Lab #1 - One Degree-of-Freedom Oscillator

Last Updated: March 4, 2009

INTRODUCTION

The mass-spring-dashpot is the prototype of all vibrating or oscillating systems. With varying degrees of approximation, car suspensions, violin strings, buildings responding to earthquakes, earthquake faults themselves, and vibrating machines are modeled as mass-spring-dashpot systems. This laboratory is aimed at demonstrating some of the basic concepts of the mass-spring-dashpot system. In this lab you will collect data on the motion of two different mass-spring-dashpot systems, and then use computer generated solutions of the equations of motion to determine system parameters. Phrases connected with some of the key ideas are: *natural frequency*, *resonance*, *forcing function*, and *frequency response*.

PRELAB QUESTIONS

Read through the laboratory instructions and then answer the following questions:

1. Find the general solution to (1.4) if the forcing term is given by $F_s(t) = 0$ and there is no damping ($c = 0$), i.e. solve $m\ddot{x} + kx = 0$. *Note: Use pencil and paper, not MATLAB.*

2. Repeat #1, this time numerically integrating the equation using *Matlab*. Choose $m = 1$, $k = 5$, and integrate over the time period $0 \leq t \leq 10$. Assume the mass starts from rest with an initial displacement of $x(0) = 1$. What is the period of the oscillation? Turn in a plot and an m-file of your code.

3. Define in your own words: *natural frequency*, *damped frequency*, *damping coefficient*, *underdamped*, *overdamped*, *resonance*, and *phase-shift*.

4. Suppose that you are measuring two sinusoidal waveforms of equal amplitude, $x_1(t)$ and $x_2(t)$, with phase difference of $\frac{\pi}{2}$. What would be the shape of the curve if you plotted $x_1(t)$ vs $x_2(t)$? What if the phase difference is zero? π? If you have trouble visualizing the situation, try calculating a few points and plotting them.

5. Find the period T, support amplitude of motion $A_{support}$, mass amplitude of motion $A_{response}$, and phase difference ϕ for the following two curves:

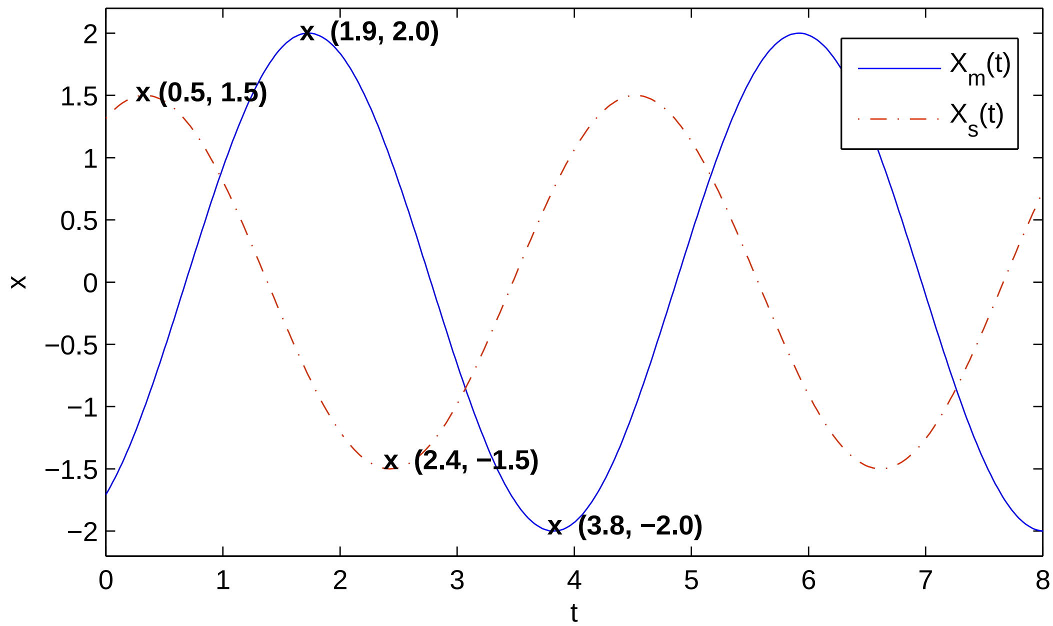

Figure 1.1: Sample lab data

THE MASS-SPRING-DASHPOT SYSTEM

The picture in Figure 1.2a shows a mathematical model of the laboratory mass-spring-dashpot, or one degree-of-freedom oscillator. A mass is supported by a spring and is constrained to move in the $\hat{\mathbf{e}}_{\mathbf{x}}$-direction. In this lab you will record the vertical motion of the mass both with a fixed support (*free vibration*) and with the support oscillating vertically (*forced vibration*). The spring is modeled as linear, i.e. the force it applies is proportional to its increase in length. The damping is also modeled as linear, i.e. the force transmitted by the dashpot is proportional to the rate at which it is being stretched. The vertical displacement of the mass is $x(t)$ and the vertical displacement of the support is $x_s(t)$ (*See Figure 1.2*).

Neglecting gravity (*Why can we neglect it?*), the mass has two forces acting on it in the $\hat{\mathbf{e}}_{\mathbf{x}}$-direction:

$$F_{sp}(t) = \text{ The spring force } = k\,(x_s - x) \tag{1.1a}$$

$$F_d(t) = \text{ The dashpot force } = c\dot{x} \tag{1.1b}$$

The system is a one degree-of-freedom system because a single coordinate is sufficient to describe the complete state of the system. (The support displacement $x_s(t)$ does not count as a degree of freedom since it is specified by the motor position and is thus considered to be a given.) From Newton's second law the equation of motion for this system is

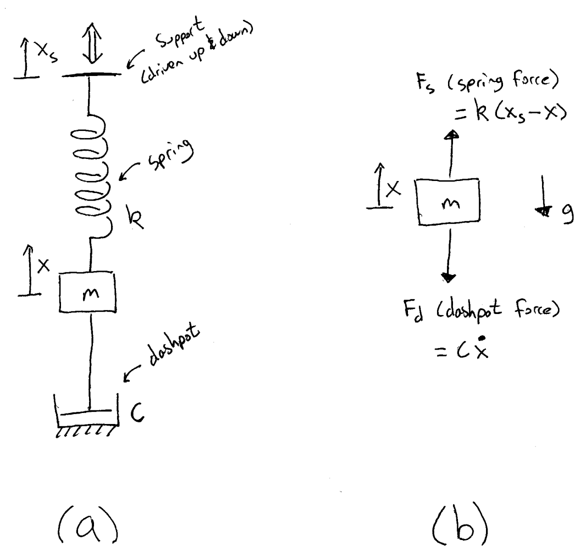

Figure 1.2: Model and free body diagram of the mass-spring-dashpot system.

$$\left\{\sum \underline{\mathbf{F}}\right\} \cdot \hat{\mathbf{e}}_{\mathbf{x}} \Rightarrow -F_d + F_{sp} = m\ddot{x} \tag{1.2}$$

and plugging in for spring and dashpot terms we get

$$m\ddot{x} = -c\dot{x} + kx_s - kx \tag{1.3}$$

and rearranging terms we get the equation of motion

$$m\ddot{x} + c\dot{x} + kx = F_s(t) \text{ with } F_s(t) = kx_s(t) \tag{1.4}$$

where $F_s(t)$ is the (presumably specified) "forcing function" due to the motion of the support. In this case the forcing function is the amount the spring is additionally stretched due the support motion multiplied by the spring stiffness.

In the first part of this experiment you will attempt to determine the value of the viscous damping constant c by measuring the rate at which oscillations decay towards zero, an experiment called a "ring-down test". In addition, the system response to both *free vibration* and *forced vibration* will be observed experimentally and through computer simulation.

A REAL-WORLD EXAMPLE: THE LOUDSPEAKER

A speaker, similar to the ones used in many home and auto speaker systems, is one of many devices which may be conveniently modeled as a one degree-of-freedom mass-spring-dashpot system. The one you will observe in this lab is typical (see Figure 1.3). It has a plastic cone supported at the edges by a roll of plastic foam (the surround), and guided at the center by a cloth bellows (the spider). It has a large magnet structure and (not visible from outside) a coil of wire attached to the point of the cone which can slide up and down inside the magnet. When you turn on your stereo, it forces a current through the coil in time with the music, causing the coil to alternately repel and attract the magnet pushing the cone up and down in its housing. This results in the vibration of the cone which you hear as sound.

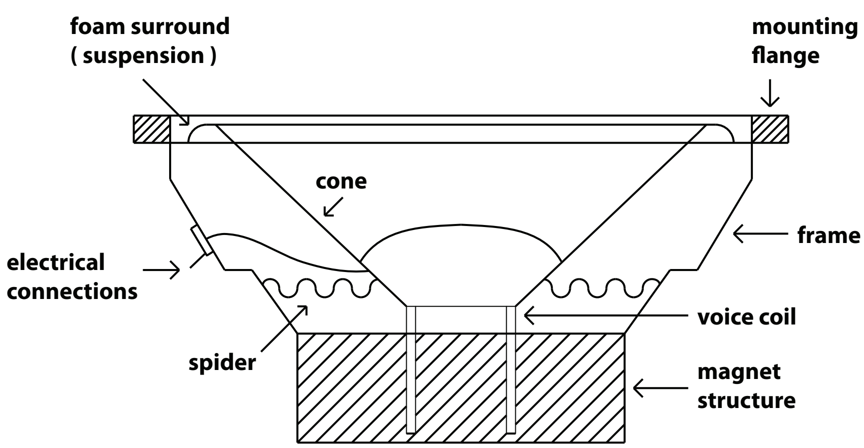

Figure 1.3: Cross-sectional view of a speaker.

A simplistic view is that the cone and coil provide inertia ($m\ddot{x}$), the foam surround and clothe bellows act as a spring (kx), viscous damping comes from the cone moving through the air ($c\dot{x}$), and the magnet provides external forcing $F_s(t)$. Putting it all together we get the familiar equation of motion of a driven mass-spring-dashpot system:

$$m\ddot{x} + c\dot{x} + kx = F_s(t) \tag{1.5}$$

In the second part of the lab, you will non-destructively measure the weight of the speaker coil and cone by examining the speakers dynamics.

SOLVING THE EQUATIONS OF MOTION

Recall that the equation of motion is given by:

$$m\ddot{x} + c\dot{x} + kx = F_s(t) \tag{1.6}$$

Our goal is to find the motion of the mass, $x(t)$, for a given forcing function $F_s(t)$. Two cases are of particular interest:

$$F_s(t) = 0 \text{ (unforced or 'free' vibration)} \tag{1.7}$$

$$F_s(t) = kx_s(t) = kA_{support}\cos\omega t \text{ (sinusoidal forcing)} \tag{1.8}$$

Equation (1.6) is a linear, second order ordinary differential equation with constant coefficients. The solution with $F_s(t)$ given either by (1.7) (homogeneous) or (1.8) (inhomogeneous) is discussed in every freshman or sophomore math text. Briefly, the solution can be found as follows:

From ordinary differential equation theory we can write the general solution to (1.6) as the sum of a complimentary (also referred to as the transient or homogeneous) solution $x_c(t)$ and a particular solution, $x_p(t)$.

$$x(t) = x_c(t) + x_p(t) \tag{1.9}$$

The homogeneous portion $x_c(t)$ is the solution to (1.8) with $F_s(t) = 0$ (*and appropriate initial conditions*). In this case, $x_c(t)$ goes to zero as $t \to \infty$ because any initial motion of the mass will eventual be damped out if there is no external forcing. Thus the particular solution $x_p(t)$ is what is left as $t \to \infty$ for any initial condition and includes the information about forcing.

In this section we are concerned with unforced vibrations, so we have $x(t) = x_c(t)$. We will deal with $x_p(t)$ later. As you may have seen in other courses, we posit the solution to be of the form $x_c(t) = Ae^{\lambda t}$ (*if this process seems unfamiliar to you, please review differential equations*). When we insert this into (1.5), we obtain the characteristic equation,

$$m\lambda^2 + c\lambda + k = 0 \tag{1.10}$$

which has roots given by the quadratic equation as,

$$\lambda_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m} \tag{1.11}$$

Now, depending on the values of the parameters c,m, and k (specifically the *discriminant* $c^2 - 4mk$), there are three situations encountered, and thus three different behaviors of the displacement solution $x_c(t)$. These situations are:

- $c^2 - 4mk > 0$: This produces two distinct real roots λ_1 and λ_2, and the solution is :

  $$x_c(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \tag{1.12}$$

  This sytem is called *overdamped*– the system will slowly settle down to $x_c(t) = 0$ with **no oscillations**.

- $c^2 - 4mk = 0$: This produces a repeated real root $\lambda_1 = -c/2m$ and the solution is:

  $$x_c(t) = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_1 t} \tag{1.13}$$

  This system is called *critically damped* - the system will quickly settle down to $x_c(t) = 0$ with **no oscillations**. *Why is the decay more rapid than the overdamped case?*

- $c^2 - 4mk < 0$: This produces a complex conjugate pair $\alpha \pm i\beta$ with $\alpha < 0$ and the solution is:

  $$x_c(t) = e^{\alpha t} \left[C_1 \cos(\beta t) + C_2 \sin(\beta t) \right] \tag{1.14}$$

  This sytem is called *underdamped*–the mass will oscillate, but the oscillations will decay with time according to the exponential factor (see Figure 1.4). This is the most common situation - most real world systems are underdamped.

Naturally, the constants C_1 and C_2 will be determined from initial conditions for the speed and displacement of the mass.

A useful quantity (you will see why), termed the *natural frequency* ω_n is defined as,

$$\omega_n = \sqrt{\frac{k}{m}} \tag{1.15}$$

As you will show in prelab question (1), this is the system's frequency of free vibration when there is no damping ($c = 0$). Additionally, instead of employing the discriminant $c^2 - 4mk$ to describe the state of the system (over/under/critically damped) , it is convenient to define a *damping factor*, ζ, as

$$\zeta = \frac{c}{2\sqrt{mk}} \tag{1.16}$$

ζ is defined in such a way that

- $\zeta > 1$ is an overdamped system

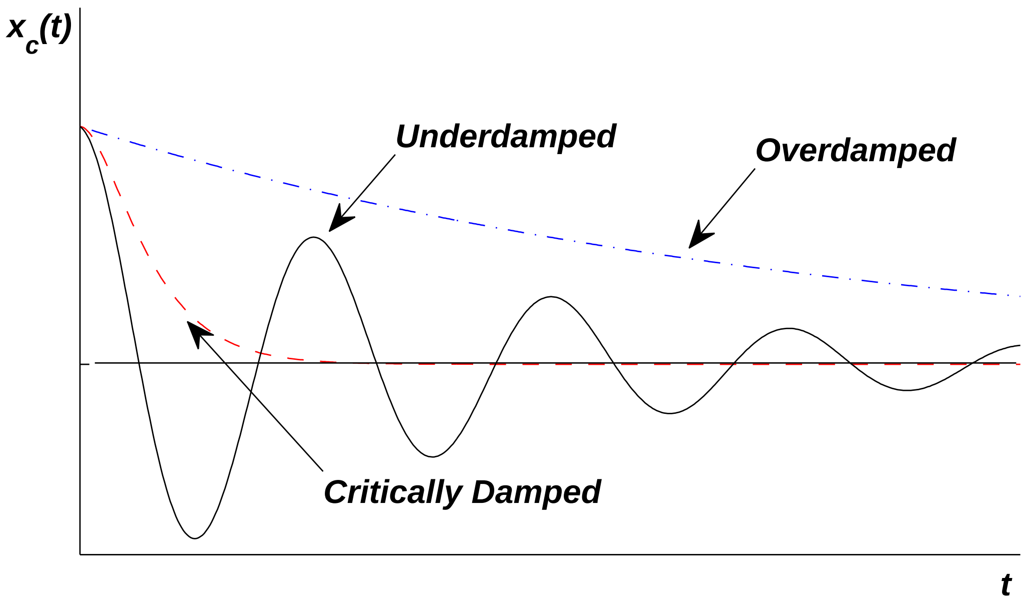

Figure 1.4: Typical solutions for *underdamped*, *overdamped*, and *critically damped* cases. Note that for *overdamped* and *critically damped* systems there are no oscillations.

- $\zeta = 1$ is a critically damped system
- $\zeta < 1$ is an underdamped system

Thus, ζ is a non-dimensional measure of the amount of damping in the system. **In this lab, we will assume that both the mass-spring-dashpot system and the speaker are underdamped.** ***In fact we will assume*** $\zeta \ll 1$***!***

Using these definitions, we can restate the quadratic equation we found above in terms of the new variables, which yields (after some algebra)

$$\lambda_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \tag{1.17}$$

Since we are studying the underdamped system in the lab, we take $\zeta < 1$ and find that the roots are

$$\lambda_{1,2} = -\zeta\omega_n \pm i\omega_d \tag{1.18}$$

where we defined the *damped natural frequency* (i.e. the frequency of oscillation *with* damping) as $\omega_d = \omega_n\sqrt{1-\zeta^2}$. Thus, the solution for the underdamped system (1.14) is,

$$x_c(t) = e^{-\zeta\omega_n t}\left[C_1\cos(\omega_d t) + C_2\sin(\omega_d t)\right] \tag{1.19}$$

which can be restated as,

$$x_c(t) = Ae^{-\zeta\omega_n t}\cos(\omega_d t - \phi) \tag{1.20}$$

where $A = \sqrt{C_1^2 + C_2^2}$, and $\phi = tan^{-1}(-C_2/C_1)$ are two constants to be determined from the initial conditions.

THE LOGARITHMIC DECREMENT METHOD

It is often important to measure how much damping there is in an engineering system. The viscous damping coefficient, c, may be determined experimentally by measuring the rate of decay of unforced oscillations - this process is called a "ring down" test. We define the logarithmic decrement, D, as the natural logarithm of the ratio of any two successive amplitudes:

$$D = \ln\left(\frac{x_n}{x_{n+1}}\right) \tag{1.21}$$

where x_n and x_{n+1} are the heights of two successive peaks in the decaying oscillation (see Figure 1.5). **Note that "x" here refers to the mass displacement $x_c(t)$ with respect to equilibrium and not the x-axis.** The larger the damping, the greater will be the rate of decay of oscillations and the bigger the logarithmic decrement, D.

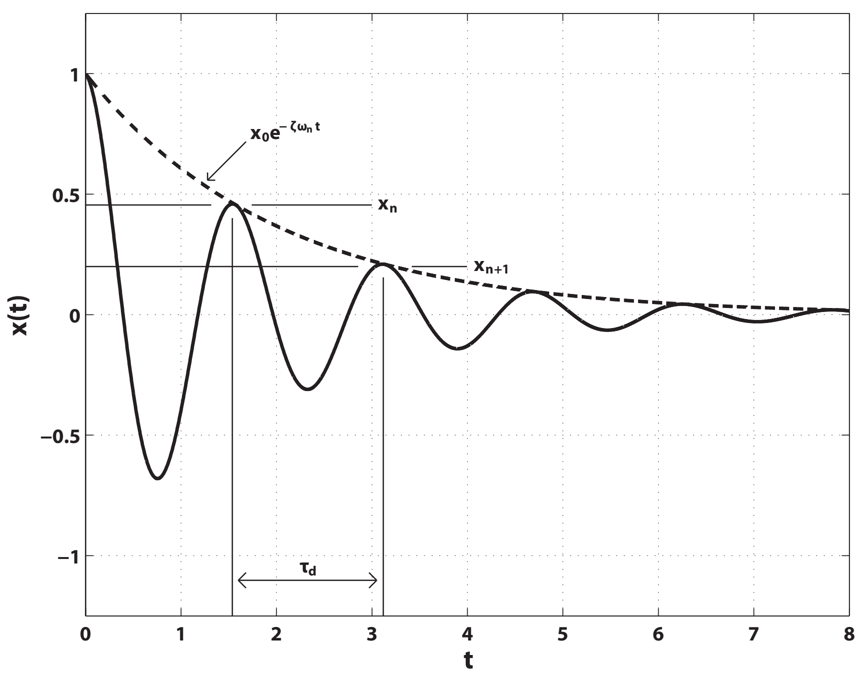

Figure 1.5: The logarithmic decrement method.

Because of the exponential envelope that this curve has (refer to (1.20)), $x_n = (Const.) * e^{-\zeta\omega_n t}$ and $x_{n+1} = (Const.) * e^{-\zeta\omega_n (t+\tau_d)}$, where τ_d is the period of the damped oscillation, i.e. $\tau_d = \frac{2\pi}{\omega_d}$. Thus

$$D = \ln\left(\frac{e^{-\zeta\omega_n t}}{e^{-\zeta\omega_n (t+\tau_d)}}\right) = \zeta\omega_n\tau_d \tag{1.22}$$

We simplify this expression by substituting in (1.16) for ζ and then solve for the damping constant c, yielding *(algebra omitted)*

$$c = \frac{2mD}{\tau_d} \tag{1.23}$$

We can also obtain an equation for k from (1.22) , yielding

$$k = \frac{c^2\left(1 + \frac{4\pi^2}{D^2}\right)}{4m} = \frac{c^2}{4m\zeta^2} \tag{1.24}$$

Thus, by doing a "ring-down" test we can experimentally measure values of D and τ_d. Then using equations (1.23) and (1.24) and given the mass m, we can calculate the damping coefficient c and spring constant k for a one degree-of-freedom oscillator.

FORCED VIBRATIONS AND FREQUENCY RESPONSE

Often a system is periodically forced and we are interested in how it will respond, e.g. the tires on your car going over evenly spaced ruts in the road jostles the car. When the forcing function is sinusoidal with frequency ω, it can be shown that the steady state solution $x_p(t)$ is sinusoidal in time with the same frequency ω. *Note: here we use 'p' to denote the particular solution.* Furthermore, the amplitude of the system's response depends on the frequency and amplitude with which we drive it. When the frequency with which we force the system ω is close to the system's natural frequency of vibration ω_n, the response has a quite large amplitude. This phenomenon, called resonance, will be discussed in the next section.

Starting with our equation of motion (1.6):

$$m\ddot{x} + c\dot{x} + kx = F_s(t) \tag{1.25}$$

If we let the forcing term be given by:

$$F_s(t) = kx_s(t) = F_{drive}\cos\omega t \tag{1.26}$$

Then we are looking for a steady state solution of the form:

$$x_p(t) = A_{response}\cos(\omega t - \phi) \tag{1.27}$$

where $A_{response}$ is the amplitude of the system response and ϕ is the phase of the response $x_p(t)$ with respect to the exciting force $F_s(t)$. *Note: the phase of a curve is a shift of one graph to the left or right with respect to another graph and has units of radians. If the phase is 0 then the response is at a maximum when the forcing is at a maximum, and if the phase is π then the response is at a minimum when the forcing is at a maximum.* Using a trick from linear algebra we can solve for how the amplitude and phase of the response depend on the driving frequency. The results are given in section (9.10) of your book, and plotted above in Figure 1.6.

Note from the plot that:

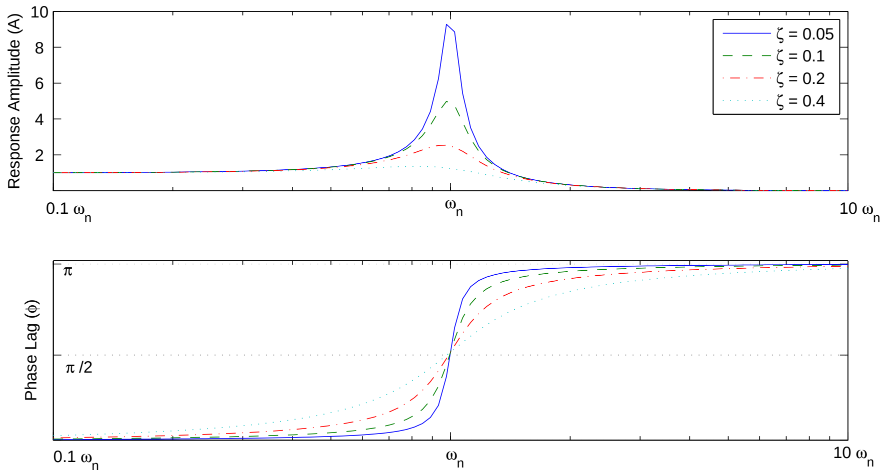

Figure 1.6: The system response $x(t) = A_{response} \cos(\omega t - \phi)$ as a function of forcing frequency ω, for various amounts of damping ζ. The forcing amplitude, F_{drive}, is fixed.

- for very low drive frequencies ($\omega \ll \omega_n$) the response is synchronized with the driving. The phase lag (ϕ) is 0, and the amplitude of vibration of the mass is the same as the amplitude of vibration of the support. *What is the physical argument for this?*

- for drive frequencies near ω_n the response amplitude is at a maximum and the phase lag is $\frac{\pi}{2}$.

- for very high drive frequencies ($\omega \gg \omega_n$) the response is completely out of phase with the driving ($\phi = \pi$) and the amplitude of vibration goes to zero. *What is the physical argument for why the amplitude vanishes?*

- the less damping there is, the sharper the change in phase is, and the greater the response near ω_n.

RESONANCE

Resonance as defined by *Merriam-Webster* is *a vibration of large amplitude in a mechanical or electrical system caused by a relatively small periodic stimulus of the same or nearly the same period as the natural vibration period of the system.* This definition confirms what we already noted in Figure (1.6), i.e. that the amplitude of response was a maximum when we drove the system at a frequency ω near the natural frequency ω_n. To find the exact resonant frequency, ω_r, we find the point on our graph of $A_{response}(\omega)$ with a slope of zero (see section (9.10) of your book for more details):

$$\left.\frac{dA_{response}}{d\omega}\right|_{\omega=\omega_r} = 0 \Rightarrow \omega_r = \omega_n\sqrt{1-2\zeta^2} \tag{1.28}$$

Note for small damping ($\zeta \ll 1$) we have $\sqrt{1-2\zeta^2} \sim 1$ and so the *resonant frequency* ω_r and the *natural frequency* ω_n are approximately equal $\omega_r \simeq \omega_n$. This supports what we observed in Figure 1.6 where the peak in the response seems to be very near to ω_n.

PHASE DIAGRAMS In our experiment we will need a way to tell if the system is near resonance. We could adjust the forcing frequency ω until the response is maximized. However, this is not a very precise method. A better way is to examine the response phase(ϕ). It can be shown that when we force the system at it's natural frequency (ω_n) that the phase is $\phi = \frac{\pi}{2}$ *(*Verify this by inspection (see Figure 1.6) or directly from the equation for ϕ in (9.10) of your book). The corresponding *phase diagram* will then be a circle *(If you are interested, further details on what a phase diagram is can be found in the appendix).* Thus when the phase diagram is a circle we are at (or very close to) resonance. Though it is more difficult to prove, we will see that when our forcing frequency ω is below resonance, the phase diagram will look like an ellipse tilted to the right, and when it is above resonance, the phase diagram will look like an ellipse tilted to the left.

LABORATORY SET-UP

- **Mass-Spring-Dashpot System**
  The apparatus consists of a laboratory-model mass-spring-dashpot system with displacement transducers (Linear Variable Differential Transformers or LVDTs) for measuring $x(t)$ and $x_s(t)$. The output from the LVDTs is communicated to the computer via the data acquisition board. An electric motor and controller, acting through a scotch yoke, enable a sinusoidal forcing function to be applied to the system. Note that the controller dial readings are arbitrary; frequency and period data must be obtained from your computer plots.

- **Loudspeaker**
  The apparatus consists of a speaker on a stand with one LVDT to measure cone displacement. Waveforms are generated by the computer, amplified, and sent through a resistor to drive the speaker. The computer is also used to measure current flow through the speaker and displacement of its cone (using the attached LVDT).

Please follow all safety precautions. Keep long hair and loose clothing well away from the electric motor, pulleys, and other moving parts.

- **Using the *LabView* Software**
  The four programs you will be using in the first part of the lab are: *FreeAcq* (Figure 1.7) for acquiring data on the unforced system; *FreeSim* (Figure 1.8) for measuring the data and simulation of the same; *ForcedAcq* (Figure 1.9) for acquiring data on the system with a sinusoidal forcing function; and *ForcedSim* (Figure 1.10) which may be used for measuring the data and simulation of the forced system. Although somewhat different

in appearance and function, the programs share many key features. The *SpeakerAcq* (Figure 1.11) program used in the second part of the lab is also similar.

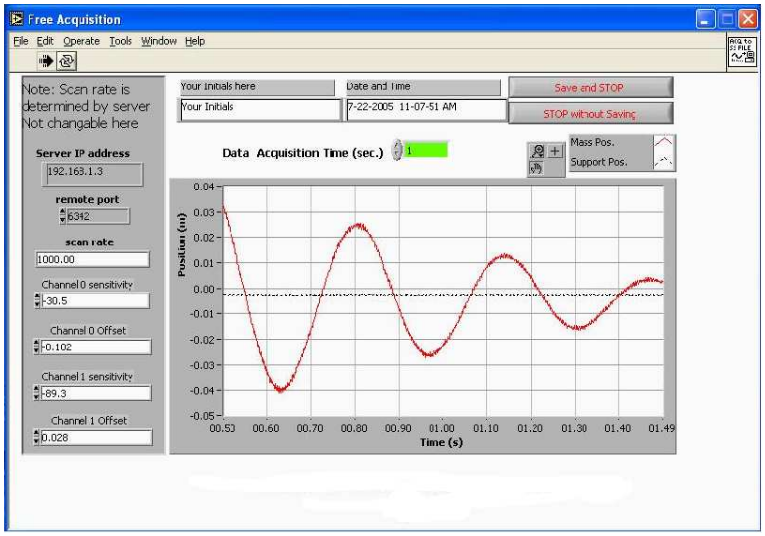

Figure 1.7: The *FreeAcq* program.

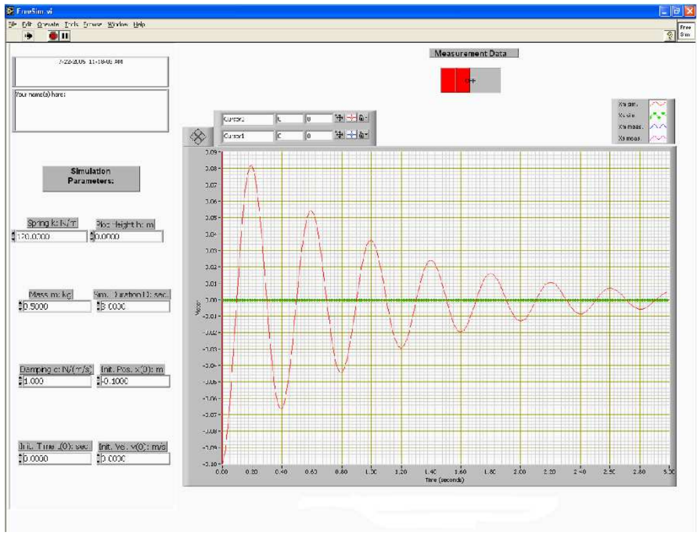

Figure 1.8: The *FreeSim* program.

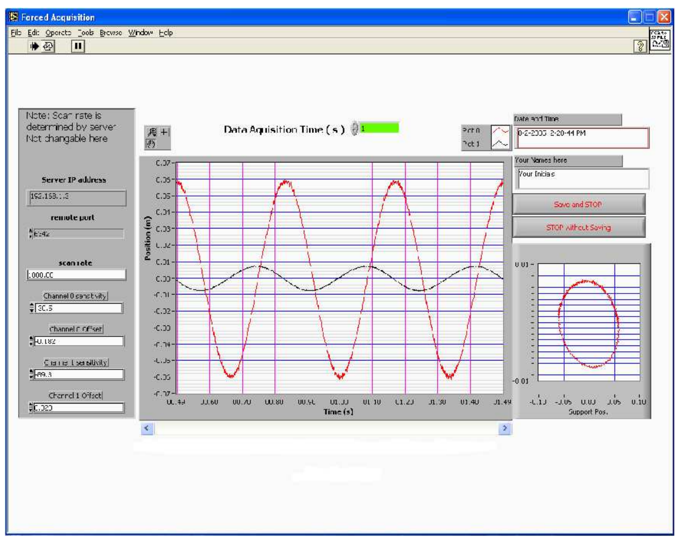

Figure 1.9: The *ForcedAcq* program.

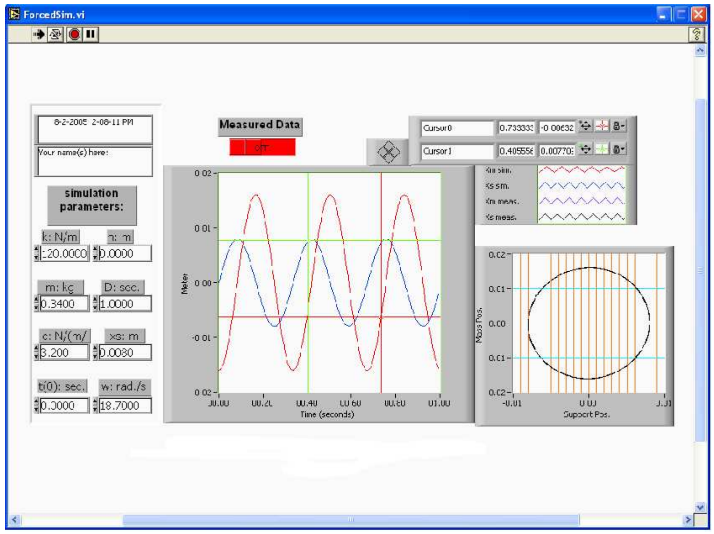

Figure 1.10: The *ForcedSim* program.

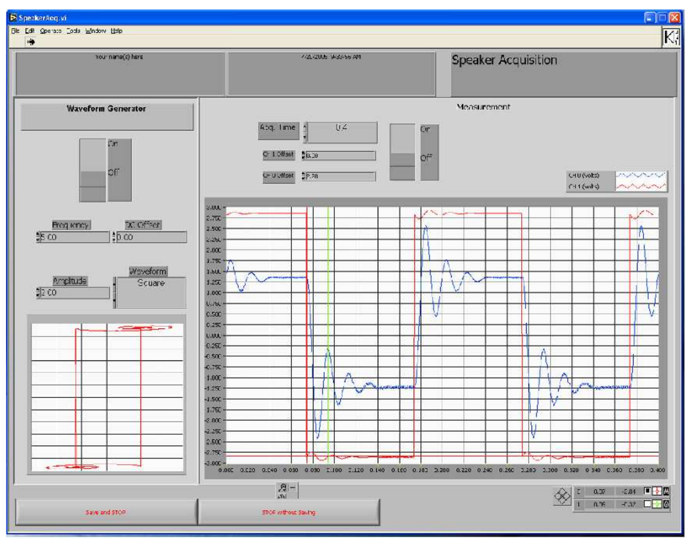

Figure 1.11: The *SpeakerAcq* program.

To run the program, you must hit the white arrow in the top left of the screen. If this arrow is black, that means that the program is already running. For the data acquisition programs, a green box on top will define the amount of time for which the program will record after hitting the arrow. To reset the data acquisition, press `STOP without Saving` and then press the white arrow to begin again.

After getting data, pressing the `Save and STOP` button stores your current data on disk. The data file is only used by the simulation programs *FreeSim* and *ForcedSim*- it is not available to the data acquisition programs.

You may find it convenient to obtain numerical data from your plots using the cursors, rather than using a ruler. Two cursors are available, one indicated by a circle and one by a square. To use a cursor, use the mouse to drag it to the point you want to measure. If your cursor has vanished off the screen, you can enter an on-screen position for it into the x and y display boxes, and it will reappear in the desired location. You can also lock the cursor to a curve by clicking the lock icon. Zoom and other features are available for the cursors and graphs; see the *LabView* manual for details.

PROCEDURE

- **Free Vibration, Mass-Spring-Dashpot**

  1. First you will measure the *free vibration* of the mass.

     – Start up the *FreeAcq* program. The data acquisition programs automatically convert the voltage output of the LVDTs to meters. To do this, they need a set of conversion factors, which are on a label on the mass-spring-dashpot base board. Make sure that the sensitivity and offset values on the left hand side of the window match the values listed on a small sheet of paper in front of the apparatus, and enter your name in the box provided. Set the data acquisition time to 6 seconds.

     – Pull down the mass and hold it still, then press the white run arrow in the top left of the toolbar, wait 1 second and then release the mass.

     – Repeat this procedure until you have a nice oscillation over the 6 seconds. Please note that the zero position is somewhat arbitrary. *You will need to take data long enough for the mass to stop oscillating in order to measure the equilibrium value.*

     – Save your best oscillation on disk by pressing the `Save and STOP` button. Save your data on the desktop with an appropriate title specific to your group.

  2. Next you will measure the logarithmic decrement D and estimate the spring stiffness k and damping coefficient c.

     – Close down the *FreeAcq* program and start the *FreeSim* program. Add the measured data to the graph by pressing the `Measurement Data` switch above the graph. Set $k = 0$ to get the simulated data out of the way, and consult the legend to make sure you know what curve you are measuring.

- Using the cursors, calculate the logarithmic decrement D and the period of the damped oscillation τ_d for each set of successive peaks - **at least 3**. *Please note that x_n and x_{n+1} in (1.21) refer to the mass displacement from equilibrium and not the "x-axis". **You will need to measure the equilibrium value and take it into account in your calculations**.*
- Using these measured values, and the mass m, calculate the damping coefficient c and spring stiffness k *(The mass of the weight and spring are written at the base of the setup. For your 'm' use the total of the weight mass and the spring mass)*.
- Make a print-out of your curve.

3. Finally, you will simulate the *free vibration* of the mass-spring-dashpot system and verify your estimate of the system parameters k, c, and m which you just calculated.
   - Input the k, c and m which you just calculated and adjust the initial condition and viewing parameters $(t(0), h, x(0), D)$ to fit your data. *Don't change k or c.*
   - Make a print-out.
   - Now see if you can adjust k and c to get a better agreement. Take note of what aspects of the graph change when you change each of the parameters k and c *independently*.
   - Make another print-out.

- **Forced Vibration, Mass-Spring-Dashpot**

  1. Here you will be recording the motion of the mass as it undergoes sinusoidal forcing.
     - Close down any other open programs and start the *ForcedAcq* program.
     - Set the acquisition time to 10 seconds, start the data acquisition and turn on the motor. Two graphs will be displayed. The left one contains two plots. One is a plot of the mass position $x(t)$ vs. time and the second one is a plot of the spring support position $x_s(t)$ vs. time. The right graph plots the phase diagram.
     - For at least five different forcing frequencies get nice plots of several cycles of motion (see instructions below). Make sure to save each data set to disk in order to analyze them in the *ForcedSim* program. Print-outs are not necessary but may be helpful.
     - To acquire data, set the data acquisition time to 10 seconds and click the arrow to run the program. Now adjust the forcing frequency until you get the desired frequency. If the data acquisition stops before you are done adjusting the forcing frequecy, you will need to click `STOP without SAVING` and then

click the arrow to run it again. Once you've got the drive frequency where you want it, reduce the data acquisition time to ~ 1 second (or atleast long enough to get one whole cycle) and then run the program again. This time hit `SAVE and STOP`. Reducing the acquisition time will reduce demand on the server and save you time doing analysis. Forcing frequencies should include:

* A low frequency for which the motor runs smoothly ($\sim \frac{1}{2}Hz$).
* A frequency just lower than resonance.
* Resonance. (*Hint: we can tell from the phase diagram that it is at resonance*)
* A frequency just higher than resonance.
* A high frequency ($\sim 3 * Resonance$).

2. Next we will measure our data in order to later calculate the response phase ϕ and amplitude $A_{response}$.
   - Close down the *ForcedAcq* program and open the *ForcedSim* program.
   - Turn on the measured data switch to view your saved data. To change the current measured data set you must close and then re-open the *ForcedSim* program. Once experimental data is loaded, make your necessary measurements (see pre-lab question) using the computer cursors.

You may also want to save the data to a USB storage device or write it to a CD for later analysis. To do this just copy the text files of the desired data onto your storage device.

- **Vibration of a Speaker**

- In the last part of the lab, you will non-destructively measure the mass of a speaker cone by measuring the shift in in its resonant frequency due to the addition of a known mass.

1. First you will find the resonant frequency of the loud speaker.
   - Set the `Waveform` control to `Sine` and the `Amplitude` control to 2. Leave the `DC Offset` control set to 0. Set the data acquisition time to 0.1 seconds. The `CH 0 Offset` and `CH 1 Offset` controls may be used to adjust the plots vertically if necessary.
   - Turn on the waveform generator and data acquisition switches and adjust the `Frequency` control value until you observe resonance of the speaker cone. **To change the frequency you must press `STOP without Saving`, enter the new desired frequency and then hit the start arrow.**
   - Make a print-out and record the resonant frequency *(note that the frequency here is given in Hz and not rad/sec)*.

Recall that the resonant frequency depends on both the mass m and spring stiffness k. By measuring the resonant frequency you cannot solve for both m and k

uniquely. However, if you also measure the resonant frequency when the mass is changed by a known amount then you will have 2 equations (i.e. (1.15), assume $\omega_r \sim \omega_n$) for 2 unknowns (m, k) in terms of measured data $(\omega 1_r, \omega 2_r, \triangle m)$. Now measure the mass of the rubber weight and then carefully press it onto the LVDT shaft. The best way is to spread the weight open, position it, and release it.

- Find the new resonant frequency, and record the mass of the rubber weight.
- Make a print-out and record the new resonant frequency.

LAB REPORT QUESTIONS

Please answer the following questions concerning the mass-spring-dashpot part of the lab within your lab report:

1. What is the spring constant k and damping coefficient c for your mass-spring-dashpot setup as calculated from your "ring-down" test? Indicate the measured data and formulas you used to calculate these values. Is the damping coefficient c really constant? How can you tell? What does this say about the air dashpot acting linearly?

2. Compare your experimental data to the simulated data for unforced motions. Comment on any similarities or differences of interest. How did changing c and k each change the simulation graph? Please attach print-outs.

3. Make a plot of the response amplitude $A(\omega)$ using your 5 data points. Make a plot of the phase-angle ϕ between $x(t)$ and $x_s(t)$ versus the forcing frequency ω. Indicate any formulas used. Do these plots match what you expect from Figure 1.6.

4. For your system, what is the theoretical percent difference between the natural frequency ω_n and the damped natural frequency ω_d (refer to 1.28)? Does the addition of a dashpot to a mass-spring system increase or decrease its oscillation frequency? Indicate any formulas used.

Please answer the following questions concerning the loudspeaker part of the lab within your lab report:

1. Calculate k and m for the speaker, using the resonant frequencies and mass you measured in lab.

2. Find another real-world vibrating system which could be reasonably modeled as a mass-spring-dashpot. Give the system a "push" and observe its response. Try applying a forcing function of various frequencies, and look for resonance.

   (a) Describe how you modeled your vibrating system as a mass-spring-dashpot. That is, what does the mass represent, what is the spring, and what is the dashpot? Be as specific as possible.

   (b) Is this system typically overdamped? Underdamped? If applicable, what was the resonant frequency (approximately)?

   (c) In what ways does the system you found most significantly differ from an ideal linear mass-spring-dashpot system?

CALCULATIONS & NOTES

Appendix: Phase Diagrams A phase diagram is a plot which contains the forcing function $F_s(t)$ on the y-axis and the response function $x(t)$ on the x-axis. The phase diagram is a graphical representation of the relative phase of the forcing and motion. Each point on the plot tells us both where we are in the drive cycle $y(t)$ and on the response cycle $x(t)$. Time is a parameter that moves us around on the diagram. Since we are only interested in the phase, we scale each term by its amplitude. Thus on our phase diagram we would plot the parametric function

$$y(t) = cos(\omega t) \tag{1.29}$$

$$x(t) = cos(\omega t - \phi) \tag{1.30}$$

When we force the system at $\omega_n \simeq$ and the phase is $\phi = \frac{\pi}{2}$ as shown before, we have:

$$y(t) = cos(\omega t) \tag{1.31}$$

$$x(t) = cos(\omega t - \frac{\pi}{2}) \tag{1.32}$$

Using the following trigonometric identities:

$$\cos^2 \omega t + \sin^2 \omega t = 1 \tag{1.33}$$

$$\cos(\omega t - \frac{\pi}{2}) = \sin \omega t \tag{1.34}$$

We can establish the following relationship for our phase plot:

$$x^2(t) + y^2(t) = 1 \tag{1.35}$$

Hopefully you will recognize this equation as the parametric form of the equation for a circle! Thus, when we force the system at its *natural frequency* which is very close to its *resonant frequency* the phase diagram is a circle. It is more difficult to show that when we forced the system below resonance the phase diagram will be an ellipse tilted to the right and above resonance an ellipse tilted to the left.