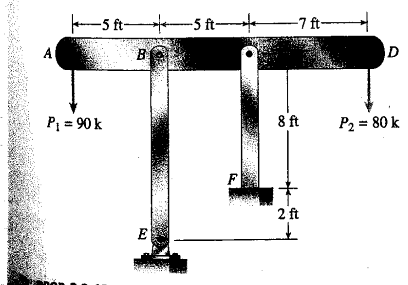
TAM 202 Spring03 HW11 soln by Tian (due 04/08)

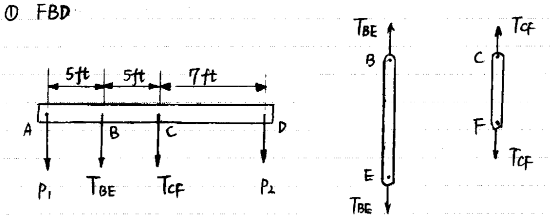
2.2-13 The horizontal rigid beam *ABCD* is supported by vertical bars *BE* and *CF* and is loaded by vertical forces $P_1 = 90$ k and $P_2 = 80$ k acting at points *A* and *D*, respectively (see figure). Bars *BE* and *CF* are made of steel ($E = 29 \times 10^6$ psi) and have cross-sectional areas $A_{BE} = 19.5$ in.^{2} and $A_{CF} = 16.8$ in.^{2} The distances between various points on the bars are shown in the figure.

Determine the vertical displacements δ_A and δ_D of points *A* and *D*, respectively.



PROB. 2.2-13

① FBD



Sign convention: tension is positive. lengthening is positive.

$\Sigma M_{/B} = 0 = P_1(5ft) - T_{CF}(5ft) - P_2(12ft)$

$\Rightarrow \quad T_{CF} = -\frac{12}{5}P_2 + P_1 = -\frac{12}{5} \times 80k + 90k = -102\ k$

$\Sigma M_{/C} = 0 = P_1(10ft) + T_{BE}(5ft) - P_2(7ft)$

$\Rightarrow \quad T_{BE} = -2P_1 + \frac{7}{5}P_2 = -2 \times 90k + \frac{7}{5} \times 80k = -68k$

② Deformation of BE & CF

$A_{BE} = 19.5\ in^2 \qquad A_{CF} = 16.8\ in^2$

$L_{BE} = 10ft = 120\ in \qquad L_{CF} = 8ft = 96\ in$

$E = 29 \times 10^6\ psi$

lengthening of BE:

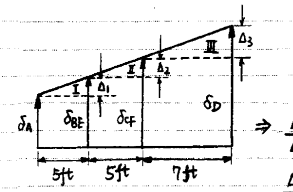
$\delta_{BE} = \frac{T_{BE}L_{BE}}{E\,A_{BE}} = \frac{(-68k)\ (120\,in)}{(29 \times 10^6\,psi)\ (19.5\,in^2)} = -0.014\ in$

(Continued)

lengthening of CF

$\delta_{CF} = \frac{T_{CF}L_{CF}}{E\,A_{CF}} = \frac{(-102k)\ (96\,in)}{(29 \times 10^6\,psi)\ (16.8\,in^2)} = -0.020\ in$

③ Displacement (beam ABCD is assumed to be rigid)



$\delta_{BE} = -0.014\ in$

$\delta_{CF} = -0.020\ in$

Similar Triangles

$\Rightarrow \frac{\Delta_1}{\Delta_2} = \frac{5ft}{5ft} = 1$ (triangle I & II)

$\frac{\Delta_2}{\Delta_3} = \frac{5ft}{7ft} = \frac{5}{7}$ (triangle II & III)

$\delta_A = \delta_{BE} - \Delta_1 = \delta_{BE} - \Delta_2$

$= \delta_{BE} - (\delta_{CF} - \delta_{BE})$

$= 2\delta_{BE} - \delta_{CF}$

$= 2 \times (-0.014\,in) - (-0.020\,in)$

$= -0.008\ in$

$\delta_D = \delta_{CF} + \Delta_3 = \delta_{CF} + \frac{7}{5}\Delta_2$

$= \delta_{CF} + \frac{7}{5}(\delta_{CF} - \delta_{BE})$

$= \frac{12}{5}\delta_{CF} - \frac{7}{5}\delta_{BE}$

$= \frac{12}{5} \times (-0.020\,in) - \frac{7}{5} \times (-0.014\,in)$

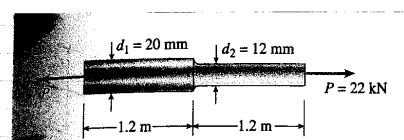
$= -0.028\,in$

Note: "−" means shortening since "+" is lengthening

2.3-6

2.3-6 A steel bar 2.4 m long has a circular cross section of diameter $d_1 = 20$ mm over one-half of its length and diameter $d_2 = 12$ mm over the other half (see figure). The modulus of elasticity $E = 205$ GPa.

(a) How much will the bar elongate under a tensile load $P = 22$ kN? (b) If the same volume of material is made into a bar of constant diameter d and length 2.4 m, what will be the elongation under the same load P?



PROB. 2.3-6

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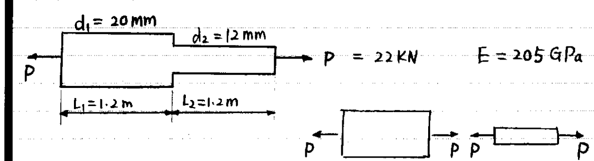
(a) Bar with two prismatic segments



elongation of the bar

$\delta = \sum_{i=1}^{2} \delta_i = \sum_{i=1}^{2} \frac{N_i L_i}{E_i A_i}$

where

$N_1 = N_2 = P = 22\ kN$

$L_1 = L_2 = 1.2\ m$

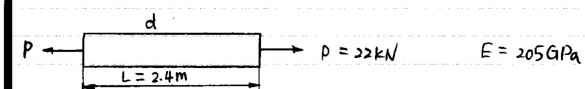
$E_1 = E_2 = E = 205\ GPa$

$A_1 = \frac{\pi}{4}d_1^2 \qquad A_2 = \frac{\pi}{4}d_2^2$

$\delta = \sum_{i=1}^{2} \frac{N_i L_i}{E_i A_i} = \frac{(22\,kN)\ (1.2m)}{(205\,GPa)}\left[\frac{1}{\frac{\pi}{4}(20mm)^2} + \frac{1}{\frac{\pi}{4}(12mm)^2}\right]$

$= 1.55\ mm$

(b) Prismatic bar



original bar $\quad V_o = \frac{\pi}{4}d_1^2 L_1 + \frac{\pi}{4}d_2^2 L_2$

prismatic bar $\quad V_p = \frac{\pi}{4}d^2 L$

$V_o = V_p \Rightarrow \frac{\pi}{4}d_1^2 L_1 + \frac{\pi}{4}d_2^2 L_2 = \frac{\pi}{4}d^2 L$

$\Rightarrow d = \sqrt{\frac{d_1^2 L_1 + d_2^2 L_2}{L}} = \sqrt{\frac{d_1^2 + d_2^2}{2}}$

$= \sqrt{\frac{(20mm)^2 + (12mm)^2}{2}}$

$= 16.49\ mm$

$\delta = \frac{PL}{EA} = \frac{PL}{E \cdot \frac{\pi}{4}d^2} = \frac{4}{\pi}\frac{(22\,kN)\ (2.4m)}{(205GPa)\ (16.49mm)^2}$

$= 1.21\ mm < 1.55\ mm$

(Continued)

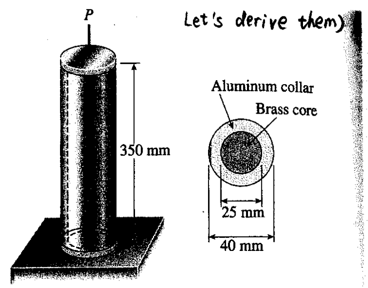
Note: A prismatic bar of the same volume will always have a smaller change in length than will a nonprismatic bar, provided the constant axial load P, modulus E, and total length L are the same

2.4-2

2.4-2 A cylindrical assembly consisting of a brass core and an aluminum collar is compressed by a load *P* (see figure). The length of the aluminum collar and brass core is 350 mm, the diameter of the core is 25 mm, and the outside diameter of the collar is 40 mm. Also, the moduli of elasticity of the aluminum and brass are 72 GPa and 100 GPa, respectively.

(a) If the length of the assembly decreases by 0.1% when the load *P* is applied, what is the magnitude of the load?

(b) What is the maximum permissible load P_{max} if the allowable stresses in the aluminum and brass are 80 MPa and 120 MPa, respectively? (*Suggestion:* Use the equations derived in Example 2-5.) (Cannot remember in exams? Let's derive them)



PROB. 2.4-2

1. Derive expressions for δ (elongation of the assembly), σ_A (stress in Al collar) and σ_B (stress in Brass core).

Sign Convention: tension is positive and lengthening is positive.

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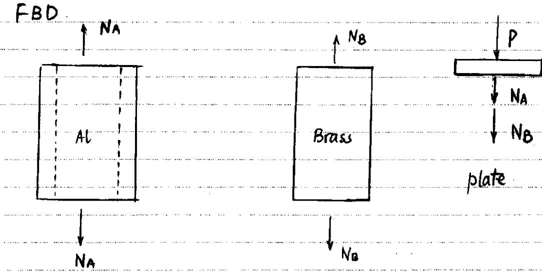
① FBD



Equilibrium of plate $\quad N_A + N_B + P = 0 \qquad (1)$

② Deformation

lengthening of Al

$$\delta_A = \frac{N_A L}{E_A A_A} \qquad (2)$$

lengthening of Brass

$$\delta_B = \frac{N_B L}{E_B A_B} \qquad (3)$$

③ Compatibility

$$\delta_A = \delta_B \equiv \delta \qquad (4)$$

From (1) – (4)

$$\Rightarrow \delta = -\frac{PL}{E_A A_A + E_B A_B} \qquad (5)$$

$$N_A = -\frac{P E_A A_A}{E_A A_A + E_B A_B} \qquad (6)$$

$$N_B = -\frac{P E_B A_B}{E_A A_A + E_B A_B} \qquad (7)$$

$\Rightarrow$

$$\sigma_A = \frac{N_A}{A_A} = -\frac{P E_A}{E_A A_A + E_B A_B} \qquad (8)$$

$$\sigma_B = \frac{N_B}{A_B} = -\frac{P E_B}{E_A A_A + E_B A_B} \qquad (9)$$

Note: "–" means the stresses in Al and Brass are both compressive.

(Continued)

2. Solve the problem

(a) the lengthening of the assembly is

$$\delta = -0.1\%\ L = -0.001 L$$

Use (5)

$$\Rightarrow -0.001 L = -\frac{PL}{E_A A_A + E_B A_B}$$

where

$E_A = 72\ GPa \qquad E_B = 100\ GPa \qquad L = 350\ mm$

$$A_A = \frac{\pi}{4}\left[(40\ mm)^2 - (25\ mm)^2\right] = 760.8\ mm^2$$

$$A_B = \frac{\pi}{4}(25\ mm)^2 = 490.9\ mm^2$$

$$\Rightarrow P = 0.001\,(E_A A_A + E_B A_B)$$

$$= 0.001\left[(72\,GPa)(765.8\,mm^2) + (100\,GPa)(490.9\,mm^2)\right]$$

$$= 0.001\,(1.042 \times 10^5\ kN)$$

$$= \underline{104.2\ kN}$$

(b) Allowable load

$\sigma_A = -80\ MPa \qquad \sigma_B = -120\ MPa$

again, "–" means compressive stress.

Use (8) for Al

$$\Rightarrow P = -\frac{\sigma_A (E_A A_A + E_B A_B)}{E_A}$$

$$= -\frac{(-80\,MPa)(1.042 \times 10^5\,kN)}{72\,GPa}$$

$$= \underline{115.8\ kN}$$

Use (9) for Brass

$$\Rightarrow P = -\frac{\sigma_B (E_A A_A + E_B A_B)}{E_B}$$

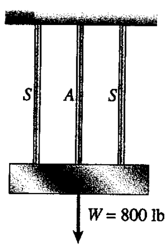
$$= -\frac{(-120\,MPa)(1.042 \times 10^5\,kN)}{100\,GPa}$$

$$= \underline{125.1\ kN}$$

So Al governs $\Rightarrow P_{max} = 115.8\ kN$

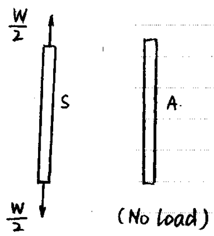
2.5-5 A rigid beam of weight $W = 800$ lb hangs from three equally spaced wires, two of steel and one of aluminum (see figure). The diameter of the wires is 1/8 in. Before they were loaded, all three wires had the same length.

What temperature increase ΔT in all three wires would result in the entire load being carried by the steel wires? (Assume $E_s = 30 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}$/°F, and $\alpha_a = 12 \times 10^{-6}$/°F.)

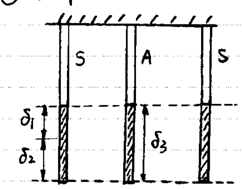


PROB. 2.5-5

① FBD (at desired state)



② Displacement



position without load or temperature change

position with load and temperature change

δ_1 = elongation of steel due to temperature change

$= \alpha_s (\Delta T) L$

δ_2 = elongation of steel due to load W/2

$= \frac{W}{2}\left(\frac{L}{E_s A_s}\right)$

δ_3 = elongation of Al due to temperature change

$= \alpha_A (\Delta T) L$ (Note: no load contribution at desired state)

③ Compatibility:

$\delta_3 = \delta_1 + \delta_2$

$\Rightarrow \alpha_A (\Delta T) L = \alpha_s (\Delta T) L + \frac{W}{2}\left(\frac{L}{E_s A_s}\right)$

$\Rightarrow \Delta T = \dfrac{W}{2 E_s A_s (\alpha_A - \alpha_s)}$

$= \dfrac{800\,lb}{2(30\times10^6\,psi)\left(\frac{\pi}{4}(\frac{1}{8}\,in)^2\right)(12\times10^{-6}/°F - 6.5\times10^{-6}/°F)}$

$= \underline{198\,°F}$

(Continued)

Note:

1. Since there is no load on the Aluminum wire, the elongation of Al is only due to the change of temperature.
2. Due to symmetry, the tension in each steel wire is $\frac{W}{2}$.
3. If the temperature increase is larger than ΔT, the Al wire would be in compression, which is not possible (Wires & Strings can only support tension). Therefore, the steel wires continue to carry all the load. If the temperature increase is less than ΔT, the Al wire will be in tension and carry part of the load.

Note: both problems in quiz 5 have been added to this homework set, and you should use two methods to solve problem 10. Please see quiz 5 solution for these two problems.