The three "spaces" on the simple board game shown are labeled "C", "T", and "D" for coin, tetrahedron, and die. On one turn a player advances clockwise a random number of spaces as determined by shaking and dropping the object on their present space (from the C position a player moves 1 or 2 spaces with equal probabilities, from the T space a player moves 1-4 spaces with equal probabilities, and from the D space a player moves 1-6 spaces with equal probabilities).

In very long games what fraction of the moves end up on the D space on average? (Hint: Use exact arithmetic rather than truncated decimal representations. The needed calculations easily fit in the space allotted.)

Gameboard

\[ A_{ij} = \begin{bmatrix} \text{Probability of switch from \( j \to i \) if at \( j \) at \( n \) move} \\ \text{switch from \( j \to i \) if at \( j \)} \end{bmatrix} = A_{ij} \]

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0</td>
<td>( \frac{1}{4} )</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>T</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{4} )</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>D</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{3} )</td>
</tr>
</tbody>
</table>

Hint: If you use this table, briefly define the entries.

Let \( V = \begin{bmatrix} V_C \\ V_T \\ V_D \end{bmatrix} \) = probabilities of being on squares C, T & D at move \( n \).

**Basic Markov eq:**

\[ V^{n+1} = AV^n \rightarrow \text{long term steady state is } AV = V \]

\[ (A-I)V = 0. \text{ Solve for } V \text{ by row operations...} \]

\[ \begin{bmatrix} -1 & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{2} & -\frac{3}{4} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & -\frac{2}{3} \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{1}{4} & \frac{1}{3} \\ 0 & -\frac{5}{8} & \frac{3}{2} \\ 0 & \frac{5}{8} & -\frac{1}{2} \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{1}{4} & \frac{1}{3} \\ 0 & -\frac{5}{8} & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix} \rightarrow V_C = \frac{5}{8} \text{ free} \]

\[ -V_C + \frac{1}{4} V_T + \frac{1}{3} S = 0 \]
\[ -\frac{5}{8} V_T + \frac{1}{3} S = 0 \rightarrow V_T = \frac{4}{5} S \]

\[ \Rightarrow V = S \begin{bmatrix} \frac{8}{15} \\ \frac{4}{5} \\ 1 \end{bmatrix} = t \begin{bmatrix} 8 \\ 12 \\ 15 \end{bmatrix} \]

\[ \Rightarrow V = \begin{bmatrix} V_C \\ V_T \\ V_D \end{bmatrix} = \frac{1}{35} \begin{bmatrix} 8 \\ 12 \\ 15 \end{bmatrix} \]

\[ \Rightarrow \text{but for sensible probability need } V_C + V_T + V_D = 1 \]
\[ \Rightarrow t = \frac{1}{8+12+15} = \frac{1}{35} \]

\[ \text{prob. of being on D in the long run} \]
\[ = V_D = \frac{15}{35} = \frac{3}{7} \]