M294 P.11 #P88#3

iii) $u_t = u_{xx} - u$

any one is O.K.

Some solutions are (depending on guess)

<u>solns indep. of $x$:</u> $u_t = -u \Rightarrow$ $u = e^{-t}$, $u = 3e^{-t}$

<u>solns indep. of $t$:</u> $0 = u_{xx} - u \Rightarrow$ $u = e^{x}$, $u = e^{-x}$, $u = 10e^{-x} - 3e^{x}$, $u = 7\sinh x + 2\cosh x$

<u>solns found by sep. of variables:</u>

$u = X(x)T(t)$, $\quad XT' = X''T - XT$

$$\frac{T'}{T} = \frac{X'' - X}{X}$$

$T = e^{\lambda t}$, $X'' = (\lambda + 1)X$

With $\lambda = 1$ you get $X'' = 2X$ so $X = e^{\pm\sqrt{2}x}$

With $\lambda = -1$ you get $X'' = 0$ so $X = ax + b$

With $\lambda = -2$ you get $X'' = -X$ so $X = \sin x, \cos x$

etc. The results are:

$$u(x,t) = \begin{cases} e^{t} e^{\sqrt{2}x} \\ e^{t} e^{-\sqrt{2}x} \\ e^{-t}(3x - 15) \\ e^{-2t} \sin x \\ e^{-2t}(3\sin x + 5\cos x) \\ \text{etc} \end{cases}$$

<u>solutions found by educated guessing:</u>

try $u(x,t) = e^{at} \sin bx$, then $u_t = u_{xx} - u$

becomes $au = (-b^2 - 1)u$

So $a = -b^2 - 1$

So

$$u(x,t) = \begin{cases} e^{-(b^2+1)t} \sin bx \\ e^{-5t} \sin 2x \\ e^{-10t} \sin 3x \\ \text{etc} \end{cases}$$

AND There are others.

M294 PIII FA93 #3

**12)** $\alpha^2 u_{xx} = u_{tt} \qquad 0<x<\ell \quad , t>0$

a.) Try $u = X(x)T(t)$

$$\alpha^2 X'' T = X T''$$

or $\frac{X''(x)}{X(x)} = \boxed{\frac{1}{\alpha^2}\frac{T''(t)}{T(t)} = -\lambda^2}$ Since LHS is $f(x)$ alone and RHS is $g(t)$ alone, they must each equal a constant which we're told is $-\lambda^2$

$\therefore \quad \frac{X''}{X} = -\lambda^2$ or $\boxed{X'' + \lambda^2 X = 0}$, just the same eqn that we found for the heat problem.

b.) Either recognize soln or Try $e^{rt}$.

Apply the B.C. $B \neq 0$

$X(x) = A\cos\lambda x + B\sin\lambda x$

$X(0) = A(1) + B(0) = A = 0$

$X(\ell) = B\sin\lambda\ell$.

For non-trivial solns $\lambda = \frac{n\pi}{\ell}$

$$\boxed{\therefore X(x) = B\sin\frac{n\pi x}{\ell}}$$

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c.) Using this $\lambda$ in the $t$ equation

$$\frac{T''}{\alpha^2 T} = -\left(\frac{n\pi}{\ell}\right)^2 \Rightarrow \boxed{T'' + \left(\frac{n\pi\alpha}{\ell}\right)^2 T = 0}$$

$$\therefore \boxed{T = C\sin\frac{n\pi\alpha}{\ell}t + D\cos\frac{n\pi\alpha}{\ell}t}$$

M294 PIII FA94 #7

13. (a) if $u = X(x)Y(y)$ then $X'Y + XY'' + XY = 0$

so $\underbrace{\frac{X'}{X}}_{c} + \underbrace{\frac{Y''}{Y}}_{-c} + 1 = 0$

$\therefore \boxed{X' = cX}$, $X = e^{cx}$

$\therefore \boxed{Y'' = (-1-c)Y}$

(b) one soln is with $Y = \cos 3y$, $c = 8$, $X = e^{8x}$, $u = e^{8x}\cos 3y$

M294 PIII FA95 #3

15) $X = \sin(\sqrt{\lambda}\, x)$ gives $X'' = -\lambda X$
and $X(0) = 0$ ( Note the $X(0) = 0$ rules out cosines )
Then $X'(1) = \sqrt{\lambda}\cos(\sqrt{\lambda})$ is 0 if $\sqrt{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$$X(x) = \sin\left(\frac{n\pi x}{2}\right)$$
$$\lambda = \left(\frac{n\pi}{2}\right)^2$$
$$n = 1, 3, 5 \ldots$$

M294 PIII FA95 #4

15) (a) $y u_{xx} + u_y = yX''Y + XY'$ is 0 iff $\frac{X''}{X} = -\frac{Y'}{yY}$

$$X'' = -cX \qquad Y' = cyY$$

(b) $X = \cos(\sqrt{c}\, x)$ $\quad Y = e^{cy^2/2}$ any $c > 0$ will work

(c) $u(x,y) = \cos(3x)e^{9y^2/2}$ will work, and many others.

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M294 P III SP96 # 2

17 $\boxed{a}$ $L = \pi$

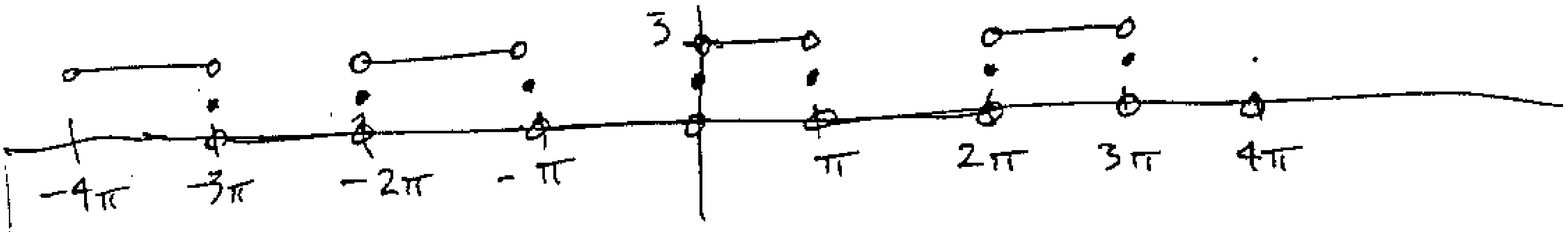
$$a_0 = \frac{1}{\pi}\int_{-\pi}^{0} 0\,dx + \frac{1}{\pi}\int_0^{\pi} 3\,dx = 3$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} 3\cos(nx)\,dx = \frac{3}{\pi}\left.\frac{\sin(nx)}{n}\right]_0^{\pi} = 0$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} 3\sin(nx)\,dx = \frac{3}{\pi}\left.\frac{\cos(nx)}{-n}\right]_0^{\pi} = \frac{3}{\pi}\,\frac{\cos(n\pi)-1}{-n}$$

$$= \begin{cases} 0 & n \text{ even} \\ \frac{6}{n\pi} & n \text{ odd} \end{cases}$$

$$f(x) \sim \frac{3}{2} + \frac{6}{\pi}\sin(x) + \frac{6}{3\pi}\sin(3x) + \cdots$$




series converges to $3/2$ at $x = 0$.

$\boxed{b}$ $u_t + u = 3u_x$ If $u(x,t) = X(x)T(t)$ then

$$XT' + XT = 3X'T$$

$\div XT$

$$\frac{T'}{T} + 1 = \frac{3X'}{X} = \text{constant } \lambda$$

$$\boxed{3X' = \lambda X} \qquad \boxed{T' + T = \lambda T}$$

M244 F SP96 #6

(a) Equation separates to $T''=-\lambda T$ and $a^2X''=(b^2-\lambda)X$; soln is

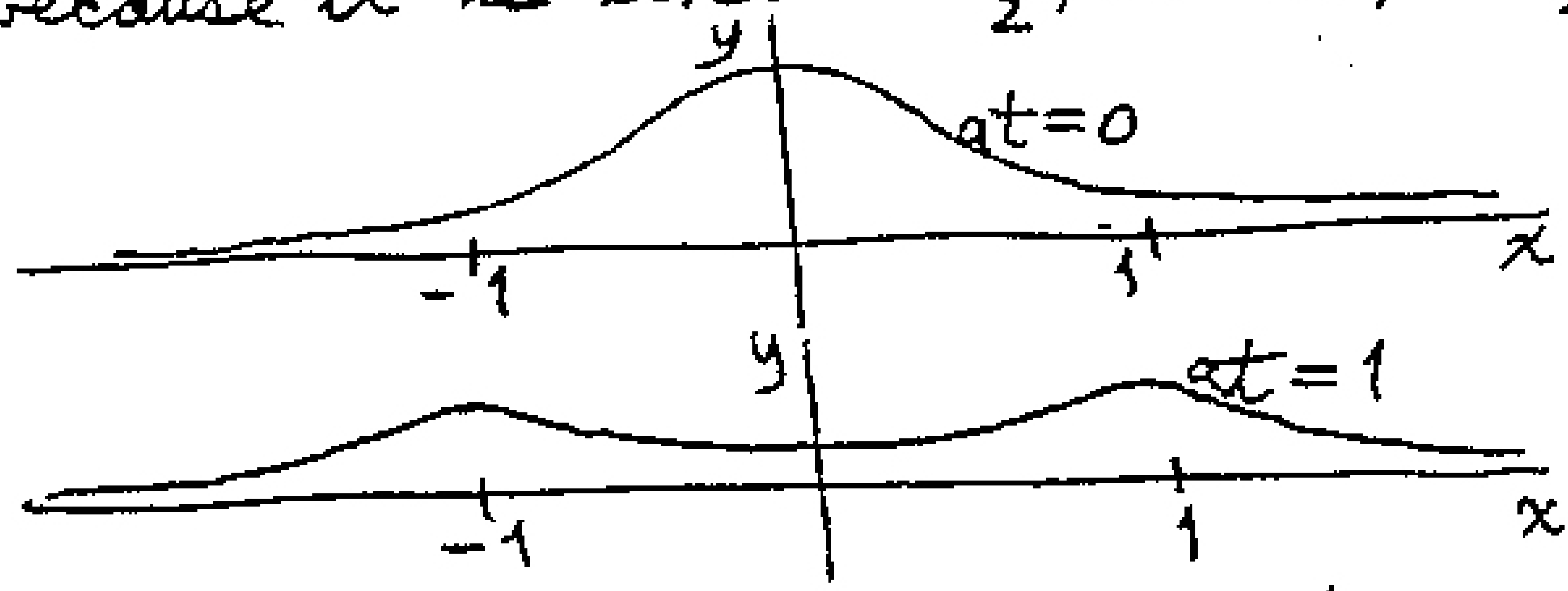
$$u(x,t)=\sum_{n=1}^{\infty} c_n\cos\left(\sqrt{b^2+\left(\frac{an\pi}{L}\right)^2}\,t\right)\sin\left(\frac{n\pi x}{L}\right)$$

$$u(x,t)=3\cos\left(\sqrt{b^2+\left(\frac{a\pi}{L}\right)^2}\,t\right)\sin\left(\frac{\pi x}{L}\right)$$

M294 F SP96 #7

) (i) (a) because it is like "$\frac{1}{2}f(x-at)+\frac{1}{2}f(x+at)$" where $f(x)=y(x,$

(d)




ii) $u(r,0)=0$ rules out cosines; $u\to 0$ at $\infty$ rules out $r^n$; $\therefore$ (a)

ii) (a) does not have 0° on boundary; (b) is not steady state
(c) does not involve $x_1$ $\therefore$ (d) none of the above

M294 F FA92 #7

(a) $-1$, $-\pi/3$, $\pi/3$; $0, 0, \pi$ (jumps); $1, 0.2, 0.2$

(b) $|x|=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos((2n-1)x)$ $\quad -\pi\le x\le\pi$ becau

M294 F FA92 #8

4)

Using $x=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos((2n-1)x)$ $\quad 0<x<\pi$

you get $3x=\frac{3\pi}{2}-\frac{12}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos((2n-1)x)$

So $\boxed{T(x,t)=\frac{3\pi}{2}-\frac{12}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos((2n-1)x)\,e^{-(2n-}}$

It works because (i) $\frac{3\pi}{2}$ and each product cos( )e
are solns to heat eqn, which is linea
(ii) $T_x \doteq \sum$ sines which are 0 at ends
and (iii) $T(x,0)=3x$ because we're given the
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