M294 P II SP87 #4

13) a)

$$\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & -1-\lambda & 4 \\ 0 & 2 & -8-\lambda \end{vmatrix} =$$

$$(1-\lambda)(\lambda^2+9\lambda+0)$$

The roots (and hence the eigenvalues) are

$\lambda = 1, 0, -9$

b) eigenvector for $\lambda = 1$:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & -2 & 4 \\ 0 & 2 & -9 \end{bmatrix} \underline{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The matrix is already reduced enough to work with.

$$\boxed{V = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}$$

The other eigenvectors are

$$\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} \ (\lambda = 0) \qquad \begin{bmatrix} -1 \\ -5 \\ 10 \end{bmatrix} \ (\lambda = -9)$$

14)

$$AV_1 = \begin{bmatrix} 5 \\ 5 \\ 5 \\ -5 \\ 5 \end{bmatrix} = 5V_1$$

So $\boxed{5 \text{ is an eigenvalue}}$

The other 2 (which are easy to find) are

8, since $AV_2 = 8V_2$

and 2, since $AV_3 = 2V_3$

M294 F SP87 #10

15)

10) $[A] = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. Find eigenvectors and eigenvalues.

$\det([A-\lambda I]) = \begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 1 = \lambda^2 - 2\lambda = 0$

$\Rightarrow \lambda = 0, 2$

For $\lambda = 0$, $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} v_1 = \vec{0}$, so take $\underline{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ } e-vectors

For $\lambda = 2$, $\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} v_2 = \vec{0}$, so take $\underline{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ }

Let $[P] = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = [\underline{v}_1 | \underline{v}_2]$

Then $[P]^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \leftarrow$ [by inspection or row operations]

So $[P]^{-1}[A][P] = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$.

Let $[K] = [P]^{-1} = \boxed{\frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = [R]}$

M294 F FA92 #6

31) gen soln $y = c_1 \cos\sqrt{\lambda}\, x + c_2 \sin\sqrt{\lambda}\, x$ (if $\lambda > 0$)

$y'(0) = c_2\sqrt{\lambda} = 0$ so $c_2 = 0$

$y(1) = c_1 \cos\sqrt{\lambda}$ will be 0 if $\sqrt{\lambda} = \frac{\pi}{2}$

$$\boxed{\begin{array}{l} y = c_1 \cos\left(\frac{(2n+1)\pi}{2} x\right) \quad n = 0, 1, 2, 3, \ldots \\ \lambda = \left(\frac{(2n+1)\pi}{2}\right)^2 \end{array}}$$

one more case ; $\lambda = 0$ gen soln $y = mx + b$

$\left.\begin{array}{l} y'(0) = m = 0 \\ y(1) = b = 0 \end{array}\right\}$ So

M293 F FA93 #6

39) (a) $\det(A-\lambda I) = -\lambda \det\begin{pmatrix} -\lambda & -1 \\ -1 & 1-\lambda \end{pmatrix} + 2\det\begin{pmatrix} -2 & 1 \\ -1 & 1-\lambda \end{pmatrix} + \det\begin{pmatrix} -2 & 1 \\ -\lambda & -1 \end{pmatrix}$

$= -\lambda(\lambda^2 - \lambda - 1) + 2(-2 + 2\lambda + 1) + 2 + \lambda$

$= -\lambda^3 + \lambda^2 + 6\lambda = -\lambda(\lambda^2 - \lambda - 6)$ ✓

(b) $\lambda = 0$, and $\lambda^2 - \lambda - 6 = (\lambda+2)(\lambda-3)$ $\lambda = -2, 3$

$\underline{\lambda = 0}$ $(A - 0I)u = \begin{bmatrix} 0 & -2 & 1 \\ -2 & 0 & -1 \\ 1 & -1 & 1 \end{bmatrix} u$ $-2u_2 + u_3 = 0$ $u_2 = \frac{1}{2}u_3$, let $u = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

$u_1 = -\frac{1}{2}u_3$ $Au = 0$

$\underline{\lambda = -2}$ $(A + 2I)v = \begin{bmatrix} 2 & -2 & 1 \\ -2 & 2 & -1 \\ 1 & -1 & 3 \end{bmatrix} v$

$2v_1 - 2v_2 + v_3 = 0$

$v_1 - v_2 + 3v_3 = 0$

$v_3 = 0$

$v_1 = v_2$

let $v = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $Av = -2v$

$\underline{\lambda = 3}$ $(A - 3I)w = \begin{bmatrix} -3 & -2 & 1 \\ -2 & -3 & -1 \\ 1 & -1 & -2 \end{bmatrix} w$

$A - 3I \sim \begin{bmatrix} 0 & -5 & -5 \\ 0 & -5 & -5 \\ 1 & -1 & -2 \end{bmatrix}$

w_3 free

$w_2 = -w_3$

$w_1 = +2w_3 + w_2 = w_3$ let $w = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$Aw = 3w$

$$\boxed{A\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} = 0 \qquad A\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad A\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 3\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}}$$

M293 F FA95 #8

40) $A = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$ $\det(A - \lambda I) = \lambda^2 - 3\lambda - 4$
$= (\lambda+1)(\lambda-4)$

(a) $\lambda = -1$

$(A - \lambda I)u = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} u$

$u_1 + 2u_2 = 0$, $u_2 = -\frac{1}{2}u_1$

let $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\lambda = 4$

$(A - \lambda I)v = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix} v$

$-4v_1 + 2v_2 = 0$

let $v = \begin{bmatrix} 1 \\ +2 \end{bmatrix}$

$$\boxed{A\begin{bmatrix} 2 \\ -1 \end{bmatrix} = -1\begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 2-2=0, A\begin{bmatrix} 1 \\ +2 \end{bmatrix} = 4\begin{bmatrix} 1 \\ +2 \end{bmatrix}}$$

(b) let $P = [u\ v] = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ then $P^{-1}AP = \begin{pmatrix} -1 & 0 \\ 0 & 4 \end{pmatrix}$

you may put other eigenvectors as the columns such as

$P = [v\ u]$ then $P^{-1}AP = \begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}$

$P = [3u\ -v]$ then $P^{-1}AP = \begin{pmatrix} -1 & 0 \\ 0 & 4 \end{pmatrix}$

etc

(c) use $P = \left[\frac{u}{\|u\|}\ \frac{v}{\|v\|}\right]$ then P is orthogonal.

M293 F SP96 #14

43) The answer is c).

M293 F SP96 #15

44) The answer is c).

M293 F SP96 #38

45) False.

M293 F SP96 #39

46) True.

M293 F SP96 #40

47) True.

M294 P III SP98 #5

56) $|A - \lambda I| = \begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & -1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2(1-\lambda) + (1-\lambda) = (1-\lambda)[1 + (\lambda-2)^2] = 0$

$\Longrightarrow \lambda_1 = 1$; $\lambda_2 = 2+i$; $\lambda_3 = 2-i$

for $\lambda_1 = 1$, $A - I \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \Rightarrow c\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ are eigenvectors

for $\lambda_2 = 2+i$, $\begin{bmatrix} 2-(2+i) & 1 & 1 \\ 0 & 1-(2+i) & 0 \\ -1 & -1 & 2-(2+i) \end{bmatrix} \to \begin{bmatrix} -i & 1 & 1 \\ 0 & -1-i & 0 \\ -1 & -1 & -i \end{bmatrix} \to \begin{bmatrix} -i & 1 & 1 \\ 0 & -1-i & 0 \\ 0 & -1+i & 0 \end{bmatrix}$

$\Rightarrow c\begin{bmatrix} 1 \\ 0 \\ i \end{bmatrix}$ are eigenvectors.

for $\overline{\lambda_2} = 2-i$, $c\begin{bmatrix} 1 \\ 0 \\ -i \end{bmatrix}$ are eigenvectors. $\left(\begin{bmatrix} 1 \\ 0 \\ i \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + i\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right.$, conjugate eigenvec is therefore $\begin{bmatrix} 1 \\ 0 \\ -i \end{bmatrix}$