MATH 293 PRELIM I SPRING 1990 #2

#2) $3\underline{u} + 4\underline{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \qquad -2\underline{u} + 7\underline{v} = \begin{bmatrix} -1 \\ 2 \\ -7 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3 & 4 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} \underline{u} \\ \underline{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 2 & -7 & 0 \end{bmatrix} \longrightarrow \left[\begin{array}{cc|cccc} 3 & 4 & 0 & 1 & 0 & 1 \\ -2 & 7 & -1 & 2 & -7 & 0 \end{array}\right] \xrightarrow[\text{ROW 2}]{\text{ROW 1 +}}$$

$$\left[\begin{array}{cc|cccc} 1 & 11 & -1 & 3 & -7 & 1 \\ -2 & 7 & -1 & 2 & -7 & 0 \end{array}\right] \xrightarrow[\text{2×ROW 1}]{\text{ROW 2 +}} \left[\begin{array}{cc|cccc} 1 & 11 & -1 & 3 & -7 & 1 \\ 0 & 29 & -3 & 8 & -21 & 2 \end{array}\right] \xrightarrow{\text{ROW 2/29}}$$

$$\left[\begin{array}{cc|cccc} 1 & 11 & -1 & 3 & -7 & 1 \\ 0 & 1 & -3/29 & 8/29 & -21/29 & 2/29 \end{array}\right] \xrightarrow[\text{11×ROW 2}]{\text{ROW 1 −}} \left[\begin{array}{cc|cccc} 1 & 0 & -1+33/29 & 3-88/29 & -7+231/29 & 1-22/29 \\ 0 & 1 & -3/29 & 8/29 & -21/29 & 2/29 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|cccc} 1 & 0 & 4/29 & -1/29 & 28/29 & 7/29 \\ 0 & 1 & -3/29 & 8/29 & -21/29 & 2/29 \end{array}\right]$$

$$\therefore \underline{u} = 1/29 \begin{bmatrix} 4 \\ -1 \\ 28 \\ 7 \end{bmatrix} \qquad \underline{v} = 1/29 \begin{bmatrix} -3 \\ 8 \\ -21 \\ 2 \end{bmatrix}$$

#6)

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SUPPOSE THE EQUATION $A\underline{x} = \underline{b}$ IS CONSISTENT FOR SOME GIVEN $\underline{b}$, AND LET $\underline{p}$ BE A SOLUTION. THEN THE SOLUTION SET OF $A\underline{x} = \underline{b}$ IS THE SET OF ALL VECTORS OF THE FORM $\underline{w} = \underline{p} + \underline{v}_h$, WHERE $\underline{v}_h$ IS ANY SOLUTION OF THE HOMOGENEOUS EQUATION $A\underline{x} = \underline{0}$

$$\underline{p} = \underline{x}_p = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad \underline{v}_h = \underline{x}_h = s\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 2 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

$\underline{x} = \underline{w}$?

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ s \\ t \end{bmatrix} = \underline{x} \ ?$$

a)

$$\underline{x} = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 3 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & 2 & 0 & 3 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 3 \end{array}\right] \xrightarrow[\text{ROW 1}]{\text{ROW } i -} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -3 & 3 \end{array}\right] \xrightarrow[\text{ROW 3}/-3]{\text{ROW 3} + \text{ROW 2},\ \text{ROW 4} + \text{ROW 3}}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[\text{ROW 1} - 2\times\text{ROW 3}]{\text{ROW 2} + 2\times\text{ROW 3}} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\text{ROW 1} - \text{ROW 2}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$\Rightarrow s = 1,\ t = -1 \quad \therefore \ \underline{x} = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 3 \end{bmatrix}$ IS A SOLUTION TO $A\underline{x} = \underline{b}$

b)

$$\underline{x} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \end{bmatrix} \qquad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & 0 & 3 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 2 \end{array}\right] \xrightarrow{\text{ROW } i - \text{ROW 1}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -3 & 1 \end{array}\right] \xrightarrow{\text{ROW 3} + \text{ROW 2}}$$

$$\left.\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 1 \end{array}\right]\right\} \Rightarrow$$ Inconsistent, $-3t = 2$ AND $-3t = 1$ CAN NOT BOTH HOLD TRUE

$\therefore \ \underline{x} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \end{bmatrix}$ IS NOT A SOLUTION TO $A\underline{x} = \underline{b}$

B. MORGAN