Your TA, Section # and Section time:

Your name:

Cornell TAM/ENGRD 2030

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

Catch-all Makeup Prelim

May 7, 2011

How to get the highest score?

Please do these things:

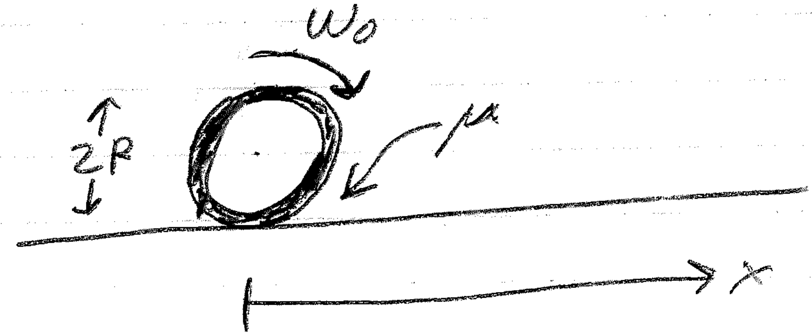
- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation.**
- A+ Be (I) neat, (II) clear and (III) well organized.
- □ TIDILY REDUCE and box in your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60 – 100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.
  Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 10: ____/25

Problem 11: ____/25

Problem 12: ____/25

1. A uniform hoop spins and then is dropped onto a horizontal surface. The hoop spins and slides for a while and eventually ends up in pure rolling.

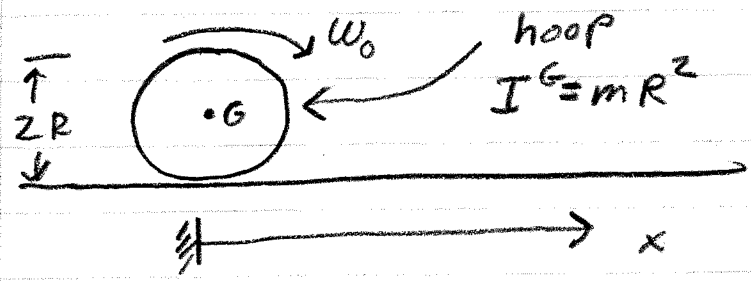


a) When the cylinder eventually rolls what is its velocity (vel. of C.O.M.)

b) What, then, is $\vec{\omega}$?

c) How far does it travel before it enters pure rolling?

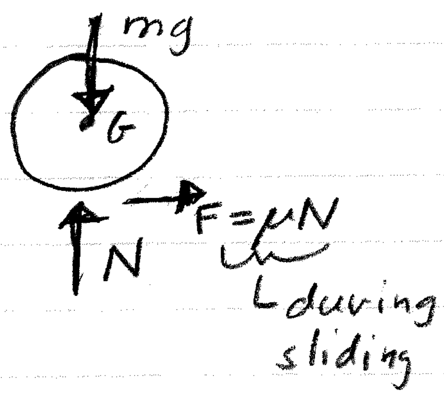
Make-up <u>prob. 1</u> "Solution"



initially:
$\vec{\omega} = -\omega_0 \hat{k}$
$\vec{v} = \vec{0}$

Find when & where rolling starts and at what v, ω?

FBD →



$\{\sum \vec{F} = m\vec{a}\} \cdot \hat{j} \Rightarrow \boxed{N = mg}$

$\Rightarrow \boxed{F = \mu mg}$

Signs: $\circlearrowleft \omega = +$, $\vec{v_x} = +$

$AMB_{/G} \Rightarrow \sum \vec{M}_{/G} = \dot{\vec{H}}_{/G} \Rightarrow R\mu mg = I^G \dot{\omega}$ ①

$\{LMB\} \cdot \hat{i} \Rightarrow F = ma_x \Rightarrow \mu mg = m\dot{v}_x$ ②

① $\Rightarrow \omega = -\omega_0 + \frac{R\mu mg}{I^G} t = -\omega_0 + \frac{\mu g}{R} t$

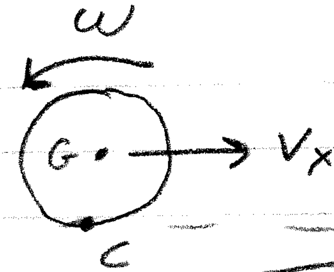
$\Downarrow$

② $\Rightarrow v_x = \mu g t$ $\qquad \omega = -\omega_0 + \frac{v_x}{R}$ ③

cont'd →

1, (cont'd)

Rolling:



$$V_c = 0 \Rightarrow V_x + \omega R = 0 \Rightarrow \boxed{\omega = -V_x/R} \quad (4)$$

Sliding turns to rolling when (3) & (4) agree:

$$\Rightarrow \frac{-V_x}{R} = -\omega_0 + \frac{V_x}{R} \Rightarrow \boxed{V_x = \frac{\omega_0 R}{2}} \quad (a)$$

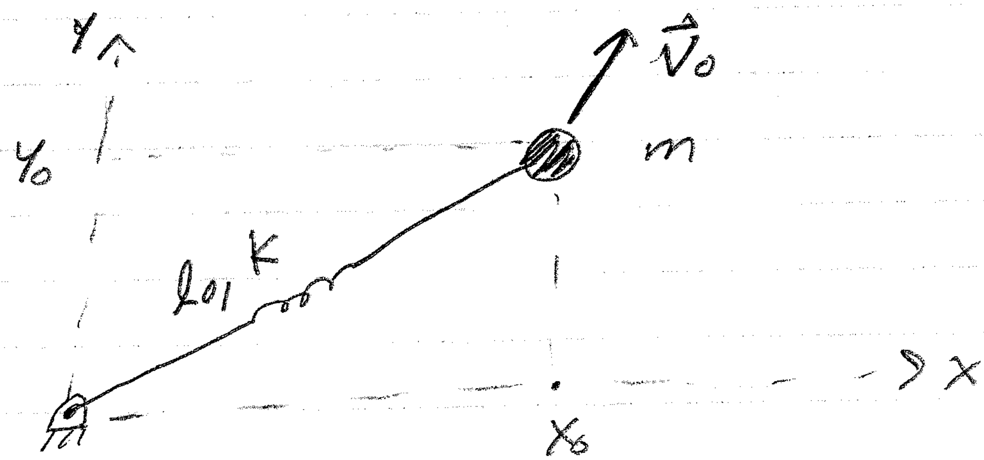
$$\Rightarrow \boxed{\omega = \frac{-\omega_0}{2}} \quad (b)$$

At transition to rolling:

$$V_x = \mu g t = \omega_0 R/2 \Rightarrow t = \frac{\omega_0 R}{2\mu g}$$

$$x = \cancel{x_0}^{\,0} + \mu g t^2/2 = \frac{\mu g}{2}\left(\frac{\omega_0 R}{2\mu g}\right)^2 = \boxed{\frac{\omega_0^2 R^2}{8\mu g}} \quad (c)$$

2) Neglect gravity, 2D.



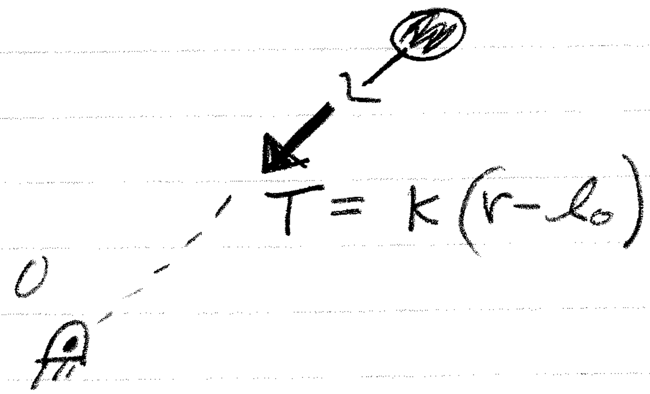
Given $m = 3$ kg
$l_0 = 2$ m
$K = 100$ N/m
$\vec{r}_0 = 5\text{m}\,\hat{i} + 4\text{m}\,\hat{j}$
$\vec{V}_0 = 6\text{ m/s}\,\hat{j}$

Write matlab code to find $x(t=3s)$.

2) "Solution"

$\hat{e}_r = \vec{r}/|\vec{r}|$

FBD



LMB: $\Sigma \vec{F} = m\vec{a}$

$$-T\hat{e}_r = m\ddot{\vec{r}}$$

$$T = K(r - \ell_o), \quad r = \sqrt{x^2 + y^2}$$

$$\hat{e}_r = \frac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}$$

$$\ddot{\vec{r}} = (\dot{v}_x\hat{i} + \dot{v}_y\hat{j})$$

(Matlab code on next page) →

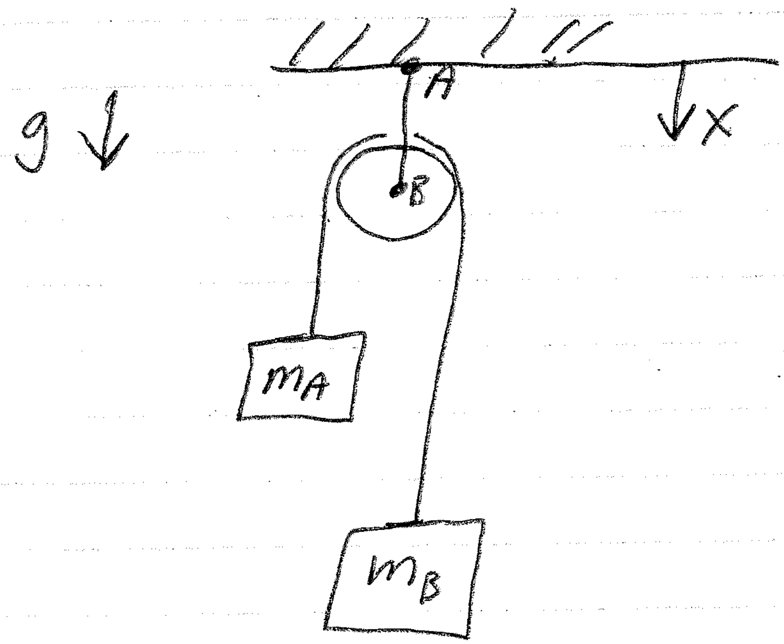
2) (cont'd)

MATLAB CODE

one text file

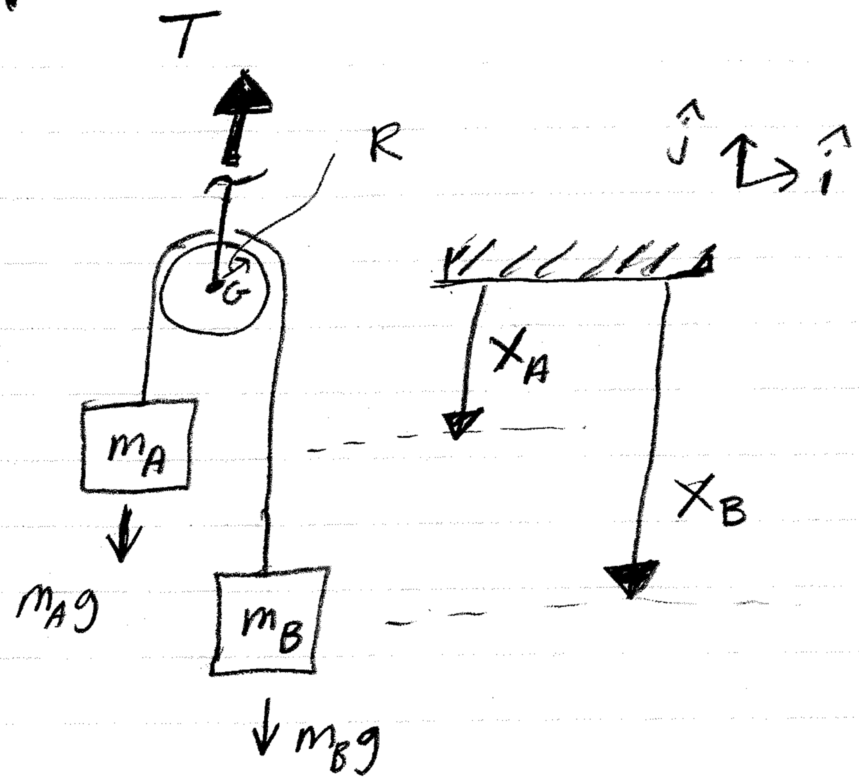
```
function makeupprob2()

r0=[5;4]; v0=[0;6]; z0=[r0;v0];
tspan = [0 3];

[t zarray] = ode45(@myrhs, tspan, z0);

lastx= zarray(:,1); % THE ANSWER

end


function zdot=myrhs(t,z)
m=3; L=2; k=100;

r = z(1:2);    v = z(3:4); magr=norm(r);

rdot= v;                   % 2 ODEs
vdot = -(1/m)*k*(magr - L)*r/magr;
                           % 2 more ODEs

zdot =[ rdot ; vdot ];
end
```

3) In terms of m_A, m_B & g find the tension in cable AB.



3) "Solution"

FBD



Kinematics:

$\ddot{X}_A = -\ddot{X}_B$

string length = const

$AMB_{/G}$: $\{\Sigma \vec{M}_{/G} = \dot{\vec{H}}_{/G}\} \cdot \hat{K}$

$$gR(m_A - m_B) = R(m_A \ddot{X}_A - m_B \ddot{X}_B)$$

$\ddot{X}_B = -\ddot{X}_A$

$$\Rightarrow \quad \ddot{X}_A = \frac{g\cancel{R}(m_A - m_B)}{\cancel{R}(m_A + m_B)} = g\left(\frac{m_A - m_B}{m_A + m_B}\right)$$

$$\Rightarrow \quad \ddot{X}_B = -g\left(\frac{m_A - m_B}{m_A + m_B}\right)$$

LMB: $\{\Sigma \vec{F} = \Sigma m_i \vec{a}_i\} \cdot \hat{j}$

$$T - m_A g - m_B g = -m_A \ddot{X}_A - m_B \ddot{X}_B$$

3) cont'd

$$T = (m_A + m_B) g - (m_A - m_B) \frac{m_A - m_B}{m_A + m_B} g$$

$$= \frac{(m_A + m_B)^2 - (m_A - m_B)^2}{m_A + m_B} g$$

$$\boxed{T_{AB} = \frac{4 m_A m_B}{m_A + m_B} g}$$

Checks

$m_A = 0 \Rightarrow T_{AB} = 0$
$m_B = 0 \Rightarrow T_{AB} = 0$
} free fall ✓

$m = m_A = m_B \Rightarrow T_{AB} = 2mg$ statics ✓