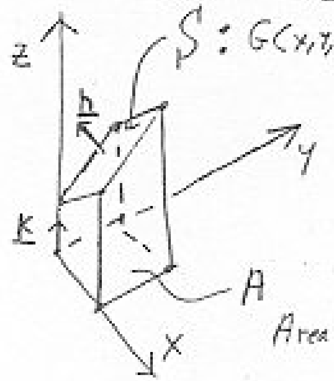
Surface Area

③ M294 SP87 P1 #7



$S: G(x,y,z) = 3z - 4x - 12y = 7$

$$\underline{n} = \frac{\nabla G}{|\nabla G|} = \frac{3\underline{k} - 4\underline{i} - 12\underline{j}}{\sqrt{9+16+144}}$$

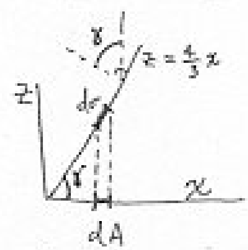
$$= \frac{-4\underline{i} - 12\underline{j} + 3\underline{k}}{13}$$

$$\text{Area} = \iint_S 1\, d\sigma = \iint_A \frac{dA}{\underline{n}\cdot\underline{k}}\, dA$$

$$= \iint_A \frac{dA}{(3/13)} = \frac{13}{3} \underbrace{\iint dA}_{\text{unit square}} = \boxed{\frac{13}{3}}$$

M294 SP88 P1 #2

5)



$\cos\gamma = 3/5$

Our basic formula for surface area is

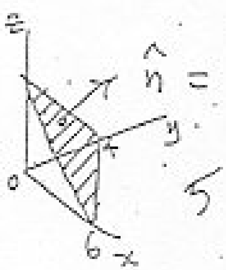
$$d\sigma = \frac{dA}{\cos\gamma} = \frac{5}{3}\, dA$$

therefore

$$\text{area of region} = \frac{5}{3}\cdot\left(\text{area of projection}\right) = \frac{5}{3}\cdot 9\pi = \boxed{15\pi}$$

M 294 FA93 P1 #9

15)



$$\hat{n} = \frac{2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}}{\sqrt{4+9+16}} \qquad d\sigma = \frac{dxdy}{|\hat{n}\cdot\hat{k}|} = \frac{\sqrt{29}}{4}\, dxdy$$

5 5

$$\text{area} = \iint \frac{\sqrt{29}}{4}\, dxdy = \boxed{\frac{\sqrt{29}}{4}\cdot\frac{1}{2}\cdot 4\cdot 6}$$

5 5

$= 3\sqrt{29}$

M294 FA93 P1 #3

16) area =

$$\iint d\sigma = \iint_{x^2+y^2 \le b^2} \frac{dx\,dy}{|\cos\gamma|}$$

for $F(x,y,z) = x^2 - y^2 - z = 0$

$\underline{\nabla} F = 2x\underline{i} - 2y\underline{j} - \underline{k}$

$$\cos\gamma = \frac{-1}{\sqrt{(2x)^2 + (2y)^2 + (-1)}}$$

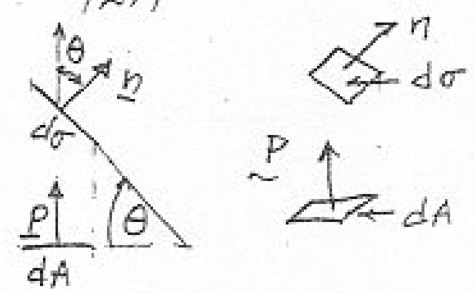
$$= \int_0^{2\pi}\int_0^b \sqrt{4r^2+1}\; r\,dr\,d\theta$$

$$= 2\pi\left[\frac{2}{3}\frac{(4r^2+1)^{3/2}}{8}\right]_0^b = \boxed{\frac{\pi}{6}\left((4b^2+1)^{3/2} - 1\right)}$$

M294 #3

18)

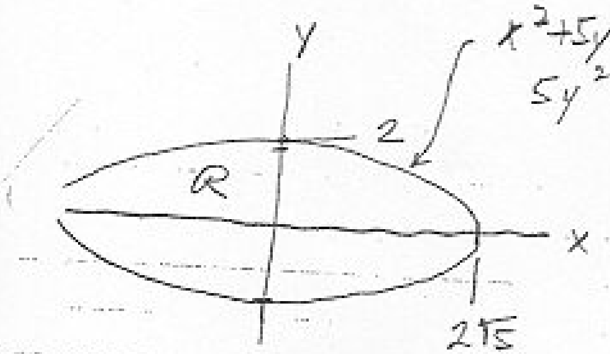
a) $\underset{\sim}{n} = \dfrac{\nabla f}{|\nabla f|}$, $f = z - x^2 - 5y^2$, $\underset{\sim}{n} = \dfrac{-2x\hat{i} - 10y\hat{j} + \hat{k}}{[4x^2+100y^2+1]^{1/2}}$



$dA = d\sigma \cdot \cos\theta$
$= d\sigma\,|\underset{\sim}{n}\cdot\underset{\sim}{p}|$

θ = angle btween $\underset{\sim}{n}$ & $\underset{\sim}{p}$

Step 1: Projection of S onto x-y plane is the elliptical region $x^2+5y^2=20$



$x^2+5y^2 = 20$
$5y^2 = 20 - x^2$

$y = \pm[4 - x^2/5]^{1/2}$

Step 2: $\dfrac{dA}{\underset{\sim}{n}\cdot\underset{\sim}{p}} = \dfrac{dx\,dy}{\underset{\sim}{n}\cdot\underset{\sim}{p}} = d\sigma$

$$\underset{\sim}{n}\cdot\underset{\sim}{p} = \underset{\sim}{n}\cdot\hat{k} = \frac{1}{[4x^2+100y^2+1]^{1/2}}$$

Step 3: $$\iint_S d\sigma = \iint_R \frac{dA}{\underset{\sim}{n}\cdot\underset{\sim}{p}} = \int_{-2\sqrt{5}}^{2\sqrt{5}}\int_{-[4-x^2/5]^{1/2}}^{+[4-x^2/5]^{1/2}} [4x^2+100y^2+1]^{1/2}\,dy\,dx$$