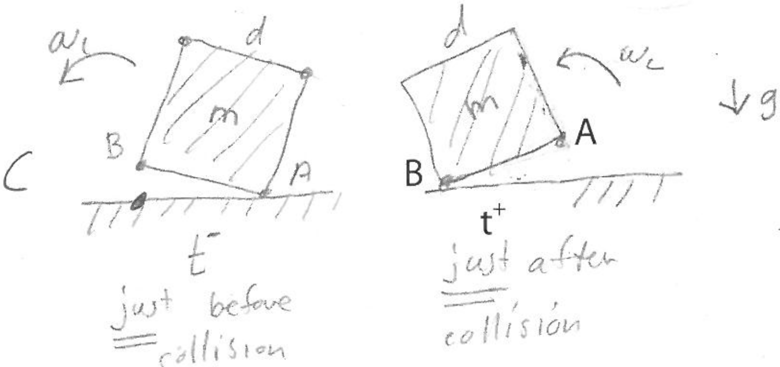
equilib stretch ⇒ $\ddot{X}_G = 0$ ⇒ $\boxed{\Delta X_G = \dfrac{mg}{4k}}$ (a)

(1) ⇒ $m\ddot{X}_G + 4kX_G = 4kX_{G0}$

⇒ $(X_G - X_{G0}) = A\sin\sqrt{\frac{4k}{m}}\,t + B\sin\sqrt{4k/m}\,t$

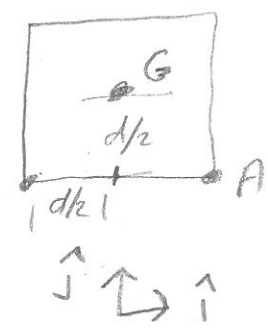
⇒ $\boxed{\omega = 2\sqrt{\frac{k}{m}} \qquad f = \frac{1}{\pi}\sqrt{\frac{k}{m}}}$ b

17) 2D, with gravity g. A uniform cube with mass m and side d rocks on edge A and tips until it has a collision with edge B. Then edge A breaks free, and then the cube rocks about edge/hinge B. *Just* before the collision, at $t = t^-$, the angular velocity of the cube is known to be $\omega_1 \hat{k}$. Just before and after the collision the tip angles are negligibly small. What is the angular velocity $\omega_2 \hat{k}$ *just* after the collision at $t = t^+$? Answer in terms of some or all of m, g, d and ω_1.



collisional FBD

geometry



$|\vec{r}_{G/C}| = \frac{\sqrt{2}}{2} d$

AMB$/_C$

$$\vec{H}_C^- = \vec{H}_C^+$$

$$\underbrace{\vec{r}_{G/C} \times m\vec{v}^-}_{= \vec{0} \text{ because } \vec{v}^- \text{ is } \parallel \text{ to } \vec{r}_{G/C}} + I\omega^- \hat{k} = \vec{r}_{G/C} \times m\vec{v}^+ + I\omega^+ \hat{k}$$

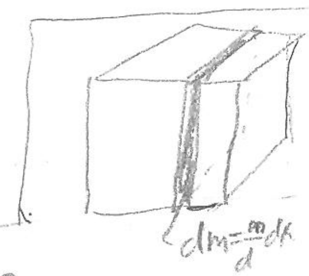
$\vec{v}^-$: $\vec{\omega}^- \times \vec{r}_{G/A}$ $\qquad$ $\vec{v}^+$: $\vec{\omega}^+ \times \vec{r}_{G/C}$

$$\Rightarrow \left\{ I\omega^- \hat{k} = |\vec{r}_{G/C}|^2 m\omega^+ \hat{k} + I\omega^+ \hat{k} \right\}$$

$$\{\ \} \cdot \hat{k} \Rightarrow \quad \omega^+ = \frac{I}{I + md^2/2} \omega^-$$

$$\omega^+ = \frac{(1/6)md^2}{(1/6)md^2 + 1/2 md^2} \omega^-$$

$$\boxed{\omega^+ = \frac{1}{4}\omega^-}$$



for cube

$I^G = \frac{1}{6} md^2$

Check: $\int r^2 dm = \int x^2 + y^2 \, dm$

$= 2\int x^2 dm$

$= 2 \int_{-d/2}^{d/2} x^2 \overbrace{\left(\frac{m}{d}\right) dx}^{dm}$

$= 2 \frac{x^3}{3}\Big|_{-d/2}^{d/2} \frac{m}{d}$

$= md^2/6$