#1. a)\[
\begin{bmatrix}
0 & -2 & -1 \\
2 & 6 & 0 \\
3 & 7 & 1
\end{bmatrix}
\xrightarrow{\text{Row 2} \leftrightarrow \text{Row 1}}
\begin{bmatrix}
0 & 1 & 0 \\
2 & 7 & 1 \\
3 & 7 & 1
\end{bmatrix}
\xrightarrow{\text{Row 3} \rightarrow \text{Row 2}}
\begin{bmatrix}
0 & 1 & 0 \\
1 & 3 & 0 \\
3 & 7 & 1
\end{bmatrix}
\xrightarrow{\text{Row 2} \rightarrow \text{Row 3}}
\begin{bmatrix}
0 & 1 & 0 \\
1 & 3 & 0 \\
0 & -3 & 2
\end{bmatrix}
\xrightarrow{\text{Row 3} \rightarrow \text{Row 2}}
\begin{bmatrix}
1 & 5 & 0 \\
0 & -2 & 1 \\
0 & 0 & 1
\end{bmatrix}
\xrightarrow{\text{Row 2} \rightarrow \text{Row 3}}
\begin{bmatrix}
1 & 5 & 0 \\
0 & -2 & 1 \\
0 & 0 & 1
\end{bmatrix}
\]

Since \(Ax = 0\) had only the trivial solution, then the columns of matrix \(A\) are linearly independent.

b) \(W = x_1 y_1 + x_2 y_2 + x_3 y_3\)

Since \(W\) spans all of \(R^3\), \(W\) is a 3-dimensional space.

#5

For \(y\) to be in span \(\{v_1, v_2, v_3, v_5\}\), it must be a linear combination of \(v_1, v_2, v_3\). Another way to look at it is \(Ax = b\) must be consistent for \(A = [v_1, v_2, v_3]\) and \(b = y\).

\[
\begin{bmatrix}
1 & -2 & 1 \\
-1 & 1 & 1 \\
-2 & -7 & 1
\end{bmatrix}
\xrightarrow{\text{Row 2} \rightarrow \text{Row 1}}
\begin{bmatrix}
1 & -2 & 1 \\
0 & 0 & 1 \\
-2 & -7 & 1
\end{bmatrix}
\xrightarrow{\text{Row 3} \rightarrow \text{Row 2}}
\begin{bmatrix}
1 & -2 & 1 \\
0 & 0 & 1 \\
-2 & -3 & 1
\end{bmatrix}
\]

\(x = 3, y = ?\)

\(y = x_1 v_1 + x_2 v_2 + x_3 v_3\)

For there to be a solution, \(h - 3 + 3 = 0\) or \(h = 0\).

#6

If a set contains more vectors than there are entries in each vector, then the set is linearly dependent.
Section 1.5

7) The answer is False.

8) FALSE; let $A$ be a $m \times n$ matrix, where $m = 6$ and $n = 4$. The only way for the columns of $A$ (vectors in $\mathbb{R}^6$) to span $\mathbb{R}^6$ is to have a pivot position in every row. This is impossible with 6 rows and 4 columns. i.e. a set of 4 vectors in $\mathbb{R}^6$ can not span $\mathbb{R}^6$.

13) a) If $A$ is a (possibly not square) matrix with $A^T A$ invertible are the columns of $A$ linearly independent? (yes, no, maybe).

Assume cols. $A$ not L.I. $\Rightarrow$ $A X = 0$ has non-trivial solns.

Multiply both sides by $A^T$ $\Rightarrow$ $A^T A X = A^T 0$

$\Rightarrow (i)$ $A^T A X = 0$ has non-trivial solns.

BUT, prob. statement said $A^T A$ was invertible.

so (i) has no non-triv. solns.

CONTRACTION $\Rightarrow$ [YES, cols. of $A$ L.I.]