4.4 General 3-D Integrals

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4.4.1 Set up iterated triple integral for the volume of the region bounded by the sphere $x^2 + y^2 + z^2 = 4$ in

a) spherical

b) cylinder

c) rectangular coordinates.

Use the following orders of integration:

a) ρ, θ, ϕ

b) z, θ, r

c) x, y, z

MATH 294 SPRING 1990 PRELIM 1 # 1 294SP90P1Q1.tex

4.4.2 Find the volume of the solid enclosed by the cylinder $x^2 + y^2 = 4$, bounded below by the plane $z = x$, and bounded above by the paraboloid $x^2 + y^2 + z = 10$.

MATH 294 SPRING 1990 PRELIM 1 # 5 294SP90P1Q5.tex

4.4.3 Determine the location of the center of mass of the solid that is enclosed between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$ and inside the cone $z = \sqrt{x^2 + y^2}$. Assume uniform density $\rho \equiv 1$.

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4.4.4 Find the volume of the region in the first octant bounded by the coordinate planes, the cylinder $x^2 + y^2 = 4$ and the plane $z = 4 - y$.

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4.4.5 **a**) Find the volume of the tetrahedron with vertices

$$A(0,0,0), \quad B(1,0,0), \quad C(0,1,0) \text{ and } D(x,y,z); \text{ in terms of } x, y \text{ and } z$$

b) For what point(s) D is this volume a minimum? What is this minimum volume? What does this mean physically?

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4.4.6 **a**) Sketch the level curve $f(x,y) = 3$, where $f(x,y) = x - y^2$. Show some point (a, b) on this curve, giving a and b explicitly. Compute $\vec{\nabla} f(a,b)$ and show it on the same figure. What is the relation between $\vec{\nabla} f$ and the level curve?

b) Evaluate $\int_{C_1} xdy - ydx$ and $\int_{C_2} xdy + ydx$ where C_1 is the unit circle counter-clockwise and C_2 is the semicircular part of C_1 where $x \geq 0$.

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4.4.7 Find I_{zz} the moment of inertia about the z axis through its center, of a solid sphere of radius $R = 1$, and density $\rho = 1$.

HINTS:

1. Definition: $I_{zz} = \int\int\int (x^2 + y^2)\delta dV$, and
2. Trig identity for help in doing integrals: $\sin^3 w = (1 - \cos^2 w)\sin w$.

MATH 293 FALL 1996 PRELIM 2 # 4 293FA96P2Q4.tex

4.4.8 Find the volume of the wedge cut from the first octant (x, y, and z are all positive) by the surface $x + y^2 = 4$ and the surface $z = 12 - 3y^2$. (make a sketch of at least the domain in the (x, y) plane; the surfaces themselves are harder to draw.)

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4.4.9 **Centroid.** Find the centroid of the solid bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1$.
[Hint: The volume of a solid cone is $\frac{1}{3}$ x (base area) x (height).]
Clear MATLAB code which would yield a numerical approximation to the desired value of $\bar{z}$ gets full credit for $\bar{z}$.

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4.4.10 Consider the sphere $x^2 + y^2 + z^2 = 25$.

a) Express the equation of the sphere in cylindrical coordinates (r, θ, z) and find the volume inside it by evaluating a triple integral in cylindrical coordinates.

b) Now consider the region that you get by starting with the solid interior of the sphere as before, and removing the points which are contained inside the cone $z = \sqrt{x^2 + y^2}$. This means that our new region consists of points having $x^2 + y^2 + z^2 \leq 25$ and $z \leq \sqrt{x^2 + y^2}$. Find the volume of this region by evaluating a triple integral in spherical coordinates (ρ, ϕ, θ).

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4.4.11 Consider the region in the first octant bounded by the coordinate planes, the plane $x + z = 1$, and the plane $y + 2z = 2$.

a) Sketch the region.

b) Set up a triple integral for the volume of the region, including all the limits.

c) Find the volume of the region.

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4.4.12 Calculate the volume of that part of the sphere $x^2 + y^2 + z^2 \leq 2$ above the plane $z = 0$ and below the cone $z = \sqrt{x^2 + y^2}$.

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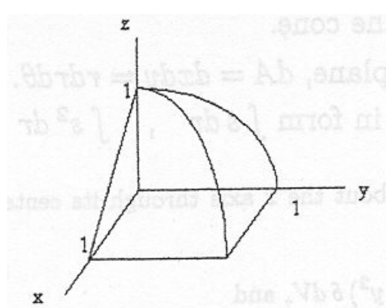
4.4.13 Find the volume of the region in the first octant bounded by the coordinate planes, the plane $x + z = 1$ and the cylinder $z = 1 - y^2$.



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4.4.14 Find the volume of the capped circular cylinder that consists of the interior of the cylinder $x^2+y^2=1$ that is bounded above and below by the sphere $x^2+y^2+z^2=4$.