M294 PI SP87 #4

8) $\vec{\nabla} f$ points in the direction of maximum increase; $\vec{\nabla} f = (-4x, -12y^3, 0)$

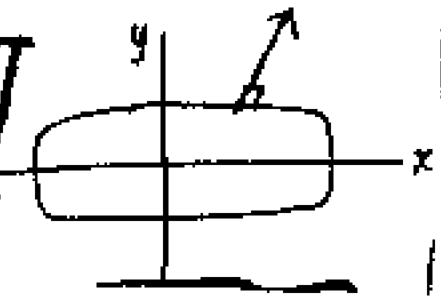
$\vec{\nabla} f(1,1,1) = (-4, -12, 0)$

But you want a unit vector so use

a) $\frac{(-4,-12,0)}{\sqrt{16+144}} = \boxed{\frac{(-1,-3,0)}{\sqrt{10}}}$

For the surface $f = -4$ (i.e. $2x^2+3y^4=5$) it is true that $\pm\vec{\nabla} f$ give normal vectors so pick a point of the surface and evaluate. I pick $(x,y,z) = (1,1,1)$ which is on the surface since the coordinates work ($2\cdot 1^2 + 3\cdot 1^4 = 5$) and from a) a unit normal vector is $\frac{(-4,-12,0)}{\sqrt{16+144}}$. To make it point

b) outward, use $\frac{(4,12,0)}{\sqrt{16+144}} = \boxed{\frac{(1,3,0)}{\sqrt{10}}}$



M294 F SP87 #6

9) $\frac{d\underline{x}}{dt} = (x+y)\hat{i} + (-x+y)\hat{j}$, $\underline{x}(0) = \binom{1}{0}$

$\dot{\underline{x}} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\underline{x}$, $\underline{x} = \binom{x}{y}$,

FINDING THE EIGENVALUES:

$\det(A-\lambda I) = 0 = (1-\lambda)^2 + 1 \Rightarrow \lambda = 1 \pm i$

CHOOSE $\lambda = 1 - i$

$(A - \lambda I)\underline{v} = 0$

$\begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix}\underline{v} = 0$, so $v = \binom{i}{1}$, so $\underline{x}(t) = c_1 Re(e^{t-it}\underline{v}) + c_2 Im(e^{t-it} v)$

$= c_1 e^t \binom{\sin t}{\cos t} + c_2 e^t \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}$

$\underline{x}(0) = \binom{1}{0} = \begin{pmatrix} c_1 \cdot 0 + c_2 \cdot 1 \\ c_1 \cdot 1 - c_2 \cdot 0 \end{pmatrix} = \binom{1}{0}$ $c_2 = 1$, $c_1 = 0$

$\underline{x}(t) = e^t \binom{\cos t}{-\sin t}$, $\boxed{\underline{x}(1) = e\begin{pmatrix} \cos 1 \\ -\sin 1 \end{pmatrix}}$

M294 F SP87 #6

M294 P II SP 88 #1

10 a) $f = xy\sin z$

$\nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$

$= y\sin z\,\hat{\imath} + x\sin z\,\hat{\jmath} + xy\cos z\,\hat{k}$

$\boxed{\nabla f(1,2,3) = 2\sin 3\,\hat{\imath} + \sin 3\,\hat{\jmath} + 2\cos 3\,\hat{k}}$

b) $\vec{F} = xy\,\hat{\imath} + e^{yz}\hat{\jmath} + z^2\hat{k}$

$\operatorname{div}\vec{F} = \frac{\partial(xy)}{\partial x} + \frac{\partial(e^{yz})}{\partial y} + \frac{\partial(z^2)}{\partial z}$

$= y + ze^{y} + 2z$

$\boxed{\operatorname{div}\vec{F}(1,2,3) = 2 + 3e^2 + 6}$

c) $\operatorname{curl}\vec{F} = \left(\frac{\partial}{\partial y}(z^2) - \frac{\partial}{\partial z}(e^{yz})\right)\hat{\imath}$

$+\left(\frac{\partial}{\partial z}(xy) - \frac{\partial}{\partial x}(z^2)\right)\hat{\jmath}$

$+\left(\frac{\partial}{\partial x}(e^{yz}) - \frac{\partial}{\partial y}(xy)\right)\hat{k}$

$= -ye^{yz}\hat{\imath} - x\hat{k}$

$\boxed{\operatorname{curl}\vec{F}(1,2,3) = -2e^6\hat{\imath} - \hat{k}}$

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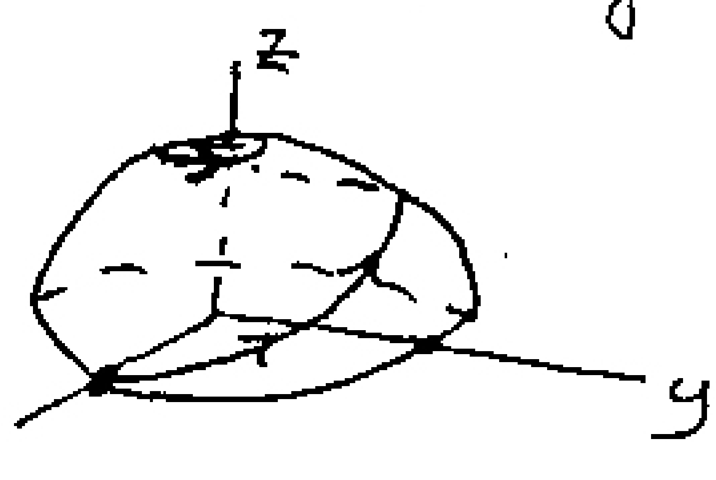
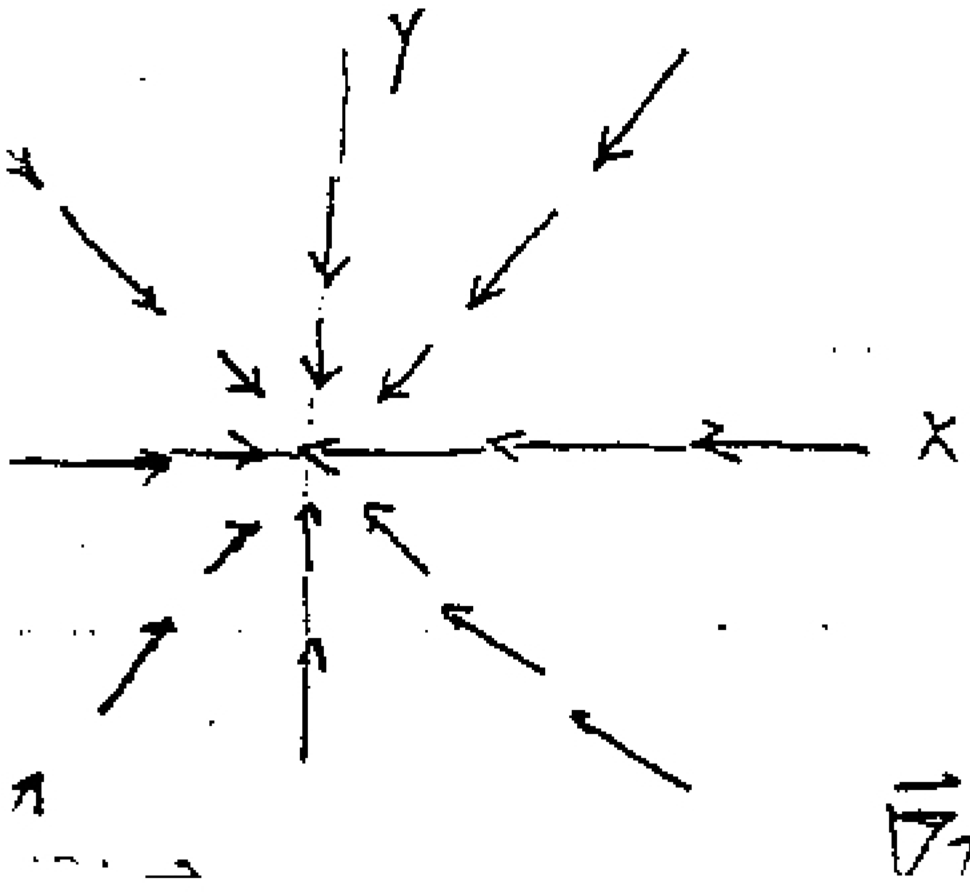
44) M294 PI FA93 #2

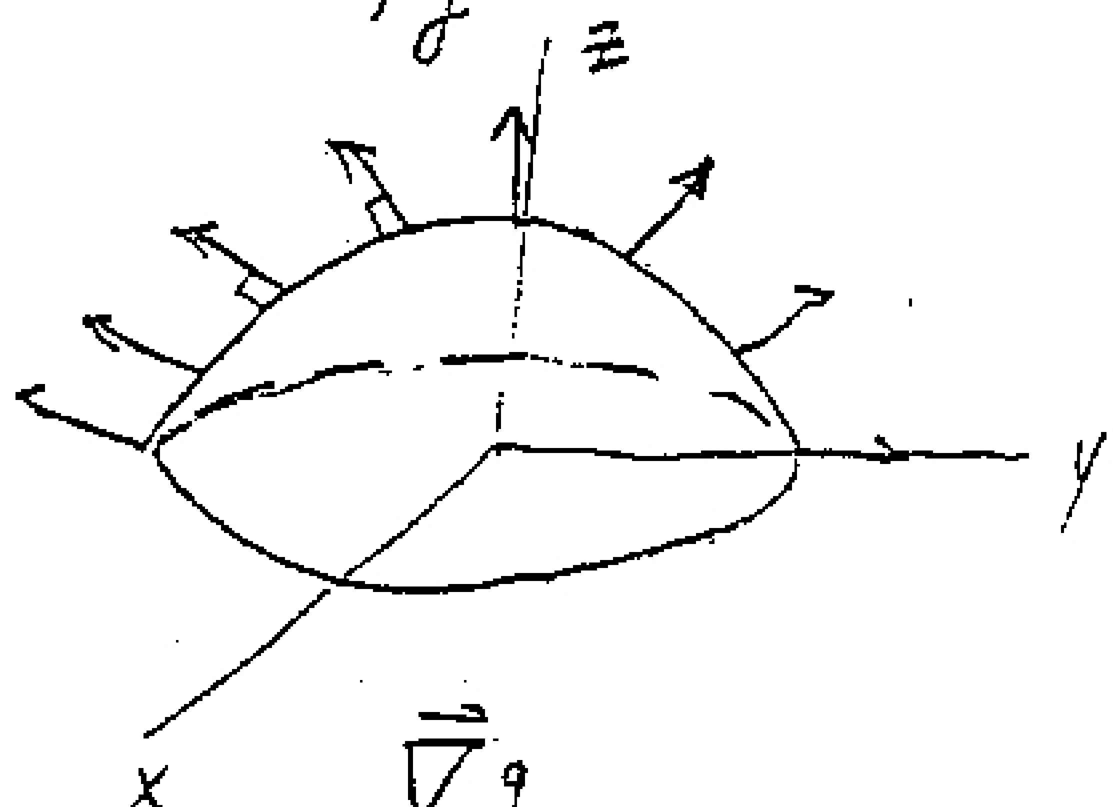
On C, $x^2+y^2+z^2 = e^{-2t}\cos^2 t + e^{-2t}\sin^2 t + (1-e^{-2t}) = 1$ so C lie on the sphere, Also $\underline{r}(0) = \underline{i}$ and the x & y coordinates circ counterclockwise and decrease toward the origin because of the e^{-t}, so it is a curve on the upper hemisphere spiraling toward the z axis point $(0,0,1)$.



M294 PI SP95 #5

3) $\vec{\nabla} f = -2x\hat{\imath} - 2y\hat{\jmath}$, $\vec{\nabla} g = 2x\hat{\imath} + 2y\hat{\jmath} + \hat{k}$





$\vec{\nabla} g$

$\vec{\nabla} f$ is normal to level curves (contour) lines of surface

M293 PI FA95#1

66) (a) $y^2 = 4x$, $2y\frac{dy}{dx} = 4$, at $P(1,2)$ you get $2\cdot 2\cdot\frac{dy}{dx} = 4$
so slope $= 1$, $\vec{t} = \boxed{\frac{\vec{i}+\vec{j}}{\sqrt{2}}}$ will work, or $\frac{-\vec{i}-\vec{j}}{\sqrt{2}}$.

(b) $x\vec{i} + y\vec{j} = (1\vec{i} + 2\vec{j}) + \frac{\vec{i}+\vec{j}}{\sqrt{2}}t$, or $\left\{\begin{array}{l} x = 1 + \frac{1}{\sqrt{2}}s \\ y = 2 + \frac{1}{\sqrt{2}}s \end{array}\right\}$ or $\boxed{y = x+1}$

(c) $\vec{n} = \vec{k}\times\vec{t}$ will do, $= \boxed{\frac{\vec{j}-\vec{i}}{\sqrt{2}}}$, or $\frac{\vec{i}-\vec{j}}{\sqrt{2}}$

(d) $x\vec{i} + y\vec{j} = (1\vec{i} + 2\vec{j}) + \frac{\vec{j}-\vec{i}}{\sqrt{2}}t$, or $\boxed{\left\{\begin{array}{l} x = 1 - \frac{1}{\sqrt{2}}s \\ y = 2 + \frac{1}{\sqrt{2}}s \end{array}\right\}}$ or $y = -x + 3$

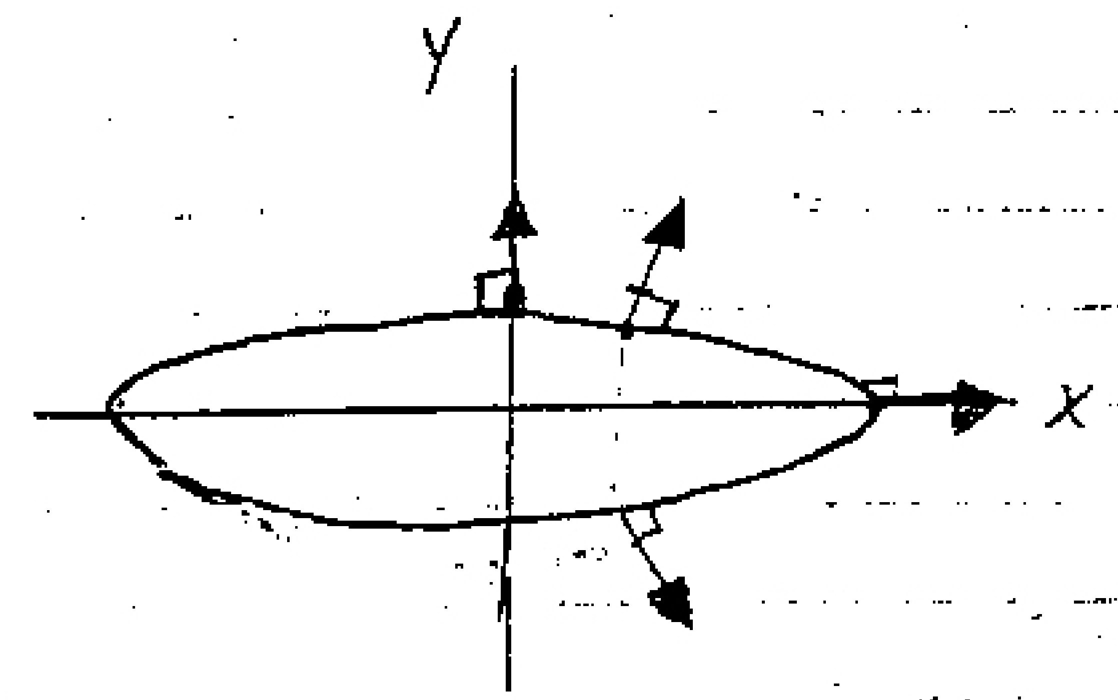
M294 PI FA95#1

67)

a) $f(4,2) = 4^2 + 8\cdot 2^2 = 16 + 32 = 48$

b) $\vec{\nabla} f = 2x\hat{\imath} + 16y\hat{\jmath}$

c) $\vec{\nabla} f(\sqrt{48}, 0) = 2\sqrt{48}\,\hat{\imath}$
$\vec{\nabla} f(4,2) = 8\hat{\imath} + 32\hat{\jmath}$
$\vec{\nabla} f(4,-2) = 8\hat{\imath} - 32\hat{\jmath}$
$\vec{\nabla} f(0,\sqrt{6}) = 16\sqrt{2}\,\hat{\jmath}$

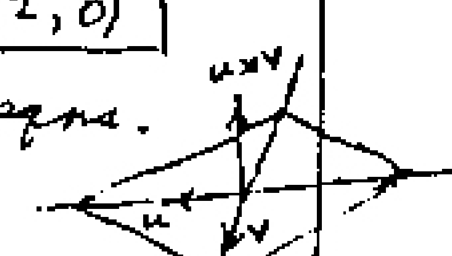


level curve

M294 PI FA95#2

68) (a) to intersect, $\left\{\begin{array}{l} x = 2+t = 3+u \\ y = 2+t = -2u \\ z = -t = 1+u \end{array}\right\}$ for some u and t. $t = 0, u = -1$
ans $\boxed{\text{yes, at } (2,2,0)}$

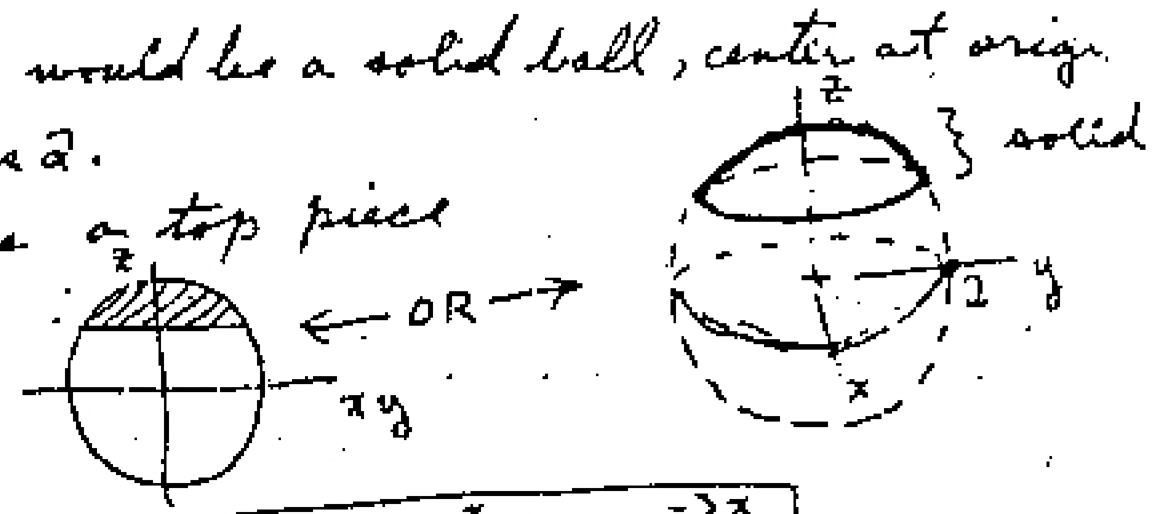
(b) $\boxed{\vec{u} = \vec{i}+\vec{j}-\vec{k}}$ and $\boxed{\vec{v} = \vec{i}-2\vec{j}+\vec{k}}$, from the parametric eqns.
Any nonzero multiples of $\vec{u}$ and $\vec{v}$ are also acceptable.



(c) Since L_1 and L_2 intersect at $(2,2,0)$ they lie in a plane through $(2,2,0)$ with normal $\vec{u}\times\vec{v} = \det\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 1 & -2 & 1 \end{pmatrix}$
$= -\vec{i} - 2\vec{j} - 3\vec{k}$; equation $-x-2y-3z = C$
$-2-2\cdot 2-3\cdot 0 = C$, $\boxed{-x-2y-3z = -6}$

M293 PI FA95#4

69) (a) $x^2+y^2+z^2 \le 4$ alone would be a solid ball, center at origin radius 2.
$z \ge 1$ takes a top piece



(b) $y'' + 2y' - 3y = 0$
$r^2 + 2r - 3 = 0$
$(r-1)(r+3) = 0$ $r = 1$ or -3 $\boxed{y = c_1e^{x} + c_2e^{-3x}}$

(c) $y' = c_1e^{x} - 3c_2e^{-3x}$
$y'' = c_1e^{x} + 9c_2e^{-3x}$
$y''+2y'-3y = (c_1 + 2c_1 - 3c_1)e^{x} + (9c_2 - 2\cdot 3c_2 - 3c_2)e^{-3x}$
$= 0 + 0$ $\boxed{\checkmark}$

M293 F FA95 #

70) (a) $t x' + x = t^2$ says $(tx)' = t^2$ or you can use integrating factor. $tx = \frac{1}{3}t^3 + C$, $x = \frac{t^2}{3} + \frac{C}{t}$

$x(0) = 0$ requires $C = 0$, so $\boxed{x(t) = t^2/3}$

(b) $y'' - 6y' + 9y = 0$

$r^2 - 6r + 9 = 0$

$(r-3)^2 = 0$ double root. $y(t) = (c_1 + c_2 t)e^{3t}$

$y(0) = 0 = c_1$

$y'(0) = c_2 \cdot 0 \cdot 3 + c_2 \cdot 1 \cdot 1 = 1$, $c_2 = 1$,

$\boxed{y(t) = t e^{3t}}$

(c) $\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} = \boxed{\frac{2}{3}t\,\vec{i} + \left(e^{3t} + 3te^{3t}\right)\vec{j}}$

M293 F SP96 #17

71) b.

M293 F SP96 #18

72) e.

M293 F SP96 #19

73) e.

M293 F SP96 #25

74) c.

M293 F SP96 #20

75) c.

M293 F SP96 #24

76) d.

M293 F SP96 #30

77) FALSE

M294 PI FA92 #1

5) a) i

b) $\int_C 3z\hat{j} \cdot d\underset{\sim}{r}$

$d\underset{\sim}{r} = \frac{d\underset{\sim}{r}}{dt}\,dt$

$\underset{\sim}{r}(t) = 2t\hat{i} + (1-t)\hat{j} + (2-t)\hat{k}$

$\frac{d\underset{\sim}{r}}{dt} = 2\hat{i} - \hat{j} - \hat{k}$

on C $3z\hat{j} \cdot d\underset{\sim}{r} = 3(2-t)\hat{j} \cdot (2\hat{i} - \hat{j} - \hat{k})\,dt$

$= -3(2-t)$

$\therefore \int_C 3z\hat{j} \cdot d\underset{\sim}{r} = \int_0^1 -3(2-t)\,dt = -6t + 3t^2/2 \Big|_0^1 = -6 + 3/2 = \boxed{-9/2}$

M294 PI SP96 #1

86)

a) $\displaystyle\int_{(0,0,0)}^{(4,0,2)} d(x^2z^3) = x^2z^3\Big|_{(0,0,0)}^{(4,0,2)} = 4^2 2^3$

b) $\vec{r}(t) = (\vec{i} + 3\vec{k}) + t\,((2-1)\vec{i} + (5-0)\vec{j} + (0-3)\vec{k})$ There are many correct solutions besides this one.