INTRODUCTION
In this lab we look at the kinematics of some mechanisms which convert rotary motion into oscillating linear motion and vice-versa. In *kinematics* we use geometry and calculus to study motion without thinking about the forces which cause it. Mostly we will look at the *slider-crank*. A slider-crank mechanism is often used in engines to convert the linear thrust of pistons into the useful rotary motion of the drive-shaft. You will look at a lawn-mower engine and at an adjustable slider-crank demonstration. You can also recall the rotary-to-straight line mechanisms used in the spring-mass and normal-modes experiments.

Who cares? One application is for finding forces. Once an object’s acceleration, \( \mathbf{a} \), has been determined through kinematics, linear momentum balance allows us to find the net force acting on the object (\( \mathbf{F} = m\mathbf{a} \)).

PRELAB QUESTIONS
Read through the laboratory instructions and then answer the following questions:

1. What data will you collect from the lawn-mower engine and what will you simulate on the computer?
2. Which parameter(s) can be adjusted on the adjustable slider-crank? Which are fixed?
3. Derive an equation relating the piston displacement \( x \) to the crankshaft speed, \( \omega \), time, \( t \), connecting rod length, \( L \), and crank radius \( R \). Do not assume \( L \gg R \).

SLIDER-CRANK KINEMATICS & INTERNAL COMBUSTION ENGINES
Figure 1.2 shows a sketch of the slider-crank mechanism. The point \( A \) is on the piston, line \( AB \) (with length \( L \)) is the *connecting rod*, line \( BC \) (with length \( R \)) is the crank, and point \( C \) is on the *crankshaft*. In an engine, a mixture of gasoline and air in the cylinder is ignited in an exothermic (heat producing) reaction. As a result, the pressure in the cylinder rises, forcing the piston out. The force transmitted through the connecting rod has a moment about the center of the crankshaft, causing the shaft to rotate. An exhaust valve releases the gas pressure once the piston is extended. Inertia of machinery (often a flywheel) connected to the crankshaft (as well as forcing from other pistons in multi-cylinder engines) forces the piston back up the cylinder. In a standard “four-cycle” engine the crankshaft makes another full revolution before the next ignition (to bring in fresh air and then compress it before ignition).

The *kinematic constraint* comes about from the geometry of the engine. If we know the angle through which the crank arm has rotated, \( \theta = \omega t \), then we can determine the piston
displacement, $x$. The crank arm $R$, connecting rod $L$, and piston displacement $x$, form a triangle containing the angle $\theta = \omega t$. Using this triangle we can find our kinematic constraint $x = f(\theta(t); R, L)$ with basic geometry. We can then take derivatives to find a relationship between the piston’s velocity or acceleration and the crankshaft’s spin rate $\omega$ and engine geometry $R, L$.

Notice that if the connecting rod $L$ is much longer then the crank arm $R$ then it will remain close to parallel to the $x$-direction throughout the stroke. The $x$ displacement of the piston will then be the $x$-displacement of the end of the crank arm plus a fixed length $L$. The former we know from circular motion, giving us a simple kinematic constraint:

$$x(t) \simeq R\cos(\omega t) + L; \quad (L >> R) \quad (1.1a)$$

$$v(t) \simeq -R\omega\sin(\omega t); \quad (L >> R) \quad (1.1b)$$

$$a(t) \simeq -R\omega^2\cos(\omega t); \quad (L >> R) \quad (1.1c)$$

If we do not assume $L >> R$ then the kinematic constraint is a little bit more complicated. You will work this out in the Pre-Lab.

In this experiment the crankshaft is driven by an electric motor at a more or less constant rate, $\omega$. This in turn, drives the piston. The same motion results as when the combustion process takes place in the piston which pushes the crankshaft at a more or less constant rate. As the crankshaft rotates the piston moves in the positive and negative $x$ direction. The
Figure 1.2: A diagram of the slider-crank system. In this cartoon the piston moves sideways. In the experimental setup the piston moves vertically.

Figure 1.3: A hand turned and adjustable slider-crank system. Turning the crank at the bottom is analogous to the turning of the crank-shaft in an engine. The knob at the lower left of the right photo turns a screw which adjusts the length of the crank. Looking down, the intersection of the long sliding rod (the engine cylinder) and the adjustable screw is the location of the center of the crank shaft.

Basic measurements in this lab are the position and velocity of the piston in the $x$ direction (which happens to be vertical in the laboratory). These measurements can be compared to those calculated by hand (if you are energetic) or to the results of a computer simulation. The simulation and the adjustable crank will allow you to see some of the effects of varying the ratio of connecting rod length $L$ to crank length $R$. 
LABORATORY SET-UP
A stripped-down lawn mower engine is driven by a variable-speed electric motor. Sensors are installed on the engine’s piston to measure displacement and velocity. A data acquisition program is used to measure, analyze, and record the piston data. Look at the engine and see how its various parts fit together. It may help to look at Figure 1.2 and at the various demonstration slider-crank systems present in the dynamics laboratory. Identify the piston, connecting rod, and crankshaft (the connecting rod won’t be visible at your lab set-up, but you can see it in the demonstration slider-crank systems). The cylinder head has been removed, exposing the top of the piston and allowing sensors to be attached.

The speed and direction of the electric motor are controlled by a knob and switch on the motor controller. The numbers on the speed controller are arbitrary; do not write them down as r.p.m. or radians per second (instead obtain angular velocity information from the data acquisition program). Does the direction of motor rotation affect the slider-crank kinematics?

The displacement and velocity data are measured using a LVDT and velocity transducer. Acceleration is calculated by the computer through numerical differentiation of the velocity data. This process magnifies any noise in the data. The computer also measures and displays the angular frequency by timing successive crossings of the zero line and converting to radians per second. The displacement, velocity, and acceleration are all plotted in LabView along with their minimum and maximum values (see Figure 1.4). A simple simulation program lets you compare your data to theoretical values and look at the effects of different slider-crank geometries.

Please follow safety precautions. The electric motor driving the lawn mower engine is powerful enough to cause serious injury if you get in its way. Keep long hair and loose clothing well away from the belt and pulleys at the back of the engine. If you need to touch the pulley, piston, or LVDT for some reason, check first that the electric motor power is off and that the speed control is set to zero. Make sure your lab partner knows what you are doing.

Using the LabView software

1. To run the software, open up the Engrd203Lab account and then open the folder Crank on the desktop. Open the program Crank. As soon as the program is running, it will ask you to move the piston to the top of its travel. Press Ready after you have done this and wait until the next pop-up comes before moving the piston again. Then once prompted move the piston to the bottom of its travel and press Ready again and allow the computer a few seconds to calibrate. This calibration procedure allows the computer to convert the output of the LVDT (in volts) into displacement (in meters). Do this carefully. It may help to rock the pulley back and forth slightly as you try to home in on the highest (or lowest) piston position. If you make a mistake, you can redo the procedure by clicking on the SET-UP button. The Crank program has a box
for the initials of your lab group. Click on the box with the mouse, type your initials, and then press the Enter key, not the Return key. Your initials will then appear on your plots, making it easier to identify them as they emerge from the laser printer.

2. When the data acquisition “switch” on the screen is turned on, the computer acquires and displays a new set of data every ten seconds or so. Allow ten or twenty seconds for the data plot to stabilize after changing the motor speed. If you have a plot that you want to keep, turn the data acquisition off. Also, turn the motor off promptly when you are not acquiring data to save wear and tear on the lab set-ups and on the nerves of other students.

The legend and scale factors for the plots are displayed in the top left corner. Multiply the y-axis reading (between -1 and 1) by the appropriate scale factor to obtain the actual measured value, in the units given in the legend. For example, if the velocity plot has a y-value of 0.5 at a particular time, and a scale factor of 4 m/s, the measured velocity at that time would then be 0.5*4 m/s = 2 m/s.

3. The SAVE button stores your data on the hard disk. The file created in this way can only be used by the simulation program (CrankSim2).

4. To exit from the program, click the “close” box in the top right corner of the window. To leave LabView completely, at any time, pull down the File menu and select Quit. If the program tells you that “Quitting now will stop all active VIs” select OK.

Figure 1.4: Using the LabView Crank program.
**PROCEDURE**

You will record and analyze $x(t)$, $v(t)$, and $a(t)$ while spinning the lawn mower engine at various speeds.

1. Check that the electric motor power switch is off, the speed control knob is at zero, and the data acquisition is on. Twist the pulley back and forth by hand and look at the resulting plot of piston position, velocity, and acceleration. If the piston moves upwards, in what direction does the plotted curve move? (i.e. how is the coordinate system defined for our system?) You will need to wait several seconds for the data to be displayed.

2. Put a penny on top of the piston, turn on the motor, and adjust the motor speed so that the penny just barely starts to bounce on top of the piston. You should be able to hear a faint clinking sound. Wait until you have a good graph of the data and then turn off first the data acquisition and then the motor.

Record the angular velocity and the minimum and maximum values for the displacement, the velocity, and the acceleration in a table. Save one set of curves. There is no need to print up your data, you will be able to examine it in CrankSim2. Check that the displacement plot makes sense, given that the crank length is known to be 0.0222 m. Check that the acceleration data makes sense - what should the acceleration be when the penny begins to leave the top of the piston? Given the coordinate system for our engine should that be the maximum or minimum value of the acceleration for this data?

3. Remove the penny and repeat the procedure above for at least four additional speeds. Try to get as wide a variety of speeds as possible. At very slow speeds the motor does not turn smoothly and the data is drowned out by noise. When using very high speeds, try to acquire data quickly, turn off the data acquisition “switch”, and shut the motor off immediately. Record your data in a table (including the penny data).

You will now simulate the slider-crank mechanism on the computer. The CrankSim2 program (Figure 1.5) will be used to compare the theoretical values for displacement, velocity, and acceleration with the values measured above. The effects of changing the crank length $R$, connecting rod length $L$, and angular velocity $\omega$ of the crankshaft may also be observed.

1. To start up the simulation program double-click on CrankSim2 in the Crank Lab folder. If you want to compare your simulation to your most recently saved data, turn the measured-data “switch” on; otherwise, turn it off to eliminate the clutter of all the extra graphs. Described below are the parameters you can change in the simulation:

   - $R$ is the crank length in meters.
   - $L$ is the connecting rod length in meters.
• $\omega$ is the angular velocity of the drive shaft in radians per second.

As with the data acquisition program, the maximum and minimum values are displayed. These are the simulation maxima and minima. Note that the displacement shown is the value $x$ in Figure 1.2 minus the connecting rod length $L$. This makes it more easily comparable to the measured data. The $x = 0$ point is thus defined to be halfway between the piston’s top and bottom positions instead of at the center of the crankshaft.

2. Set up the simulation with the crank length and connecting rod length of the lawn mower engine. Enter the angular velocity from the saved penny data and turn the Measured Data switch on. Make a printout of your data.

3. Switch off the measured-data curve. Now simulate slider-cranks with different geometries by varying the crank length $R$ and the connecting rod length $L$. Observe and record velocities and accelerations for the following cases. Please make print-outs to support your observations and conclusions.

- $L$ is much greater than $R$ - i.e. $L$ of 10 m, and $R$ of 0.0223 m
- $R$ is increased, but still much smaller than $L$ - i.e. $L$ of 10 m and $R$ of 0.223 m
- $L$ is decreased, but still much larger than $R$ - i.e. $L$ of 1 m and $R$ of 0.0223 m
- $R$ and $L$ equal the values for the lawn mower engine - i.e. $L$ of 0.089 m and $R$ of 0.0223
L is only slightly greater than $R$ - i.e. $L$ of 0.0224 m, and $R$ of 0.0223 m What happens physically when $R$ is greater than $L$?

Next you will work with the adjustable slider-crank. This device allows you to adjust the crank length to connecting rod ratio $\frac{R}{L}$ from zero to slightly more than one, using an adjustment knob which changes the effective crank length. A handle is located underneath to rotate the apparatus by hand. **Please be gentle with it!** Large forces can be generated with even a small input torque when the ratio is close to 1. If you see things bending, back off. When turning the hand crank, do it slowly. You can also push and pull on the masses at the end of the “piston” to look at the way it converts linear to rotary motion. Be sure you can identify the crank, connecting rod, and piston on the adjustable crank apparatus as first appearances may be misleading. Here is a hint: the long thin rod with a weight on each end is the piston. Compare the shapes of the curves you saw in the simulation above to what you observe and feel with the adjustable crank.

The slider-crank is just one of many devices that have been invented to convert linear to rotational motion or vice-versa. The scotch yoke, the cam, and the four-bar linkage are some others.

1. Look over the scotch yoke mechanism, which is driven by an electric motor and gearbox. Try it at different speeds and (with the motor off) push and pull on its various parts. Rotate the pulley by hand while watching the motion of the rod. Take measurements or make a drawing if you wish. Be prepared to find a kinematical equation relating disk rotation to yoke displacement and think about the advantages and disadvantages of the scotch yoke relative to the slider-crank.

2. Cam-and-follower mechanisms are a particularly versatile way to convert rotary to linear motion because you can select the type of motion you want by changing the shape of the cam. For example, cams are used in an internal combustion engine to open and close the intake and exhaust valves. Cam shapes are chosen to optimize fuel economy, power, and emission control. The cam in this lab is a simple eccentric disk - i.e. a circle rotating about a point other than its center. Try out the cam mechanism by turning it with your hand. Feel the output from the follower as the cam is rotated and then try rotating the cam by pushing and pulling on the follower.
LAB REPORT QUESTIONS

1. It is claimed that the peak vertical velocity scales with the angular velocity. Does your data support this claim? (A graph would be nice here).

2. It is claimed that the peak acceleration is proportional to the square of the angular velocity. Does your data support this claim? (A graph would be nice here).

3. How well does the acceleration you measured with the penny correspond to what you would expect? What do you think are plausible sources of your error here?

4. From your experimental data, what is the crankshaft angular velocity for which an ant standing on the top of the piston would start to need sticky feet in order to not lose contact with the piston? Explain.

5. Using your simulation data, how does the length of the connecting rod, relative to the crank length, affect the shape of the displacement, velocity, and acceleration curves?

6. The lawn mower engine piston weighs 0.175 kg. Suppose that the net force on the piston should not exceed 10 kN. Using the equation you found in Part 1 of the lab report, what is the maximum crankshaft angular velocity for which the engine can safely run.

7. Argue for or against the following point: for all slider-cranks the peak velocity occurs exactly at the midpoint of the stroke. Back up your arguments with either your print-outs for Part 3 of the procedure, or any other appropriate analysis and logic. Make sure you consider the case when \((L \simeq R)\).

8. What are the pluses and minuses of the various mechanisms that convert rotary motion to straight line motion that you have seen in these labs?