5.5 Wave

**MATH 294 SPRING 1983 FINAL # 5**

5.5.1 a) Solve the wave equation with wave speed $c = 1$, boundary conditions: $u(0, t) = u(6, t) = 0$ and initial conditions $u(x, 0) = 0$, $u_t(x, 0) = 5 \sin \left(\frac{\pi x}{3}\right)$.

b) Make a clearly labeled graph of $u(3, t)$ vs. $t$ for your solution in part (a) above.

**MATH 294 SPRING 1994 FINAL # 14**

5.5.2 Verify that $u(t, x) = \frac{1}{2} [f(x+t) + f(x-t)]$ solves the initial value problem:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad t > 0, \quad -\infty < x < \infty,$$

$$u(t, 0) = f(x), \quad u_t(t, 0) = 0.$$

**MATH 294 FALL 1986 FINAL # 9**

5.5.3 a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $(0 < x < 1, 0 < y < 1)$ where $u = u(x, y)$ and $u(0, y) = 0$, $u(1, y) = 0$, $u(x, 0) = 0$, $u(x, 1) = 2 \sin(2\pi x)$.

b) Use your result from part (a) to solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (0 < x < 1, 0 < y < 1)$$

where $u = u(x, y)$ and $u(0, y) = 0$, $u(1, y) = 2 \sin(2\pi y)$, $u(x, 0) = 0$, $u(x, 1) = 0$.

c) Use your result from part (a) and (b) to solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (0 < x < 1, 0 < y < 1)$$

where $u = u(x, y)$ and $u(0, y) = 0$, $u(x, 0) = 0$, $u(1, y) = 2 \sin(2\pi y)$, $u(x, 1) = 0$, $u(x, 0) = 2 \sin(2\pi x)$.

**MATH 294 FALL 1986 FINAL # 12**

5.5.4 Find the solution to the initial/boundary value problem

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad t > 0$$

$$u(0, t) = u(L, t) = 0, \quad t > 0$$

$$u(x, 0) = 0, \quad 0 < x < L$$

$$u_t(x, 0) = \sin \left(36\pi \frac{x}{L}\right), \quad 0 < x < L.$$
MATH 294 SPRING 1987 PRELIM 2 # 2
5.5.5 Find the value of \( u \) at \( x = t = 1 \) if \( u(x,t) \) satisfies:

\[
\frac{\partial^2 u}{\partial t^2} = 2 \frac{\partial^2 u}{\partial x^2}
\]

\[0 = u(0, t) = u(3\pi, t)\]

with

\[u(x, 0) = \sin(5x)\]

\[\frac{\partial u}{\partial t}(x, 0) = \sin x\]

MATH 294 SPRING 1987 FINAL # 7
5.5.6 Find any non-zero solution \( u(x,t) \) to

\[
\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2} \text{ with } 0 = \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(1, t)
\]

and the extra restriction that \( u(0, 0) \neq u(1, 0) \).

MATH 294 FALL 1987 PRELIM 2 # 3
5.5.7 Find the solution of the initial-boundary-value problem

\[
\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2} \quad 0 < x < 1 \quad t > 0
\]

\[u(0, t) = u(1, t) = 0 \quad t > 0\]

\[u(x, 0) = 0\]

\[\frac{\partial u}{\partial t}(x, 0) = x. \quad 0 < x < 1\]

MATH 294 SPRING 1988 PRELIM 2 # 5
5.5.8 Once released, the deflection \( u \) of a taught string satisfies the wave equation

\[
\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}
\]

where \( x \) is position along the string and \( t \) is time. It is held fixed (no deflection) at its ends at \( x = 0 \) and \( x = 2 \). At time \( t = 0 \) it is released from rest with the deflected shape \( u = 3\sin \left( \frac{\pi x}{2} \right) \). Make a plot of \( u(1, t) \) versus \( t \) for \( 0 \leq t \leq 2 \). Label the axes at points of intersection with the curve. (You may quote any results that you remember.)
The displacement $u(x, y)$ of a vibrating string satisfies
\[
\frac{\partial^2 x}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0
\]
in $0 \leq x \leq 4$, $t \geq 0$ and the boundary and initial conditions
\[
u(0, t) = 0, \quad u(4, t) = 0, \quad u(x, 0) = 0, \quad \frac{\partial u}{\partial t}(x, 0) = f(x),
\]
where
\[
f(x) = \begin{cases} 
1, & \text{when } 0 \leq x \leq 2 \\
0, & \text{when } 2 \leq x \leq 4
\end{cases}
\]
a) Find a series representation for the solution.
b) Write down the equation for the displacement of the string at $t = 4$.
5.5. WAVE

MATH 294 SPRING 1993 FINAL # 15

5.5.12 a) The solution to
\[ u_{tt} = u_{xx} - \infty < x < \infty \]
\[ u(x, 0) = e^{-x^2} \]
\[ u_t(x, 0) = 0 \] is of the form \( u(x, t) = \varphi(x + t) + \varphi(x - t) \). Find the solution without using Fourier series.

b) Find the solution of
\[ u_{xx} = u_t \quad 0 \leq x \leq 1 \]
\[ u(0, t) = 1 \]
\[ u(1, t) = 2 \]
\[ u(x, 0) = 1 + x \]
Hint: The solution may be time-independent.

MATH 294 FALL 1995 FINAL # 15

5.5.13 If \( u(x, t) = F(x + t) + G(x - t) \) for some functions \( F \) and \( G \),
a) Find expressions for \( u(x, 0) \) and \( u_t(x, 0) \) in terms of \( F \) and \( G \).

b) If also \( \begin{cases} u_{tt} = u_{xx} \quad -\infty < x < \infty \\ u(x, 0) = e^{-x^2} \\ u_t(x, 0) = 0 \end{cases} \) find expressions for \( F \) and \( G \), and sketch the graph of \( u(x, t) \) when \( t = 0, 1, \) and \( 2 \).

MATH 294 SPRING 1998 PRELIM 1 # 4

5.5.14 Consider the following partial differential equation for \( u(x, t) \),
\[ \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 \leq x \leq 1, \]
with boundary conditions \( u(0, t) = u(1, t) = 0, \quad t > 0 \),
and initial conditions \( u(x, 0) = f(x) \) and \( \frac{\partial u}{\partial t}(x, 0) = 0, \quad 0 \leq x \leq 1 \).

which, if any, of the functions below is a solution to the initial/boundary-value problem? Justify your answer.

a) \( u(x, t) = \sum_{n=1}^{\infty} b_n e^{-\pi^2 n^2 t} \sin n\pi x, \quad b_n = 2 \int_0^1 f(x) \sin n\pi x dx \)

b) \( u(x, t) = \sum_{n=1}^{\infty} b_n \cos n\pi t \sin n\pi x, \quad b_n = 2 \int_0^1 f(x) \sin n\pi x dx \)
MATH 294 SUMMER 1990 PRELIM 2 # 5

5.5.15 Consider the partial differential equation

\[(*) \frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}, \text{ for } 0 \leq x \leq L,\]

with conditions
i) \(u(0, t) = 0,\)
ii) \(u(L, t) = 0\)
iii) \(\frac{\partial u}{\partial x}(x, 0) = 0,\)
and
iv) \(u(x, 0) = f(x).\)

a) Verify that \(u(x, t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi ct}{L}\right)\) is a solution to \((*)\) and the conditions (i), (ii), and (iii).

b) Suppose \(f(x) = \sin\left(\frac{\pi x}{L}\right).\) What values for the \(C_n\)'s will satisfy condition (iv)?

c) For a general piecewise smooth function \(f(x),\) Determine the formula for the \(C_n\) so that condition (iv) is satisfied.

MATH 294 FALL 1992 UNKNOWN # 4

5.5.16 Solve the initial-boundary-value problem

\[u_{tt} = u_{xx}, \quad 0 < x < 1, \quad t > 0,\]
\[u(0, t) = u(1, t) = 0, \quad t > 0,\]
\[u(x, 0) = 8 \sin 13\pi x - 2 \sin 31\pi x,\]
\[u_t(x, 0) = -\sin 8\pi x + 12 \sin 88\pi x.\]

MATH 294 SPRING 1996 FINAL # 5 MAKE-UP

5.5.17 Consider \(u(x, y, z, t) = w(ax + by + cz + dt)\) where \(w\) is some differentiable function of one variable, and the expression \(ax + by + cz + dt\) has been substituted for that variable.

a) Find restrictions on the constants \(a, b, c,\) and \(d\) so that \(u\) will be a solution to the three dimensional wave equation \(u_{xx} + u_{yy} + u_{zz} = u_{tt}\)

b) Find a solution to the wave equation if (a) having \(u(x, y, z, 0) = 5 \cos x\) and \(u_t(x, y, z, 0) = 0.\)