M294 PII SP87#3

7)

$$\begin{aligned} x \quad\quad &+ z = 0 \\ -y &+ 4z = 0 \\ 2y &- 8z = 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 4 \\ 0 & 2 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 2 & -8 & 0 \end{bmatrix} \xrightarrow{-1R_2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 2 & -8 & 0 \end{bmatrix} \xrightarrow{-2R_2+R_3} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad x + z = 0 \Rightarrow \boxed{x = -c}$$

$$y - 4z = 0 \Rightarrow \boxed{y = 4c}$$

$$\boxed{z = c}$$

SP90 PI MATH 293 #1

12) a.

$$\begin{aligned}2x_1-4x_2-2x_3&=0\\5x_1-x_2-x_3&=6\\-3x_1+2x_2+x_3&=-2\end{aligned} \Rightarrow \begin{bmatrix}2&-4&-2\\5&-1&-1\\-3&2&1\end{bmatrix}\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=\begin{bmatrix}0\\6\\-2\end{bmatrix}$$

$$[A|b]=\left[\begin{array}{ccc|c}2&-4&-2&0\\5&-1&-1&6\\-3&2&1&-2\end{array}\right]\xrightarrow[\text{new } r_3=r_3+\frac{3}{2}r_1]{\text{new } r_2=r_2-\frac{5}{2}r_1}\left[\begin{array}{ccc|c}2&-4&-2&0\\0&9&4&6\\0&-4&-2&-2\end{array}\right]\xrightarrow[\text{new } r_3=r_3+\frac{4}{9}r_2]{\text{new } r_1=r_1+\frac{4}{9}r_2}$$

$$\left[\begin{array}{ccc|c}2&0&-2/9&24/9\\0&9&4&6\\0&0&-2/9&6/9\end{array}\right]\xrightarrow[\text{new } r_2=r_2+4(\frac{9}{2})r_3]{\text{new } r_1=r_1-r_3}\left[\begin{array}{ccc|c}2&0&0&18/9\\0&9&0&18\\0&0&-2/9&6/9\end{array}\right]\xrightarrow[\text{new } r_3=-\frac{9}{2}r_3]{\text{new } r_1=\frac{1}{2}r_1,\ \text{new } r_2=\frac{1}{9}r_2}$$

$$\left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&2\\0&0&1&-3\end{array}\right]\Rightarrow \vec{x}=\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=\boxed{\begin{Bmatrix}1\\2\\-3\end{Bmatrix}} \leftarrow \text{ans.}$$

b.

$$\begin{aligned}-x_1+3x_2+2x_3&=1\\3x_1-2x_2-x_3&=3\\x_1+4x_2+3x_3&=5\end{aligned} \Rightarrow \begin{bmatrix}-1&3&2\\3&-2&-1\\1&4&3\end{bmatrix}\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=\begin{bmatrix}1\\3\\5\end{bmatrix}$$

$$[A|b]=\left[\begin{array}{ccc|c}-1&3&2&1\\3&-2&-1&3\\1&4&3&5\end{array}\right]\xrightarrow[\text{new } r_3=r_3+r_1]{\text{new } r_2=r_2+3r_1}\left[\begin{array}{ccc|c}-1&3&2&1\\0&7&5&6\\0&7&5&6\end{array}\right]\xrightarrow[\text{new } r_3=r_3-r_2]{\text{new } r_1=r_1-\frac{3}{7}r_2}$$

$$\left[\begin{array}{ccc|c}-1&0&-1/7&-11/7\\0&7&5&6\\0&0&0&0\end{array}\right]\xrightarrow[\text{new } r_2=\frac{1}{7}r_2]{\text{new } r_1=-r_1}\left[\begin{array}{ccc|c}1&0&1/7&11/7\\0&1&5/7&6/7\\0&0&0&0\end{array}\right]\Rightarrow\begin{cases}x_1+\frac{1}{7}x_3=\frac{11}{7}\Rightarrow x_1=\frac{11}{7}-\frac{1}{7}x_3\\x_2+\frac{5}{7}x_3=\frac{6}{7}\Rightarrow x_2=\frac{6}{7}-\frac{5}{7}x_3\\x_3\text{ is free}\end{cases}$$

$$\vec{x}=\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=\begin{Bmatrix}\frac{11}{7}-\frac{1}{7}x_3\\\frac{6}{7}-\frac{5}{7}x_3\\x_3\end{Bmatrix}=\boxed{\begin{Bmatrix}\frac{11}{7}\\\frac{6}{7}\\0\end{Bmatrix}+\begin{Bmatrix}-\frac{1}{7}\\-\frac{5}{7}\\1\end{Bmatrix}x_3} \leftarrow \text{ans.}$$

c.

$$\begin{aligned}x_1+3x_2-4x_3&=0\\2x_1-x_2-x_3&=0\\3x_1-4x_2+x_3&=3\end{aligned} \Rightarrow \begin{bmatrix}1&3&-4\\2&-1&-1\\3&-4&-1\end{bmatrix}\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=\begin{bmatrix}0\\0\\3\end{bmatrix}$$

$$[A|b]=\left[\begin{array}{ccc|c}1&3&-4&0\\2&-1&-1&0\\3&-4&-1&3\end{array}\right]\xrightarrow[\text{new } r_3=r_3-3r_1]{\text{new } r_2=r_2-2r_1}\left[\begin{array}{ccc|c}1&3&4&0\\0&-7&7&0\\0&-13&13&3\end{array}\right]\xrightarrow[\text{new } r_3=r_3-\frac{13}{7}r_2]{\text{new } r_1=r_1+\frac{3}{7}r_2}$$

$$\left[\begin{array}{ccc|c}1&0&-1&0\\0&1&-1&0\\0&0&0&3\end{array}\right]\Rightarrow\begin{cases}x_1-x_3=0\\x_2-x_3=0\\0=3\leftarrow 0\neq 3\text{, inconsistent system (no solution)}\end{cases}$$

T.D.

A2.

21) M293 SP94 P2 #3

$$\begin{aligned} x-y+2z-2w&=-2\\ 2x+0y+3z-4w&=-1\\ x-3y-z-2w&=-5 \end{aligned} \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & -2 & -2\\ 2 & 0 & 3 & -4 & -1\\ 1 & -3 & -1 & -2 & -5 \end{array}\right]$$

$\therefore$ This corresponds to columns 1,2,3,4, and 6 of the larger system.

Therefore the rref for the smaller system is

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -1/2\\ 0 & 1 & 0 & 0 & 3/2\\ 0 & 0 & 1 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} x-2w=-1/2 \Rightarrow x=-1/2+2w\\ y=3/2\\ z=0\\ w \text{ is free} \end{cases}$$

$$\begin{Bmatrix} x\\ y\\ z\\ w \end{Bmatrix} = \begin{Bmatrix} -1/2+2w\\ 3/2\\ 0\\ w \end{Bmatrix} = \begin{Bmatrix} -1/2\\ 3/2\\ 0\\ 0 \end{Bmatrix} + \begin{Bmatrix} 2\\ 0\\ 0\\ 1 \end{Bmatrix} w$$

And if $w=t$,

$$\boxed{\text{b) } \frac{1}{2}\begin{Bmatrix} -1\\ 3\\ 0\\ 0 \end{Bmatrix} + \begin{Bmatrix} 2\\ 0\\ 0\\ 1 \end{Bmatrix} t} \quad \leftarrow \text{ans.}$$

SP94 P II M293 #4

22)

$$\begin{bmatrix} 1 & 0 & -1 & 3\\ 2 & 2 & 0 & 4\\ 1 & 4 & 3 & -1 \end{bmatrix} \xrightarrow[\text{new } r_3 = r_3 - r_1]{\text{new } r_2 = r_2 - 2r_1} \begin{bmatrix} 1 & 0 & -1 & 3\\ 0 & 2 & 2 & -2\\ 0 & 4 & 4 & -4 \end{bmatrix} \xrightarrow{\text{new } r_3 = r_3 - 2r_2}$$

$$\begin{bmatrix} 1 & 0 & -1 & 3\\ 0 & 2 & 2 & -2\\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{new } r_2 = \frac{1}{2} r_2} \boxed{\text{d) } \begin{bmatrix} 1 & 0 & -1 & 3\\ 0 & 1 & 1 & -1\\ 0 & 0 & 0 & 0 \end{bmatrix}} \quad \leftarrow \text{ans.}$$

F95 P II MATH 293 #1

27) a.

$$\begin{aligned} 2x_1 + 4x_3 &= 10 \\ 2x_1 + x_2 + 3x_3 &= 14 \\ 4x_1 + x_2 + 7x_3 + x_4 &= 27 \\ -2x_1 + 2x_2 - 6x_3 + x_4 &= 1 \end{aligned} \Rightarrow \begin{bmatrix} 2 & 0 & 4 & 0 \\ 2 & 1 & 3 & 0 \\ 4 & 1 & 7 & 1 \\ -2 & 2 & -6 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{bmatrix} 10 \\ 14 \\ 27 \\ 1 \end{bmatrix}$$

$$[A|b] = \left[\begin{array}{cccc|c} 2 & 0 & 4 & 0 & 10 \\ 2 & 1 & 3 & 0 & 14 \\ 4 & 1 & 7 & 1 & 27 \\ -2 & 2 & -6 & 1 & 1 \end{array}\right] \xrightarrow[\text{new } r_4 = r_4 + r_1]{\text{new } r_2 = r_2 - r_1,\ \text{new } r_3 = r_3 - 2r_1} \left[\begin{array}{cccc|c} 2 & 0 & 4 & 0 & 10 \\ 0 & 1 & -1 & 0 & 4 \\ 0 & 1 & -1 & 1 & 7 \\ 0 & 2 & -2 & 1 & 11 \end{array}\right] \xrightarrow[\text{new } r_4 = r_4 - 2r_1]{\text{new } r_3 = r_3 - r_2}$$

$$\left[\begin{array}{cccc|c} 2 & 0 & 4 & 0 & 10 \\ 0 & 1 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[\text{new } r_4 = r_4 - r_3]{\text{new } r_1 = \frac{1}{2} r_1} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 5 \\ 0 & 1 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left\{ \begin{aligned} & x_1 + 2x_3 = 5 \Rightarrow x_1 = 5 - 2x_3 \\ & x_2 - x_3 = 4 \Rightarrow x_2 = 4 + x_3 \\ & x_4 = 3 \\ & x_3 \text{ is free} \end{aligned} \right. \Rightarrow \vec{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 5-2x_3 \\ 4+x_3 \\ 3 \\ x_3 \end{Bmatrix} = \boxed{\begin{Bmatrix} 5 \\ 4 \\ 3 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{Bmatrix} x_3} \leftarrow \text{ans.}$$

b. We can check our answer by plugging it into the original set of equations.

$$\begin{aligned} 2x_1 + 4x_3 &\overset{?}{=} 10 \\ 2(5-2x_3) + 4x_3 &\overset{?}{=} 10 \\ 10 - 4x_3 + 4x_3 &\overset{?}{=} 10 \\ 10 &= 10 \checkmark \end{aligned}$$

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &\overset{?}{=} 14 \\ 2(5-2x_3) + (4+x_3) + 3x_3 &\overset{?}{=} 14 \\ 10 - 4x_3 + 4 + x_3 + 3x_3 &\overset{?}{=} 14 \\ 14 &= 14 \checkmark \end{aligned}$$

$$\begin{aligned} 4x_1 + x_2 + 7x_3 + x_4 &\overset{?}{=} 27 \\ 4(5-2x_3) + (4+x_3) + 7x_3 + 3 &\overset{?}{=} 27 \\ 20 - 8x_3 + 4 + x_3 + 7x_3 + 3 &\overset{?}{=} 27 \\ 27 &= 27 \checkmark \end{aligned}$$

$$\begin{aligned} -2x_1 + 2x_2 - 6x_3 + x_4 &\overset{?}{=} 1 \\ -2(5-2x_3) + 2(4+x_3) - 6x_3 + 3 &\overset{?}{=} 1 \\ -10 + 4x_3 + 8 + 2x_3 - 6x_3 + 3 &\overset{?}{=} 1 \\ 1 &= 1 \checkmark \end{aligned}$$

F95 FINAL MATH 293 #3

28) a.

$$[A|b] = \left[\begin{array}{cccc|c} 0 & 1 & 2 & 1 & 1 \\ 1 & 2 & 0 & 1 & 3 \\ 1 & 4 & 4 & 3 & 5 \\ 0 & -2 & -4 & -2 & -2 \end{array}\right] \xrightarrow{\text{switch } r_1 \leftrightarrow r_2} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 3 \\ 0 & 1 & 2 & 1 & 1 \\ 1 & 4 & 4 & 3 & 5 \\ 0 & -2 & -4 & -2 & -2 \end{array}\right] \xrightarrow{\text{new } r_3 = r_3 - r_1}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 3 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 2 & 4 & 2 & 2 \\ 0 & -2 & -4 & -2 & -2 \end{array}\right] \xrightarrow[\text{new } r_4 = r_4 + 2r_2]{\substack{\text{new } r_1 = r_1 - 2r_2 \\ \text{new } r_3 = r_3 - 2r_2}} \left[\begin{array}{cccc|c} 1 & 0 & -4 & -1 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} x_1 - 4x_3 - x_4 = 1 \Rightarrow x_1 = 1 + 4x_3 + x_4 \\ x_2 + 2x_3 + x_4 = 1 \Rightarrow x_2 = 1 - 2x_3 - x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \end{cases}$$

$$\vec{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 1 + 4x_3 + x_4 \\ 1 - 2x_3 - x_4 \\ x_3 \\ x_4 \end{Bmatrix} = \boxed{\begin{Bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 4 \\ -2 \\ 1 \\ 0 \end{Bmatrix} x_3 + \begin{Bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{Bmatrix} x_4} \leftarrow \text{ans.}$$

b. We can verify the solution by plugging it into the original system of equations.

$0x_1 + 1x_2 + 2x_3 + 1x_4 \stackrel{?}{=} 1$ ← row1
$1 - 2x_3 - x_4 + 2x_3 + x_4 \stackrel{?}{=} 1$
$1 = 1$ ✓

$1x_1 + 2x_2 + 0x_3 + x_4 \stackrel{?}{=} 3$ ← row2
$(1 + 4x_3 + x_4) + 2(1 - 2x_3 - x_4) + x_4 \stackrel{?}{=} 3$
$1 + 2 + 4x_3 + x_4 - 4x_3 - x_4 + x_4 \stackrel{?}{=} 3$
$3 = 3$ ✓

$1x_1 + 4x_2 + 4x_3 + 3x_4 \stackrel{?}{=} 5$ ← row3
$(1 + 4x_3 + x_4) + 4(1 - 2x_3 - x_4) + 4x_3 + 3x_4 \stackrel{?}{=} 5$
$1 + 4 + 4x_3 + x_4 - 8x_3 - 4x_4 + 4x_3 + 3x_4 \stackrel{?}{=} 5$
$5 = 5$ ✓

$0x_1 - 2x_2 - 4x_3 - 2x_4 \stackrel{?}{=} -2$
$-2(1 - 2x_3 - x_4) - 4x_3 - 2x_4 \stackrel{?}{=} -2$
$-2 + 4x_3 + 2x_4 - 4x_3 - 2x_4 \stackrel{?}{=} -2$
$-2 = -2$ ✓

M293 F SP96 #2

29) The answer is d).

SP 96 FINAL MATH 293 #3

30)

$$\begin{aligned} x + z &= 4 \\ 2x + y + 3z &= 5 \\ -3x - 3y + (a^2-5a)z &= a-8 \end{aligned} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ -3 & -3 & a^2-5a \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} 4 \\ 5 \\ a-8 \end{bmatrix}$$

$$[A|b] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 2 & 1 & 3 & 5 \\ -3 & -3 & a^2-5a & a-8 \end{array}\right] \xrightarrow[\text{new } r_3 = r_3 + 3r_1]{\text{new } r_2 = r_2 - 2r_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & -3 & a^2-5a+3 & a+4 \end{array}\right] \xrightarrow{\text{new } r_3 = r_3 + 3r_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & a^2-5a+6 & a-5 \end{array}\right]$$

For there to be an infinitely number of solutions:

1) there must be a free variable, $a^2-5a+6=0$
2) the solution must be consistent, $a-5=0$

$a^2-5a+6=0 \Rightarrow (a-2)(a-3)=0$

$a=2$ or 3 for a free var.

$a-5=0 \Rightarrow a=5$

$a=5$ for a consistent system.

There is no value for 'a' which will give an infinite number of solutions. The answer is c).

M293 PII FA96 #2

31) a) $[A \vdots b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -1 \end{bmatrix} \xrightarrow{\text{row 2} - \text{row 1}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & 1 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2nd eqn: $x_2 = -1 - 2x_3$ 1st eqn: $x_1 = 1 - x_2 - x_3 = 2 + x_3$

$\boxed{x = \begin{bmatrix} 2+x_3 \\ -1-2x_3 \\ x_3 \end{bmatrix}}$ check: $Ax = \begin{bmatrix} (2+x_3)+(-1-2x_3)+(x_3) \\ (2+x_3) - (x_3) \\ (-1-2x_3)+2(x_3) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ ✓

b) $[A \vdots c] = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & 1 & 2 & 2 \end{bmatrix}$ } rows 2 & 3 now contradict each other

$\therefore$ $\boxed{\text{no soln}}$

c) A^{-1} cannot exist because $Ax=c$ would have a soln if it did.

SP97 PII MATH293 #2

32) ~~a~~.C.

$$[A|b] = \left[\begin{array}{ccc|c} 9 & 0 & 0 & 1 \\ 1 & 0 & -2 & 1 \\ 1 & 2 & 0 & 0 \end{array}\right] \xrightarrow[\text{new } r_3 = r_3 - \frac{1}{9} r_1]{\text{new } r_2 = r_2 - \frac{1}{9} r_1} \left[\begin{array}{ccc|c} 9 & 0 & 0 & 1 \\ 0 & 0 & -2 & 8/9 \\ 0 & 2 & 0 & -1/9 \end{array}\right] \xrightarrow{\text{switch } r_2 + r_3}$$

$$\left[\begin{array}{ccc|c} 9 & 0 & 0 & 1 \\ 0 & 2 & 0 & -1/9 \\ 0 & 0 & -2 & 8/9 \end{array}\right] \xrightarrow[\text{new } r_3 = (-1/2) r_3]{\substack{\text{new } r_1 = (1/9) r_1 \\ \text{new } r_2 = (1/2) r_2}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1/18 \\ 0 & 0 & 1 & -4/9 \end{array}\right] \Rightarrow \boxed{\underline{x} = \begin{Bmatrix} 1 \\ -1/18 \\ -4/9 \end{Bmatrix}} \leftarrow \text{ans.}$$

M294 PII SP98 #3

38)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 1 \\ 2 & 4 & 0 & 0 & 2 & 4 \\ 1 & 2 & 2 & 0 & 1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 1 \\ 0 & 0 & -2 & 0 & 0 & 2 \\ 1 & 2 & 2 & 0 & 1 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 1 \\ 0 & 0 & -2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\therefore \begin{bmatrix} -2x_2 \\ x_2 \\ 0 \end{bmatrix} = c \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \quad \text{for (a)}$$

and no solution for (c),

$$\begin{bmatrix} 1-2x_2 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-2c \\ c \\ 0 \end{bmatrix} \quad \text{for (b)}$$

#

M294PII FA98 #4d,e

41)

d) Find a solution to $Ax = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$. ← This is the first column of A.

$$\Rightarrow A\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \boxed{\underline{X} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ solves } A\underline{X} = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}}$$

e) Find the general solution to $Ax = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$.

see (d) above

$\underline{0}$, since nul $A = \underline{0}$

$$\underline{X}_{gen} = \underline{X}_p + \underline{X}_h = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \underline{0} \Rightarrow \boxed{\underline{X}_{gen} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$

SP? FINAL MATH 293 #1

43)

$$\begin{aligned} 2x-3y+0z-w&=-8\\ -5x+2y-3z+2w&=-2\\ 2x+0y+2z-w&=4\\ x-y+z-w&=-2 \end{aligned} \Rightarrow \begin{bmatrix} 2&-3&0&-1\\ -5&2&-3&2\\ 2&0&2&-1\\ 1&-1&1&-1 \end{bmatrix} \begin{Bmatrix} x\\ y\\ z\\ w \end{Bmatrix} = \begin{bmatrix} -8\\ -2\\ 4\\ -2 \end{bmatrix}$$

$$[A|b]=\left[\begin{array}{cccc|c} 2&-3&0&-1&-8\\ -5&2&-3&2&-2\\ 2&0&2&-1&4\\ 1&-1&1&-1&-2 \end{array}\right] \xrightarrow{\text{switch } r_1+r_4} \left[\begin{array}{cccc|c} 1&-1&1&-1&-2\\ -5&2&-3&2&-2\\ 2&0&2&-1&4\\ 2&-3&0&-1&-8 \end{array}\right] \xrightarrow[\text{new } r_4=r_4-2r_1]{\text{new } r_2=r_2+5r_1,\ \text{new } r_3=r_3-2r_1}$$

$$\left[\begin{array}{cccc|c} 1&-1&1&-1&-2\\ 0&-3&2&-3&-12\\ 0&2&0&1&8\\ 0&-1&-2&1&-4 \end{array}\right] \xrightarrow[\text{new } r_3=r_3+2r_4]{\text{new } r_1=r_1-r_4,\ \text{new } r_2=r_2-3r_4} \left[\begin{array}{cccc|c} 1&0&3&-2&2\\ 0&0&8&-6&0\\ 0&0&-4&-3&0\\ 0&-1&-2&1&-4 \end{array}\right] \xrightarrow[\text{new } r_4=r_4+\frac{1}{4}r_2]{\text{new } r_1=r_1-\frac{3}{8}r_2,\ \text{new } r_3=r_3+\frac{1}{2}r_2}$$

$$\left[\begin{array}{cccc|c} 1&0&0&\frac{2}{8}&2\\ 0&0&8&-6&0\\ 0&0&0&0&0\\ 0&-1&0&-\frac{1}{2}&4 \end{array}\right] \xrightarrow{\text{move rows}} \left[\begin{array}{cccc|c} 1&0&0&1/4&2\\ 0&-1&0&-1/2&4\\ 0&0&8&-6&0\\ 0&0&0&0&0 \end{array}\right] \xrightarrow[\text{new } r_3=\frac{1}{8}r_3]{\text{new } r_2=-r_2}$$

$$\left[\begin{array}{cccc|c} 1&0&0&1/4&2\\ 0&1&0&1/2&-4\\ 0&0&1&-3/4&0\\ 0&0&0&0&0 \end{array}\right] \Rightarrow \begin{cases} x_1+\frac{1}{4}x_4=2 \Rightarrow x_1=2-\frac{1}{4}x_4\\ x_2+\frac{1}{2}x_4=-4 \Rightarrow x_2=-4-\frac{1}{2}x_4\\ x_3-\frac{3}{4}x_4=0 \Rightarrow x_3=\frac{3}{4}x_4\\ x_4 \text{ is free} \end{cases}$$

$$\vec{x}=\begin{Bmatrix} x_1\\ x_2\\ x_3\\ x_4 \end{Bmatrix} = \begin{Bmatrix} 2-\frac{1}{4}x_4\\ -4-\frac{1}{2}x_4\\ \frac{3}{4}x_4\\ x_4 \end{Bmatrix} = \boxed{\begin{Bmatrix} 2\\ -4\\ 3/4\\ 0 \end{Bmatrix} + \begin{Bmatrix} -1/4\\ -1/2\\ 3/4\\ 1 \end{Bmatrix} x_4} \leftarrow \text{ans.}$$

? FINAL MATH 293 #3

44) a.

$$\left[\begin{array}{ccc|c} 6&0&4&1\\ 5&-1&5&-1\\ 1&0&3&2 \end{array}\right] \xrightarrow{\text{switch } r_1+r_3} \left[\begin{array}{ccc|c} 1&0&3&2\\ 5&-1&5&-1\\ 6&0&4&1 \end{array}\right] \xrightarrow[\text{new } r_3=r_3-6r_1]{\text{new } r_2=r_2-5r_1}$$

$$\left[\begin{array}{ccc|c} 1&0&3&2\\ 0&-1&-10&-11\\ 0&0&-14&-11 \end{array}\right] \xrightarrow[\text{new } r_2=r_2-\frac{10}{14}r_3]{\text{new } r_1=r_1+\frac{3}{14}r_3} \left[\begin{array}{ccc|c} 1&0&0&-5/14\\ 0&-1&0&-22/7\\ 0&0&-14&-11 \end{array}\right] \xrightarrow[\text{new } r_3=-\frac{1}{14}r_3]{\text{new } r_2=-r_2}$$

$$\left[\begin{array}{ccc|c} 1&0&0&-5/14\\ 0&1&0&22/7\\ 0&0&1&11/14 \end{array}\right] \Rightarrow \boxed{\vec{x}=\begin{Bmatrix} -5/14\\ 22/7\\ 11/14 \end{Bmatrix}} \leftarrow \text{ans.}$$

SP92 PII MATH 293 #2

47)

$$\begin{aligned} x+2y+2z-w&=1\\ 3x+6y+z+2w&=3\\ -x-2y+z-2w&=-1\end{aligned} \Rightarrow \begin{bmatrix}1&2&2&-1\\3&6&1&2\\-1&-2&1&-2\end{bmatrix}\begin{Bmatrix}x\\y\\z\\w\end{Bmatrix}=\begin{bmatrix}1\\3\\-1\end{bmatrix}$$

$$[A|b]=\left[\begin{array}{cccc|c}1&2&2&-1&1\\3&6&1&2&3\\-1&-2&1&-2&-1\end{array}\right]\xrightarrow[\text{new } r_3=r_3+r_1]{\text{new } r_2=r_2-3r_1}\left[\begin{array}{cccc|c}1&2&2&-1&1\\0&0&-5&5&0\\0&0&3&-3&0\end{array}\right]\xrightarrow[\text{new } r_3=r_3+\frac{3}{5}r_2]{\text{new } r_1=r_1+\frac{2}{5}r_2}$$

$$\left[\begin{array}{cccc|c}1&2&0&1&1\\0&0&-5&5&0\\0&0&0&0&0\end{array}\right]\xrightarrow{\text{new } r_2=-\frac{1}{5}r_2}\left[\begin{array}{cccc|c}1&2&0&1&1\\0&0&1&-1&0\\0&0&0&0&0\end{array}\right]\Rightarrow\begin{cases}x+2y+w=1\Rightarrow x=1-2y-w\\ z-w=0\Rightarrow z=w\\ y \text{ is free}\\ w \text{ is free}\end{cases}$$

$$\begin{Bmatrix}x\\y\\z\\w\end{Bmatrix}=\begin{Bmatrix}1-2y-w\\y\\w\\w\end{Bmatrix}=\boxed{\begin{Bmatrix}1\\0\\0\\0\end{Bmatrix}+\begin{Bmatrix}-2\\1\\0\\0\end{Bmatrix}y+\begin{Bmatrix}-1\\0\\1\\1\end{Bmatrix}w} \leftarrow \text{ans.}$$

SU92 FINAL MATH 293 #1

49) a.

$$[A|b]=\left[\begin{array}{cccc|c}1&-3&4&-2&5\\0&2&5&1&2\\0&1&-3&0&4\end{array}\right]\xrightarrow[\text{new } r_3=r_3-\frac{1}{2}r_2]{\text{new } r_1=r_1+\frac{3}{2}r_2}\left[\begin{array}{cccc|c}1&0&\frac{23}{2}&-\frac{1}{2}&8\\0&2&5&1&2\\0&0&-\frac{11}{2}&-\frac{1}{2}&3\end{array}\right]\xrightarrow[\text{new } r_2=r_2+5(\frac{2}{11})r_3]{\text{new } r_1=r_1+\frac{23}{2}(\frac{2}{11})r_3}$$

$$\left[\begin{array}{cccc|c}1&0&0&-34/22&157/11\\0&2&0&6/11&52/11\\0&0&-\frac{11}{2}&-1/2&3\end{array}\right]\xrightarrow[\text{new } r_3=-\frac{2}{11}r_3]{\text{new } r_2=\frac{1}{2}r_2}\left[\begin{array}{cccc|c}1&0&0&-17/11&157/11\\0&1&0&3/11&26/11\\0&0&1&1/11&-6/11\end{array}\right]$$

$$\begin{cases}x_1-\frac{17}{11}x_4=157/11\Rightarrow x_1=\frac{157}{11}+\frac{17}{11}x_4\\ x_2+\frac{3}{11}x_4=26/11\Rightarrow x_2=\frac{26}{11}-\frac{3}{11}x_4\\ x_3+\frac{1}{11}x_4=-6/11\Rightarrow x_3=\frac{-6}{11}-\frac{1}{11}x_4\\ x_4 \text{ is free}\end{cases}\Rightarrow \vec{x}=\begin{Bmatrix}x_1\\x_2\\x_3\\x_4\end{Bmatrix}=\begin{Bmatrix}\frac{157}{11}+\frac{17}{11}x_4\\ \frac{26}{11}-\frac{3}{11}x_4\\ \frac{-6}{11}-\frac{1}{11}x_4\\ x_4\end{Bmatrix}=\boxed{\begin{Bmatrix}157/11\\26/11\\-6/11\\0\end{Bmatrix}+\begin{Bmatrix}17/11\\-3/11\\-1/11\\1\end{Bmatrix}x_4}$$

ans.

b.

$$[A|B]=\left[\begin{array}{ccc|ccc}-1&2&-3&1&2&3\\2&1&0&3&2&1\\4&-2&5&1&2&3\end{array}\right]\xrightarrow[\text{new } r_3=r_3+4r_1]{\text{new } r_2=r_2+2r_1}\left[\begin{array}{ccc|ccc}-1&2&3&1&2&3\\0&5&-6&5&6&7\\0&6&-7&5&11&14\end{array}\right]\xrightarrow[\text{new } r_3=r_3-\frac{6}{5}r_2]{\text{new } r_1=r_1-\frac{2}{5}r_2}$$

$$\left[\begin{array}{ccc|ccc}-1&0&-3/5&-1&-2/5&1/5\\0&5&-6&5&6&7\\0&0&1/5&-1&19/5&28/5\end{array}\right]\xrightarrow[\text{new } r_2=r_2+6(\frac{5}{1})r_3]{\text{new } r_1=r_1+3r_3}\left[\begin{array}{ccc|ccc}-1&0&0&-4&55/5&85/5\\0&5&0&-25&120&175\\0&0&1/5&-1&19/5&28/5\end{array}\right]$$

$$\xrightarrow[\substack{\text{new } r_2=\frac{1}{5}r_2\\ \text{new } r_3=5r_3}]{\text{new } r_1=-r_1}\left[\begin{array}{ccc|ccc}1&0&0&4&-11&-17\\0&1&0&-5&24&35\\0&0&1&-5&19&28\end{array}\right]=[I|B']$$

, where B' is the B matrix, after the row operations

$$AX=B\Rightarrow IX=B'$$

$$\boxed{X=\begin{bmatrix}4&-11&-17\\-5&24&35\\-5&19&28\end{bmatrix}} \leftarrow \text{ans.}$$