M294 P III SP96 #3

$$\boxed{u(x,t) = e^{-0.04\left(\frac{\pi}{2}\right)^2 t} \sin\left(\frac{\pi x}{2}\right) - \frac{1}{2} e^{-0.04\pi^2 t} \sin(\pi x)}$$

$$u_t = -0.04\left(\frac{\pi}{2}\right)^2 e^{-0.04\left(\frac{\pi}{2}\right)^2 t} \sin\left(\frac{\pi x}{2}\right) + \frac{0.04}{2}\pi^2 e^{-0.04\pi^2 t} \sin(\pi x)$$

$$u_{xx} = e^{-0.04\left(\frac{\pi}{2}\right)^2 t}\left(-\left(\frac{\pi}{2}\right)^2\right) \sin\left(\frac{\pi x}{2}\right) - \frac{1}{2} e^{-0.04\pi^2 t}(-\pi^2) \sin(\pi x)$$

$$= \frac{u_t}{0.04} \quad \boxed{\checkmark}$$

$$u(x,0) = \underbrace{e^0}_{1} \sin\left(\frac{\pi x}{2}\right) - \frac{1}{2}\underbrace{e^0}_{1} \sin(\pi x) \quad \boxed{\checkmark}$$

$$u(0,t) = e^{-0.04\left(\frac{\pi}{2}\right)^2 t} \sin(0) - \frac{1}{2} e^{\cdots} \sin(0) = 0 \quad \boxed{\checkmark}$$

$$u(2,t) = e^{\cdots} \sin(\pi) - \frac{1}{2} e^{\cdots} \sin(2\pi) = 0 \quad \boxed{\checkmark}$$

M294 PI SP87 #8

4) Guess: $u = \sin(ax)e^{-bt}$ (cause I remember soln. looks something like this)

$u(5,t)=0 \Rightarrow 5a = n\pi$ say $n=1 \Rightarrow a = \frac{\pi}{5}$

$\Rightarrow u = e^{-bt}\sin\left(\frac{\pi x}{5}\right)$ plug in to $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$

$\Rightarrow -b\sin(ax)e^{-bt} = 3a^2\sin(ax)e^{-bt} \Rightarrow b = 3a^2 = \frac{3\pi^2}{25}$

$\Rightarrow$ $\boxed{u(x,t) = \sin\left(\frac{\pi}{5}x\right)e^{\frac{-3\pi^2}{25}t}}$ solves PDE & BCs.

23) Heat $u_{xx} = u_t$, $u_x(0,t) = 0 = u_x(1,t)$

M294 PII SP95 #2

$0 \le x \le 1$, $t \ge 0$, $u(x,0) = f(x)$.

a) Suppose $u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n e^{-n^2\pi^2 t}\cos(n\pi x)$.

Then $u(x,0) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x)$.

Can we get any piecewise cont. function $f(x)$ with a piecewise cont. f' on $[0,1]$ this way? Yes, the series will be the Fourier Series of the even extension of $f(x)$ with period 2, (which gives $f(x)$ by the Fourier Series Theorem.)

Is $u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n e^{-n^2\pi^2 t}\cos(n\pi x)$ a solution to the heat equation?

Yes, by differentiating term-by-term.

b) if $f(x) = 2 + 5\cos 3\pi x$ then $u(x,t) = 2 + 5\cos(3\pi x)e^{-9\ldots}$

This is the heat equation, which we have solved in class by separation of variables for these boundary conditions.

We found $u(x,t) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{L}\, e^{-\frac{n^2\pi^2}{L^2}kt}$

The heat eqn is $k u_{xx} = u_t$,
Thus We have $k=1$, $L=\pi$ (given in eqn & seen in the B.C.)

$$\therefore \boxed{u(x,t) = \sum a_n \sin nx\, e^{-n^2 t}}$$

The a_n come from the Fourier expansion of the initial condition. Fortunately From problem 2, we have $x = 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx$ for the Fourier sine series.

Thus, this is the expression of the initial condition $\left\{ u(x,0) = \sum_{n=1}^{\infty} a_n \sin nx \right. \Rightarrow a_n = \frac{2(-1)^{n+1}}{n}$

$$\therefore \boxed{u(x,t) = 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx\, e^{-n^2 t}}$$

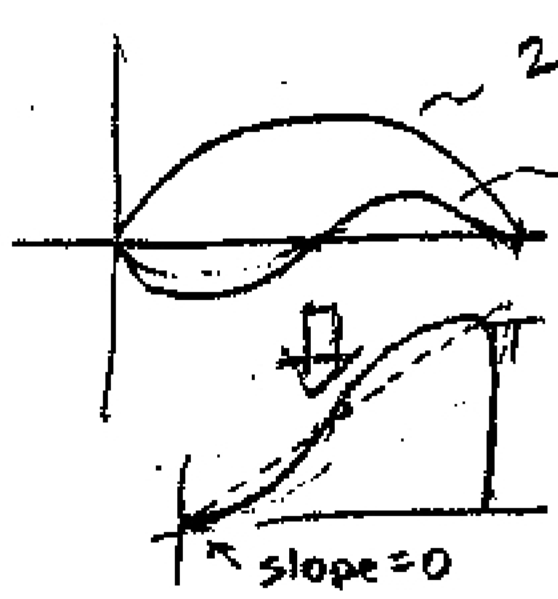
The first 2 terms are $u = \frac{2}{1}(-1)^2 \sin x\, e^{-t} + \frac{2}{2}(-1)^3 \sin 2x\, e^{-4t}$

In general, $u(x,t) = 2\sin x\, e^{-t} - \sin 2x\, e^{-4t} + \cdots$

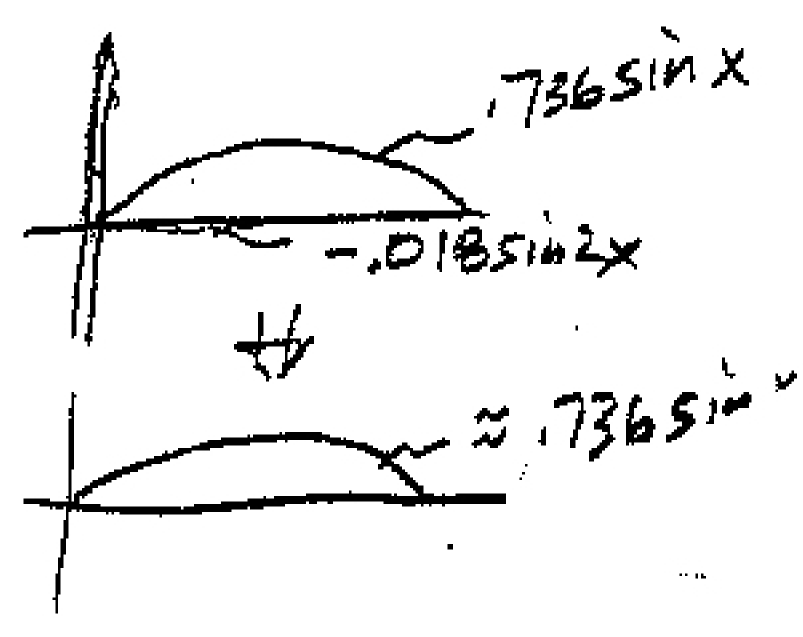
at $t=0$ $\quad u(x,0) = 2\sin x - \sin 2x + \cdots$

$t=1$ $\quad u(x,1) = 2e^{-1}\sin x - e^{-4}\sin 2x + \cdots$
$\quad = .736 \sin x - .018 \sin 2x + \cdots$

$\underline{t=0}$



$\underline{t=1}$



Because of rapid decay of 2nd term, soln color @ $t=1$ is essentially just 1st term.