M294 SP87 F #4

9) Method 1: Note $\underline{F} = x\underline{i} + y\underline{j} + z\underline{k} = \nabla\left(\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2}\right)$

$\Rightarrow \int_a^b \underline{F}\cdot d\underline{R} = f(b) - f(a) = \frac{2^2+0^2+4^2}{2} - \frac{2^2+0^2+0^2}{2} = \boxed{8}$

Method 2: $\underline{R} = 2\cos(2\pi t)\underline{i} + 2\sin(2\pi t)\underline{j} + 4t\,\underline{k} \quad 0 \le t \le 1$

$d\underline{R} = [-4\pi\sin(2\pi t)\underline{i} + 4\pi\cos(2\pi t)\underline{j} + 4\underline{k}]dt$

$\underline{F} = \underline{R}$

$\int_a^b \underline{F}\cdot d\underline{R} = \int_0^1 \{8\pi\overbrace{[-\cos(2\pi t)\sin(2\pi t) + \sin(2\pi t)\cos 2\pi t]}^{0} + 16t\}dt$

$= 16\frac{t^2}{2}\Big|_0^1 = \boxed{8}$

M294 FA92 F #2

29) (a) $\vec{r} = t\cos t\,\hat{i} + t\sin t\,\hat{j} + t\hat{k} \quad 0 \le t \le 4\pi$

$\vec{r}\,' = (\cos t - t\sin t)\hat{i} + (\sin t + t\cos t)\hat{j} + \hat{k}$

length $= \int_0^{4\pi} |\vec{r}\,'(t)|dt = \boxed{\int_0^{4\pi}\sqrt{1+t^2+1}\;dt}$

(b) $\int_C (y+z)dx + (z+x)dy + (x+y)dz = \int_C d(xy+yz+zx)$

$= \left[xy+yz+zx\right]_{(0,0,0)}^{(4\pi,0,4\pi)} = \boxed{4^2\pi^2}$

M294 FA93 P1 #6

31) $\int_{(2,3,-1)}^{0} 2xyz\,dx + x^2z\,dy + x^2y\,dz = \left[x^2yz\right]_{(2,3,-1)}^{0}$ by the Fundamen Theorem

$= \boxed{2^2\cdot 3}$

M294 FA94 P1 #1

35) a) $\hat{i}$

b) $\int_C 3z\hat{j}\cdot d\mathbf{r}$ $\qquad d\mathbf{r} = \frac{d\mathbf{r}}{dt}\,dt \qquad \mathbf{r}(t) = 2t\hat{i} + (1-t)\hat{j} + (2-t)\hat{k}$

$$\frac{d\mathbf{r}}{dt} = 2\hat{i} - \hat{j} - \hat{k}$$

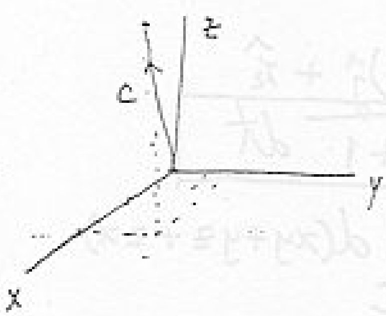
on C $\quad 3z\hat{j}\cdot d\mathbf{r} = 3(2-t)\hat{j}\cdot(2\hat{i}-\hat{j}-\hat{k})\,dt$

$$= -3(2-t)$$

$$\therefore \int_C 3z\hat{j}\cdot d\mathbf{r} = \int_0^1 -3(2-t)\,dt = -6t + 3t^2/2\Big|_0^1 = -6+3/2 = \boxed{-9/2}$$

M294 FA94 F #1

36)



Since we don't know C, one might suspect that this is a path independent integral, of the form

$$\int_C \mathbf{F}\cdot d\mathbf{r} = f(B) - f(A),$$

where A is starting point, B is ending point of C, and $\nabla f = \mathbf{F}$

Let $\mathbf{F} = \cos y\,\hat{i} - x\sin y\,\hat{j} + \hat{k}$, $\quad d\mathbf{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

Try $\quad f = x\cos y + z$

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k} = \cos y\,\hat{i} - x\sin y\,\hat{j} + \hat{k} \checkmark$$

$$\therefore \int_C \mathbf{F}\cdot d\mathbf{r} = f(2, \pi/2, 5) - f(0) = 2\cdot\cos\frac{\pi}{2} + 5 - 0 = \boxed{5}$$

37) M294 SP95 P1 #1

b) $\partial f/\partial x = 2xyz + \sin x \xrightarrow{\text{integrate}} f = x^2yz - \cos x + g(y,z)$

$\partial f/\partial y = x^2 z = x^2 z + \partial g/\partial y \Rightarrow \partial g/\partial y = 0$
$\partial f/\partial z = x^2 y = x^2 z + \partial g/\partial z \Rightarrow \partial g/\partial z = 0$ thus g = constant $\therefore \boxed{f = x^2yz - \cos x}$

b) $\int_C \vec{F}\cdot d\vec{r} = f(1,1,\pi) - f(\pi,0,0) = \pi - \cos 1 - (0 - \cos\pi) = 1 + \pi - \cos 1$

~~$= 1 - \cos\pi - (0 - \cos\pi)$~~
~~$= 1 - (-1) - (-(-1)) = 1$~~

M294 SP95 F #1b

38) (b) $\int_C \vec{F}\cdot d\vec{r}$, $\vec{F}(x,y) = y\vec{i} + x\vec{j}$, $C: \vec{r}(t) = e^{\sin t}\vec{i} + t\vec{j}$, $0 \le t \le \pi$.

$F = \nabla f \qquad f(x,y) = xy.$

$$\int_C \vec{F}\cdot d\vec{r} = f(\vec{r}(\pi)) - f(\vec{r}(0)) = \pi$$

M294 SP96 PI #1

41) a) $\displaystyle\int_{(0,0,0)}^{(4,0,2)} d(x^2z^3) = x^2z^3\Big|_{(0,0,0)}^{(4,0,2)} = 4^2 2^3$

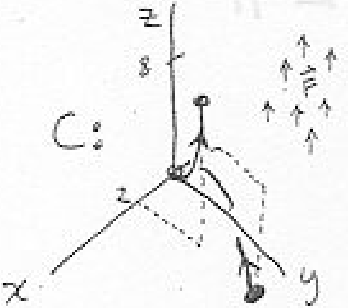
b) $\vec{r}(t) = (\vec{i} + 3\vec{k}) + t\left((2-1)\vec{i} + (5-0)\vec{j} + (0-3)\vec{k}\right)$ there are many correct solutions besides this one.

M293 FA96 P3 #3

45) (a) $\vec{r}(\pm 2) = \pm 2\vec{i} + 4\vec{j} \pm 8\vec{k}$

$\vec{r}(0) = \vec{0}$

C:



(b) $\vec{F} = \frac{1}{5}\vec{k}$ points "up" and C moves upward, so $\vec{F}\cdot d\vec{r}$ is positive (except at origin) $\therefore$ integral > 0.

(c) $\displaystyle\int_C \vec{F}\cdot d\vec{r} = \int_{-2}^{2} \frac{1}{5}\vec{k}\cdot\left(dt\,\vec{i} + 2t\,dt\,\vec{j} + 3t^2\,dt\,\vec{k}\right)$

$\displaystyle = \int_{-2}^{2} \frac{3}{5}t^2\,dt = \boxed{\frac{16}{5}}$