4.3 Green's Theorem

MATH 294 FALL 1982 FINAL # 8b 294FA82FQ8b.tex

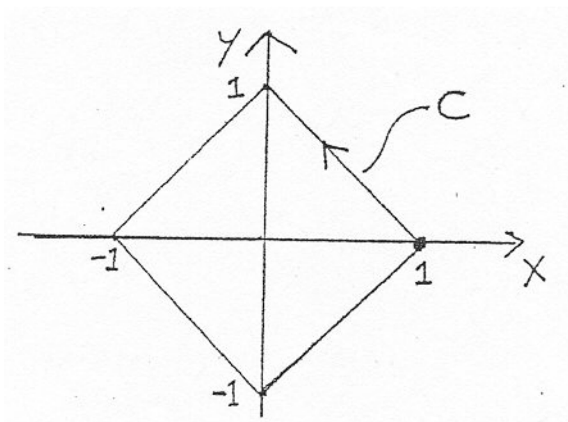
4.3.1 Use Green's theorem to find the area of the ellipse given parametrically by

$$x = a\cos\theta,\ \ y = b\sin\theta,\ \ 0 \le \theta \le 2\pi.$$

MATH 294 SPRING 1983 FINAL # 7 294SP83FQ7.tex

4.3.2 Evaluate the following integral for the path on the $x - y$ plane shown

$$\oint_C \underline{F} \cdot d\underline{R} \text{ for } \underline{F} = (3+2y)\underline{i} + (4-5x)\underline{j}$$



MATH 294 FALL 1986 FINAL # 6 294FA86FQ6.tex

4.3.3 In the xy-plane, let c_0 be the circle $x^2+y^2=25$, let c_1 be the circle $(x-2)^2+y^2=1$, let c_2 be the circle $(x+2)^2+y^2=1$, and let R be the region inside c_0 but outside c_1 and c_2. $(R = (x,y)|x^2+y^2 \leq 25,\ (x-2)^2+y^2 \geq 1, (x+2)^2+y^2 \geq 1)$. Suppose $M, N, \dfrac{\delta M}{\delta y}, \dfrac{\delta N}{\delta x}$ are continuous and that $\dfrac{\delta M}{\delta y} = \dfrac{\delta N}{\delta x}$ throughout the region R. Now let c_3 be the circle $x^2+(y+2)^2=1$, c_4 the circle $x^2+y^2=16$. Indicate which of the following statements are necessarily true.

a) $\displaystyle\oint_{c_2} Mdx + Ndy = 0$

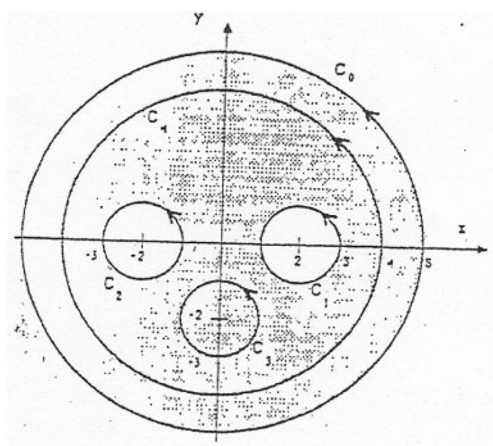
b) $\displaystyle\oint_{c_3} Mdx + Ndy = 0$

c) $\displaystyle\oint_{c_4} Mdx + Ndy = 0$

d) $\displaystyle\oint_{c_3} Mdx + Ndy = \oint_{c_4} Mdx + Ndy$

e) $\displaystyle\oint_{c_2} Mdx + Ndy = \oint_{c_1} Mdx + Ndy$

f) $\displaystyle\oint_{c_4} Mdx + Ndy = \oint_{c_1} Mdx + Ndy + \oint_{c_2} Mdx + Ndy$

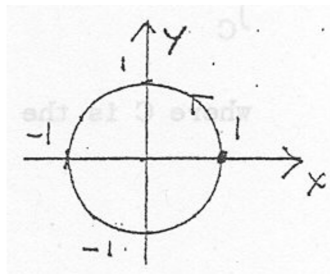


MATH 294 SPRING 1987 PRELIM 1 # 3 294SP87P1Q3.tex

4.3.4 Evaluate (by any means) $\oint \mathbf{F} \cdot d\mathbf{R}$ for the closed circular path shown using

$$\mathbf{F} = (xy - x)\hat{\imath} + (\frac{-x^2}{2})\hat{\jmath}$$

Unitcircle :



MATH 294 FALL 1987 PRELIM 1 # 1 294FA87P1Q1.tex

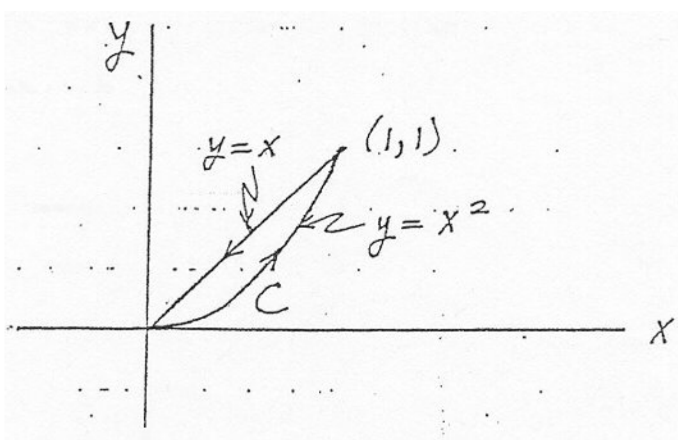
4.3.5 Evaluate

$$\int_C (x^2 - 2y)dx + (y^3 + 2x)dy$$

where C is the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

MATH 294 FALL 19876 MAKE UP PRELIM 1 # 2 294FA87MUP1Q2.tex

4.3.6 Evaluate $\oint_C xy^2dx + 2x^2dy$, where C is the closed curve sketched below



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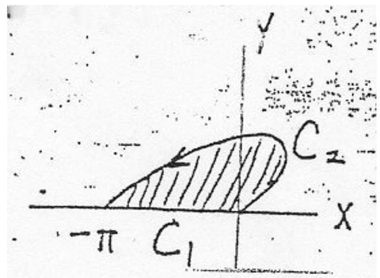
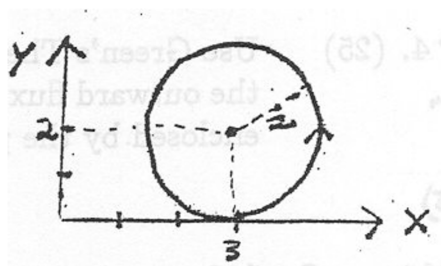
4.3.7 **a)** Compute the area of the shaded region shown below, whose boundary consists of C_1 = the line segment $(-\pi,0)$ on the x-axis together with the curve $C_2 : \underline{r}(t) = t\cos t\hat{i} + 2t\sin t\hat{j},\ 0 \leq t \leq \pi$



b) For $\underline{F} = 14x^4\hat{i} - 3x\hat{j}$, compute $\int_C \underline{F} \cdot d\underline{r}$, where $C = C_1 + C_2$ is the boundary of the region discussed in (a).

MATH 294 SPRING 1988 PRELIM 1 # 3 294SP88P1Q3.tex

4.3.8 Evaluate the path integral $\oint_C \mathbf{F} \cdot d\mathbf{R}$ for



$\mathbf{F} = \sqrt{1 + x^7}\mathbf{i} + [x + \sin(y^2)]\mathbf{j}$

on the closed curve shown.

MATH 294 FALL 1989 PRELIM 1 # 3 294FA89P1Q3.tex

4.3.9 Calculate the circulation of the vector field

$$\mathbf{F} = (\frac{y}{(x^2 + y^2)})\mathbf{i} + x\mathbf{j}$$

around the circle $x^2+y^2 = 2$. (Note that $2cos^2\theta = 1+\cos 2\theta$ and $2sin^2\theta = 1-\cos 2\theta$).

MATH 294 FALL 1989 PRELIM 1 # 4 294FA89P1Q4.tex

4.3.10 Show that the value of

$$\oint xy^2 dx + (x^2y + 2x)dy$$

around any square depends only on the area of the square and not on its location in the plane.

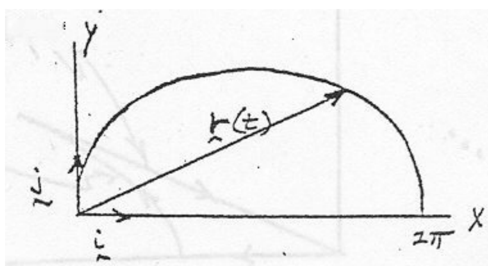
$$Circulation = \oint \mathbf{F} \cdot d\mathbf{R} = \oint M dx + N dy$$

MATH 294 SPRING 1990 FINAL # 2 294SP90FQ2.tex

4.3.11 Determine the area enclosed by the curve (cycloid) parameterized by

$$\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j},\ 0 \le t \le 2\pi,$$

and the x-axis, as depicted below. (Hint: Green)



MATH 294 SPRING 1990 FINAL # 9 294SP90FQ9.tex

4.3.12 Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = 4z\mathbf{i} - 2x\mathbf{j} + 2x\mathbf{k}$, and C is the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = y + 1$. When viewed from above (looking down the z-axis), C has clockwise orientation.

MATH 294 SUMMER 1990 PRELIM 1 # 4 294SU90P1Q4.tex

4.3.13 Use Green's theorem in the plane to show that the circulation of the vector field $\mathbf{F} = xy^2\mathbf{i} + (x^2y + x)\mathbf{j}$ about any smooth curve in the plane is equal to the area enclosed by the curve.

MATH 294 SPRING 1990 PRELIM 1 # 4 294SP90P1Q4.tex

4.3.14 Use Green's Theorem in the plane to find the counterclockwise circulation and the outward flux of the field $\mathbf{F(x, y) = xyi + y^2j}$ over the boundary of the region enclosed by the parabola $y = x^2$ and the line $y = x$ in the first quadrant.

MATH 294 SPRING 1992 PRELIM 3 # 6 294SP92P3Q6.tex

4.3.15 Apply Green's theorem to find the counterclockwise circulation for the field

$$\vec{F} = (x - y)\hat{i} + (x + y)\hat{j}$$

around the boundary of the unit square $0 \le x \le 1,\ 0 \le y \le 1$.

MATH 294 SPRING 1992 FINAL # 8 294SP92FQ8.tex

4.3.16 Let S be the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

a) Compute the outward flux of the vector field

$$\vec{F}(x, y, z) = 3y\hat{\imath} - xz\hat{\jmath} + yz^2\hat{k}$$

across the surface S.

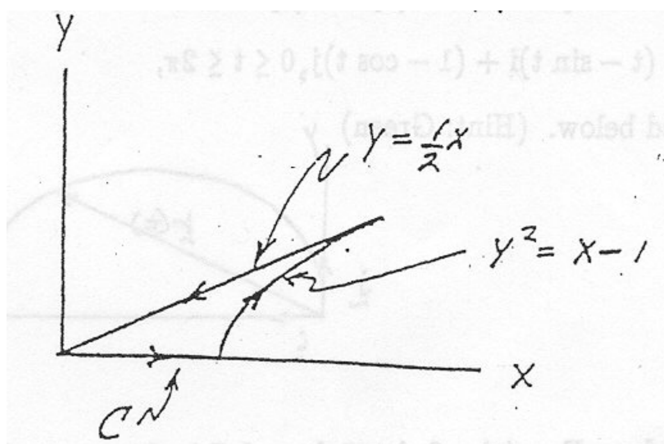
b) Compute the circulation of $\vec{F}$ around the circle $x^2 + y^2 = 4$, $z = 4$ in the counterclockwise direction as viewed from above.

MATH 294 SPRING 1992 PRELIM 2 # 4 294SP92P2Q4.tex

4.3.17 Given the vector field $\mathbf{F}(x, y) = x^2\mathbf{i} + xy\mathbf{j}$ and the closed curve C shown below, compute:

a) the (counter-clockwise) circulation of $\mathbf{F}$ around C;

b) the flux of $\mathbf{F}$ across C.



MATH 294 FALL 1993 PRELIM 1 # 1 294FA93P1Q1.tex

4.3.18 Evaluate $\int_C dx + x^2 dy$ where C is the counterclockwise boundary of the rectangle $0 \le x \le 2$, $0 \le y \le 1$.

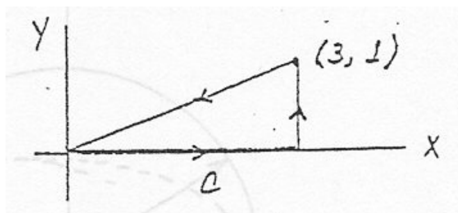
MATH 294 SPRING 1994 FINAL # 2 294SP94FQu2.tex

4.3.19 Let D be the quarter disk $x^2 + y^2 \le 1$, $x \ge 0$, $y \ge 0$ and C be its boundary curve. Use Green's Theorem to find

$$\oint_C x dx, \quad \oint_C y dx.$$

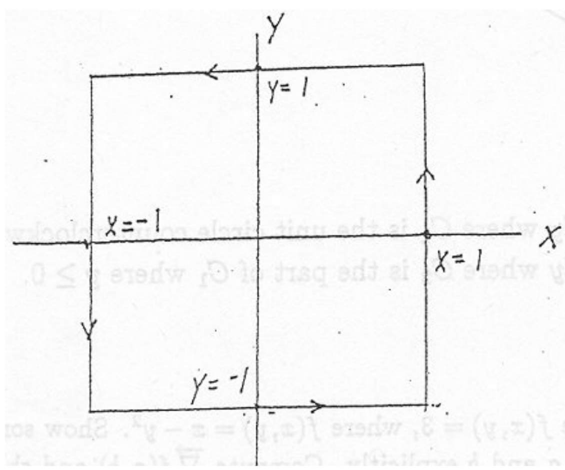
MATH 294 FALL 1994 PRELIM 1 # 4 294FA94P1Q4.tex

4.3.20 Evaluate $\oint_C xdx + xydy$ where C is the triangle shown.
You may use Green's Theorem.



MATH 294 SPRING 1995 PRELIM 1 # 4 294SP95P1Q4.tex

4.3.21 Evaluate $\oint_C (ydx - xdy)$ over the counterclockwise path shown below.



MATH 294 SPRING 1995 FINAL # 1 294SP95FQ1a.tex

4.3.22 **a**) Find

$$\int_C (x^2 + y^2 + z^2)ds$$

where C is the curve given by $\vec{r}(t) = \sin(t)\hat{\imath} + \cos(t)\hat{\jmath} + 8\hat{k},\ 0 \le t \le \pi$. Sketch the curve C.

b) Evaluate

$$\oint_C \vec{F} \cdot d\vec{r}$$

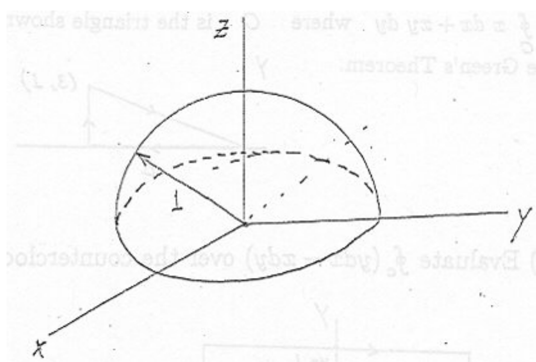
where C is the circle $x^2+y^2 = 1,\ z = 0$, and $\vec{F}(x, y, z) = e^{\sin(x+y+z)}\hat{\imath} + e^{\sin(x+y+z)}\hat{\jmath} + arccot(\cosh(xy))\hat{k}$.

MATH 294 SUMMER 1995 QUIZ 2 # 1 294SU95P2Q1.tex

4.3.23 Evaluate the line integral

$$\oint_C dx + xdy + 3dz, \text{ where } C = \text{boundary of the hemisphere}$$

$$x^2 + y^2 + z^2 = 1,\ z \ge 1.$$



MATH 294 FALL 1995 PRELIM 1 # 2 294FA95P1Q2.tex

4.3.24 **a**) Evaluate $\oint_{C_1} 2dx + xdy$ where C_1 is the unit circle counterclockwise.

b) Evaluate $\oint_{C_2} 2dx + xdy$ where C_2 is the part of C_1 where $y \ge 0$.

MATH 294 SPRING 1996 PRELIM 1 # 4 294SP96P1Q4.tex

4.3.25 **a**) Sketch the level curve $f(x, y) = 3$, where $f(x, y) = x - y^2$. Show some point (a, b) on this curve, giving a and b explicitly. Compute $\vec{\nabla} f(a, b)$ and show it on the same figure. What is the relation between $\vec{\nabla} f$ and the level curve?

b) Evaluate $\int_{C_1} xdy - ydx$ and $\int_{C_2} xdy + ydx$ where C_1 is the unit circle counterclockwise and C_2 is the semicircular part of C_1 where $x \ge 0$.

MATH 294 SPRING 1996 FINAL # 1 294SP96FQ1.tex

4.3.26 You might need Green's Theorem somewhere in this problem.

a) If $u(x, y)$ is a solution to the Laplace Equation in the plan, what is the value of the line integral $\int_C -u_y dx + u_x dy$ when C is a simple closed curve oriented counterclockwise?

b) Evaluate $\vec{\nabla}(x^3 - 3xy^2)$ and $\nabla^2(x^3 - 3xy^2)$.

c) Evaluate

$$\int_{C_1} 6xy dx + (3x^2 - 3y^2) dy$$

where C_1 is the unit circle oriented counterclockwise.

MATH 293 FALL 1998 FINAL # 3 293FA98FQ3.tex

4.3.27 Use Green's Theorem to calculate the counterclockwise circulation of the vector field

$$\mathbf{F} = (y + e^x \ln y)\mathbf{i} + \frac{e^x}{y}\mathbf{j}$$

around the boundary of the region that is bounded above by the curve $y = 3 - x^2$ and below by the curve $y = 1 + x^2$.