M294 PIII SP87 #8

1)

$$3 = \left\langle \begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}, v_3 \right\rangle = \langle c_1v_1+c_2v_2+c_3v_3+c_4v_4 \, , \, v_3\rangle = c_1\cancelto{0}{\langle v_1, v_3\rangle} + c_2\cancelto{0}{\langle v_2, v_3\rangle} + c_3\langle v_3, v_3\rangle + c_4\cancelto{0}{\langle v_4, v_3\rangle}$$

$$= c_3\langle v_3, v_3\rangle$$

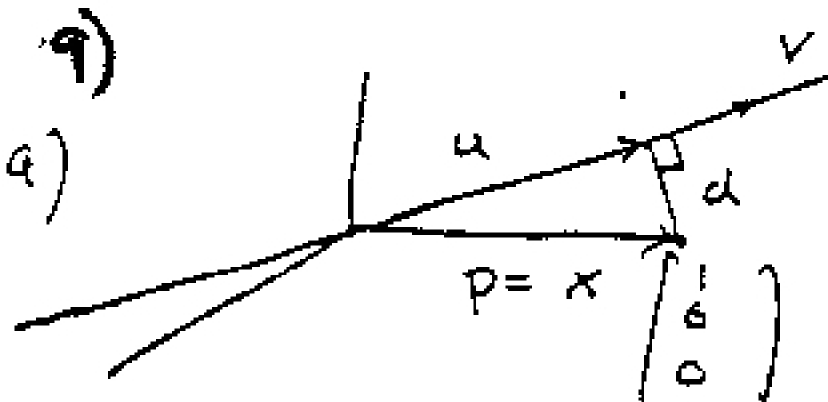
$$= c_3\,(9+4+9+16+16) = 54\,c_3$$

$$c_3 = \frac{3}{54} = \boxed{\frac{1}{18}}$$

M293 F FA95 #5

9)

a)



$L = Sp\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\right\}$

let $v = \begin{pmatrix}1\\1\\1\end{pmatrix}$

$\begin{bmatrix}1\\0\\0\end{bmatrix}$

let $u = \left(\frac{x\cdot v}{v\cdot v}\right)v = \left(\frac{1}{3}\right)\begin{pmatrix}1\\1\\1\end{pmatrix}$ = projection of P onto v

$\|u\| = \sqrt{\frac{1}{9}+\frac{1}{9}+\frac{1}{9}} = \frac{1}{\sqrt{3}}$

$\therefore d = \sqrt{1-\frac{1}{3}} = \sqrt{\frac{2}{3}}$

Alternative: by calculus, d^2 = (distance)^2 from P to typical point of L

$d^2(s) = (s-1)^2+s^2+s^2 = 3s^2-2s+1$

$\frac{d}{ds}\left[d^2\right] = 6s-2 = 0, \quad s = \frac{1}{3}$

$\therefore d^2 = \frac{1}{3} - \frac{2}{3} + 1 = \frac{2}{3}, \quad d = \sqrt{\frac{2}{3}}$

b) $u = \left(\frac{x\cdot v}{v\cdot v}\right)v = \left(\frac{1}{4}\right)\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$

$\|u\| = \sqrt{\frac{4}{16}} = \frac{1}{2}, \quad d = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}}$

or $d^2 = (s-1)^2 + 3s^2 = 4s^2-2s+1$

$\frac{d}{ds}(d^2) = 0 = 8s-2, \quad s = \frac{1}{4}$

$d^2 = \frac{4}{16} - \frac{1}{2} + 1 = \frac{3}{4}, \quad d = \sqrt{3/4}$

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10) (a) $\begin{bmatrix} c_1 - c_2 \\ c_1 \\ 2c_2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

call b_1, b_2 then $\{b_1, b_2\}$ is a basis

Use $u_1 = b_1$, and choose α so that $u_2 = b_2 - \alpha b_1$ is $\perp u_1$: $u_1 \cdot u_2 = b_1 \cdot b_2 - \alpha b_1 \cdot b_1 = -1 - \alpha \cdot 2$

so $\alpha = -1/2$, $u_2 = b_2 + \frac{1}{2} b_1 = \begin{bmatrix} -1/2 \\ 1/2 \\ 2 \end{bmatrix}$

then $\frac{u_1}{\|u_1\|} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$, $\frac{u_2}{\|u_2\|} = \frac{1}{\sqrt{\frac{1}{4}+\frac{1}{4}+4}} \begin{bmatrix} -1/2 \\ 1/2 \\ 2 \end{bmatrix}$ is an orthonormal basis

(b) $c_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ \sqrt{5} \end{bmatrix} \cdot b_2 = 1/2$, and $1 = \|b_2\|^2 = \frac{1}{4} + \alpha^2$ so $\alpha = \pm \frac{\sqrt{3}}{2}$

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11) The answer is b).