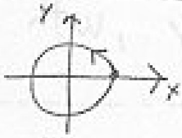
M294 SP87 P1 #3

4) Method 1:
(Do the path integral)



$x = \cos(t) \quad 0 \le t \le 2\pi$

$y = \sin(t)$

$dx = -\sin(t)\,dt$

$dy = \cos(t)\,dt$

$M = xy - x, \quad N = \frac{-x^2}{2}$

$$\underline{F}\cdot d\underline{R} = M\,dx + N\,dy = [\cos(t)\sin(t) - \cos(t)](-\sin(t))dt - \cos^3(t)\,dt$$

$$\oint \underline{F}\cdot d\underline{R} = -\int_0^{2\pi} \underbrace{\sin^2(t)}_{u^2}\underbrace{\cos(t)\,dt}_{du} + \int_0^{2\pi} \underbrace{\sin(t)}_{u}\underbrace{\cos(t)\,dt}_{du} - \int_0^{2\pi} \underbrace{\cos^3(t)\,dt}_{(1-\sin^2(t))\cos(t)\,dt}$$

$$= -\frac{\sin^3(t)}{3}\Big|_0^{2\pi} + \frac{\sin^2(t)}{2}\Big|_0^{2\pi} - \sin(t)\Big|_0^{2\pi} + \frac{\sin^3(t)}{3}\Big|_0^{2\pi}$$

$$= 0 + 0 - 0 + 0 = \boxed{0}$$

Method 2:
(GREEN THM)

$$\underline{F} = \underbrace{(xy - x)}_{M}\,\underline{i} + \underbrace{(-x^2/2)}_{N}\,\underline{j}$$

$$\oint M\,dx + N\,dy = \iint_A \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \iint_A -x - x\,dA$$

$$= -2\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} x\,dx\,dy = -2\int_{-1}^{1} \frac{x^2}{2}\Big|_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dy = -2\int_{-1}^{1} 0\,dy = \boxed{0}$$

(note: odd function integrated on even interval gives 0)

M294 FA94 P1 #4

$$\oint_C \underbrace{x\,dx + xy\,dy}_{M\,dx + N\,dy} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy = \iint_R (1+x)\,dx\,dy$$

$$= \int_0^3 \int_0^{x/3} (1+x)\,dy\,dx = \int_0^3 (1+x)\frac{x}{3}\,dx = \int_0^3 \left(\frac{x}{3} + \frac{x^2}{3}\right) dx$$

$$= \frac{x^2}{6} + \frac{x^3}{9}\Big|_0^3 = \frac{9}{6} + \frac{27}{9} = \frac{3}{2} + \frac{3}{1}\cdot\frac{2}{2} = \boxed{9/2}$$

M294 SP95 P1 #4

21) This integral can be evaluated directly or use Green's Theorem

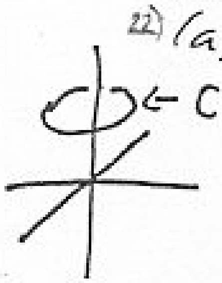
$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy \quad \text{, with } M = y \;,\; \partial M/\partial y = 1 \qquad N = -x \;,\; \partial N/\partial x = -1$$

$$\therefore \oint y\,dx - x\,dy = \iint_R (-1-1)\,dx\,dy = -2(\text{Area of } R) = -2(4) = \boxed{-8}$$

M294 SP95 F #1

22) (a) $\displaystyle\int_C (x^2+y^2+z^2)\,ds \qquad C: \vec{r}(t) = \sin t\,\vec{i} + \cos t\,\vec{j} + 8\vec{k}$

$0 \le t \le \pi$



$$= \int_0^{\pi} (\sin^2 t + \cos^2 t + 64)\sqrt{\cos^2 t + \sin^2 t}\,dt = 65\pi$$

(c) $\displaystyle\int_C \vec{F}\cdot d\vec{r}$, $C: x^2+y^2=1,\ z=0.$ $\vec{F}(x,y,z) =$

$$e^{\sin(x+y+z)}\vec{i} + e^{\sin(x+y+z)}\vec{j} +$$

$$\operatorname{arccot}(\cosh(xy))\,\vec{k}$$

$$= \iint_D \nabla\times\vec{F}\cdot\vec{k}\,dx\,dy \quad , D = x^2+y^2 \le 1,\ z=0.$$

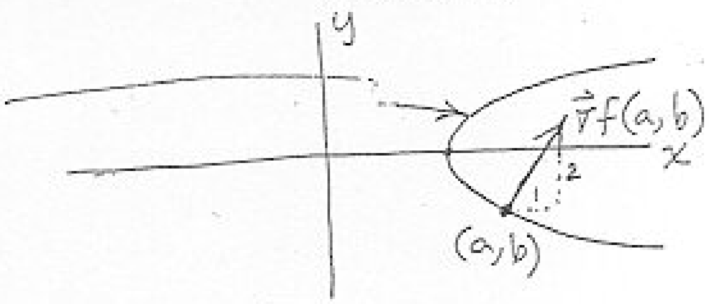
$$(\nabla\times\vec{F}\cdot\vec{k} = \frac{\partial}{\partial x}\left(e^{\sin(x+y+z)}\right) - \frac{\partial}{\partial y}\left(e^{\sin(x+y+z)}\right) = 0.$$

M294 FA95 P1 #2a

24 a) $\displaystyle\oint_{C_1} 2\,dx + x\,dy$ can do directly or use Green's Theorem, let's take circulation-curl form $\displaystyle\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$ $\qquad M = 2,\ \partial M/\partial y = 0 \qquad N = x\ \ \partial N/\partial x = 1$

$$\therefore \oint_{C_1} 2\,dx + x\,dy = \iint_R (1-0)\,dA = \text{Area of unit circle} = \boxed{\pi.}$$

M294 SP96 P1 # 4



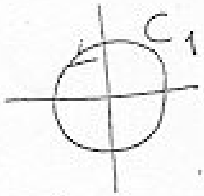
25) a) $f(x,y) = 3$ is $x - y^2 = 3$

I pick $(a,b) = (4,-1)$

$\vec{\nabla} f = \vec{\imath} - 2y\vec{\jmath}$

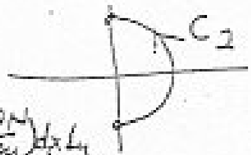
$\vec{\nabla} f(a,b) = \vec{\imath} + 2\vec{\jmath}$ is $\perp$ the curve at (a,b).

b)



Green's Theorem:

$$\oint_C M\,dy - N\,dx = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\,dy$$



$$\int_{C_1} x\,dy - y\,dx = \iint_{x^2+y^2 \le 1} \left(\frac{\partial(x)}{\partial x} - \frac{\partial(-y)}{\partial y}\right) dx\,dy$$

$$= \iint 2\,dx\,dy = 2\pi$$

$$\int_{C_2} x\,dy + y\,dx = \int_{C_2} d(xy) = xy\Big|_{(0,-1)}^{(0,1)} = 0$$

You cannot directly apply $\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$

because C_2 is not closed.

M294 SP96 F # 1

26) (a) 0 by Green's Theorem (b) $(3x^2 - 3y^2)\vec{\imath} + (-6xy)\vec{\jmath}$, 0 (c) 0