4.2 Line Integrals

MATH 294 FALL 1982 FINAL # 7 294FA82FQ7.tex

4.2.1 Consider the curve given parametrically by

$$x = \cos\frac{\pi t}{2}, \quad y = \sin\frac{\pi t}{2}, \quad z = t$$

a) Determine the work done by the force field

$$\mathbf{F_1} = y\mathbf{i} - \mathbf{j} + x\mathbf{k}$$

along this curve from (1,0,0) to (0,1,1).

b) Determine the work done along the same part of this curve by the field

$$\mathbf{F_2} = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$$

MATH 294 SPRING 1983 FINAL # 9 294SP83FQ9.tex

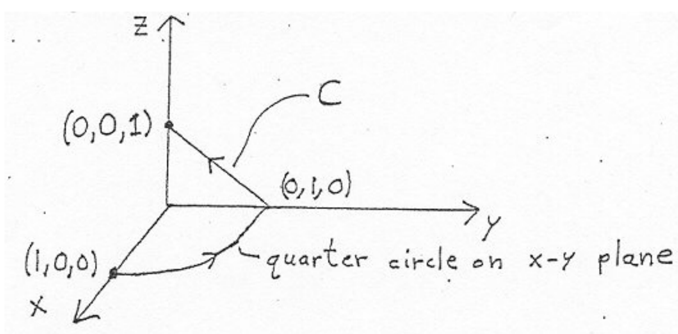
4.2.2 Consider the function $f(x, y, z) = x^2 + y + xz$.

a) What is $\underline{F} = \text{grad}(f) = \underline{\nabla} f$?

b) What is div $\underline{F} = \underline{\nabla} \cdot \underline{F}$? ($\underline{F}$ from part (a) above.)

c) What is curl $\underline{F} = \underline{\nabla} \times \underline{F}$? ($\underline{F}$ from part (a) above.)

d) Evaluate $\int_C \underline{F} \cdot d\underline{R}$ for $\underline{F}$ in part(a) above and C the curve shown:



MATH 294 SPRING 1984 FINAL # 8 294SP84FQ8.tex

4.2.3 $\mathbf{F} = 4x^3y^4\mathbf{i} + 4x^4y^3\mathbf{j}$. Find a potential function for $\mathbf{F}$ and use it to evaluate the line integral of $\mathbf{F}$ over any convenient path from (1,2) to (-3,4).

MATH 294 SPRING 1984 FINAL # 9 294SP84FQ9.tex

4.2.4 Evaluate $\int_C \mathbf{F} \cdot \mathbf{dR}$ where $\mathbf{F} = -\cos x\mathbf{i} - 2y^2\mathbf{k}$ and $C : x = t,\ y = \pi,\ z = 3t^2\ t : 1 \to 2$.

MATH 294 FALL 1984 FINAL # 3 294FA84FQ3.tex

4.2.5 Let a force field $\mathbf{F}$ be given by:

$$\mathbf{F} = 2\mathbf{i} + z^2\mathbf{j} + 2yz\mathbf{k}$$

Evaluate

$$\int_C \mathbf{F} \cdot \mathbf{T} ds$$

if $\mathbf{T}$ is the unit tangent vector along the curve C defined by

$$C: \ x = \cos t, \ y = \sin t, \ z = t,$$

and t runs from zero to 2π.

MATH 294 SPRING 1985 FINAL # 18 294SP85FQ18.tex

4.2.6 Find the work done in moving from $P_0 = (0,0)$ to $P_1 = (\pi, 0)$ along the path $y = \sin x$ in the force field $\mathbf{F}(x,y) = x\mathbf{i} + y\mathbf{j}$.

a) 0
b) $\frac{1}{2}$
c) $\frac{\pi^2}{2}$
d) none of these.

MATH 294 FALL 1986 FINAL # 10 294FA86FQ10.tex

4.2.7 The curve C is the polygonal path (5 straight line segments) which begins at (1,0,1), passes consecutively through (2,-1,3),(3,-2,4),(0,3,7), (2,1,4), and ends at (1,1,1). $\vec{F}$ is the vector field $\vec{F}(x,y,z) = y^2\vec{i} + (2xy+z)\vec{j} + y\vec{k}$. Evaluate $\int_C \vec{F} \cdot d\vec{R}$

MATH 294 FALL 1987 PRELIM 1 # 3 294FA87P1Q3.tex

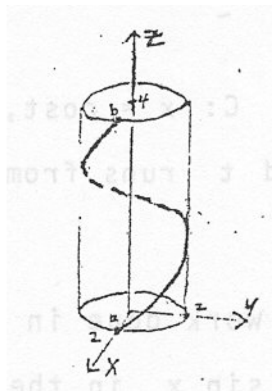
4.2.8 Evaluate $\int_C \mathbf{F} \cdot \mathbf{dr}$ where

$$\mathbf{F}(x,y,z) = \frac{-y}{x^2+y^2}\mathbf{i} + \frac{x}{x^2+y^2}\mathbf{j} + \mathbf{k}$$

and C is the curve $x = \cos t, \ y = \sin t, \ z = t \ 0 \leq t \leq 2\pi$

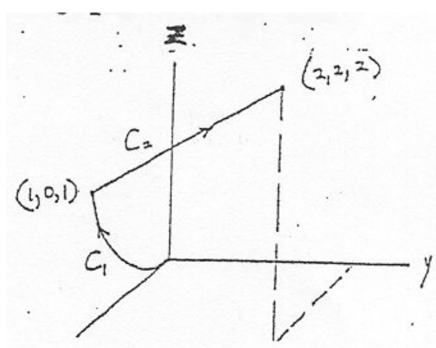
MATH 294 SPRING 1989 FINAL # 4 294SP89FQ4.tex

4.2.9 Evaluate, by any means, $\int_a^b \mathbf{F}\cdot d\mathbf{R}$ where the path is the helix shown from a at (2,0,0) to b at (2,0,4). The vector field $\mathbf{F}$ is given by $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.



MATH 294 FALL 1987 PRELIM 1 # 6 294FA87P1Q6.tex

4.2.10 For $\mathbf{F} = \frac{1}{xyz+1}(yz\mathbf{i} + xzbfj + xy\mathbf{k})$.
evaluate $int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the piece-wise smooth curve comprising two smooth curves: $C_1 : z = x^2,\ y = 0$ from (0,0,0) to (1,0,1); and C_2: the straight line from (1,0,1) to (2,2,2), as shown below.



MATH 294 FALL 1987 MAKE UP PRELIM 1 # 3 294FA87MUP1Q3.tex

4.2.11 Compute $\int_C \vec{F}\cdot d\vec{r}$ for

$$\vec{F}(x,y,z) = 2xy^3z^4\hat{\imath} + 3x^2y^2z^4\hat{\jmath} + 4x^2y^3z^3\hat{k}$$

and C given parametrically by:

$$\vec{r}(t) = \cos\pi t\hat{\imath} + e^{-t^2}\sin\frac{\pi}{2}t\hat{\jmath} + (2t-t^2)\cos\pi t\hat{k}, \quad \text{for } 0 < t < 1.$$

MATH 294 FALL 1987 MAKE UP PRELIM 1 # 4 294FA87MUP1Q4.tex

4.2.12 Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where

$$\vec{F}(x,y,z) = -y\hat{i} + x\hat{j} + \frac{z}{x^2+1}\hat{k}$$

$$C : \vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k},\ 0 \le t \le 2\pi.$$

MATH 294 FALL 1987 MAKE UP FINAL # 2 294FA87MUFQ2.tex

4.2.13 Evaluate $\int_C ydx + xdy + \frac{z}{z^2+1}dz$ where

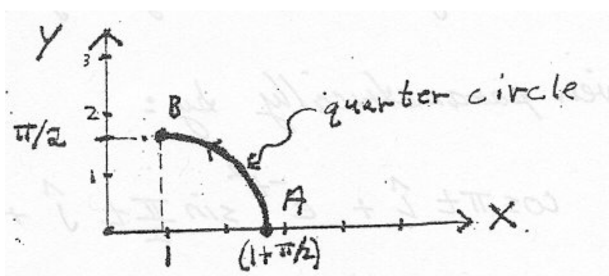
$$C : \cos t^3\hat{i} + \sin t\hat{j} + 2t\sin t\hat{k},\ 0 \le t \le \frac{\pi}{2}.$$

MATH 294 SPRING 1988 PRELIM 1 # 4 294SP88P1Q4.tex

4.2.14 Evaluate the integral $\int_A^B \mathbf{F}\cdot d\mathbf{R}$ for the vector field

$$\mathbf{F} = [\sin(y)e^{x\sin y}]\mathbf{i} + [x\cos(y)e^{x\sin y}]\mathbf{j}$$

for the path shown below between the points A and B.
[HINT !!: $\frac{\delta}{\delta x}[e^{x\sin y}] = [\sin y e^{x\sin y}]$ and $\frac{\delta}{\delta y}[e^{x\sin y}] = [x\cos y e^{x\sin y}]$.]



MATH 294 SPRING 1988 PRELIM 1 # 5 294SP88P1Q5.tex

4.2.15 Evaluate the path integral $\oint_C \mathbf{F}\cdot d\mathbf{R}$ for the vector field

$$\mathbf{F} = z\mathbf{j}$$

and the closed curve which is the intersection of the plane $z = \frac{4}{3}x$ and the circular cylinder $x^2 + y^2 = 9$.

MATH 294 FALL 1989 PRELIM 2 # 2 294FA89P2Q2.tex

4.2.16 Let $\mathbf{F} = z^2\hat{\imath} + y^2\hat{\jmath} + 2xz\hat{k}$.

a) Check that curl $\mathbf{F} = 0$.

b) Find a potential function for $\mathbf{F}$.

c) Calculate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the curve

$$\mathbf{r} = (\sin t)\hat{\imath} + (\frac{4t^2}{\tau^2})\hat{\jmath} + (1 - \cos t)\hat{k}$$

as t ranges from 0 to $\frac{\tau}{2}$.

MATH 294 FALL 1989 FINAL # 6 294FA89FQ6.tex

4.2.17 Consider the vector fields

$$\mathbf{F}(x, y, z) = \beta z^2\hat{\imath} + 2y\hat{\jmath} + xz\hat{k},\ (x, y, z) \epsilon \mathrm{R}^3.$$

depending on the real number (parameter) β.

a) Show that $\mathbf{F}$ is conservative if, and only if, $\beta = \frac{1}{2}$

b) For $\beta = \frac{1}{2}$, find a potential function.

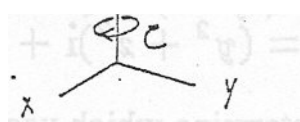
c) For $\beta = \frac{1}{2}$, evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{R},$$

where C is the straight line from the origin to the point (1,1,1).

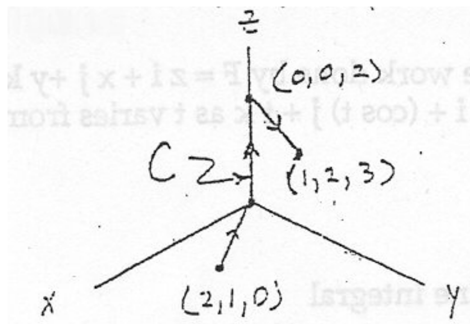
MATH 294 SPRING 1990 PRELIM 2 # 5 294SP90P2Q5.tex

4.2.18 Given $\mathbf{F}(x, y, z) = (-y + \sin(x^3 z))\hat{\imath} + (x + \ln(1 + y^2))\hat{\jmath} - ze^{xy}\hat{k}$, compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where C is the (closed) curve of intersection of the hemisphere $z = (5 - x^2 - y^2)^{\frac{1}{2}}$ and the cylinder $x^2 + y^2 = 4$, oriented as shown.



MATH 294 SPRING 1990 PRELIM 2 # 6 294SP90P2Q6.tex

4.2.19 Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z) = ze^{xz}\hat{\imath} - \hat{\jmath} + xe^{xz}\hat{k}$ and C is a path made up of the straight-line segments (2,1,0) to (0,0,0) ,(0,0,0) to (0,0,2), and (0,0,2) to (1,2,3), joined end-to-end as shown below.



MATH 294 SPRING 1990 FINAL # 1 294SP90FQ1.tex

4.2.20 Evaluate

$$\int_C (y+z)dx + (z+x)dy + (x+y)dz,$$

where C is the curve parameterized by $\vec{r}(t) = e^t \cos \pi t\hat{\imath} + (t^3+1)^{\frac{1}{2}}(\hat{\jmath}+\hat{k}),\ 0 \le t \le 2.$

MATH 294 FALL 1990 PRELIM 1 # 3 294FA90P1Q3.tex

4.2.21 **a)** Integrate the function $f(x,y,z) = xy + y + z$ over the path $\mathbf{R}(t) = \sin t\hat{\imath}\cos t\hat{\jmath} - 2t\hat{k}$, where $0 \le t \le \frac{\pi}{2}$.

b) Determine the work done by the force $\mathbf{F}(x,y,z) = x\hat{\imath} - y\hat{\jmath} + z\hat{k}$ along this path as it traveled from (0,1,0) to (1,0,$-\pi$).

MATH 294 FALL 1990 MAKE UP FINAL # 1 294FA90MUFQ1.tex

4.2.22 Given the surface $z = x^2 + 2y^2$. At the point (1,1) in the $x-y$ plane:

a) determine the direction of greatest increase of z.

b) determine a unit normal to the surface.

Given the vector field $\mathbf{F} = 2y^2z\hat{\imath} + 4xyz\hat{\jmath} + \alpha xy^2\hat{k}$,

c) find the value of α for $\mathbf{F}$ to be conservative and then determine its potential.

d) determine the work of the conservative vector field along the straight line from the point (1,2,3) to the point (3,4,5).

MATH 294 SPRING 1991 PRELIM 3 # 4 294SP91P3Q4.tex

4.2.23 one of the following vector fields is conservative:

i) $\mathbf{F} = (2y+z)\mathbf{i} + (x+z)\mathbf{j} + (x+y)\mathbf{k}$

ii) $\mathbf{F} = (y^2+z^2)\mathbf{i} + (x^2+z^2)\mathbf{j} + (x^2+y^2)\mathbf{k}$.

a) Determine which vector field is conservative.

b) Find a potential function for the conservative field.

c) Evaluate the line integral of the conservative vector field along an arbitrary path from the origin (0,0,0) to the point (1,1,1).

MATH 294 FALL 1991 PRELIM 3 # 1 294FA91P3Q1.tex

4.2.24 Calculate the work done by $\mathbf{F} = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$ along the path $\mathbf{R}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + t\mathbf{k}$ as t varies from 0 to 2π.

MATH 294 FALL 1991 PRELIM 3 # 3 294FA91P3Q3.tex

4.2.25 Evaluate the line integral

$$\int_C [(x^2 - y^2)dx - 2xdy]$$

along each of the following paths:

i) $C_1 : \ y = 2x^2$, from (0,0) to (1,2);

ii) $C_2 : \ x = t^2, \ y = 2t$, from $t = 0$ to $t = 1$;

iii) along C_1 from (0,0) to (1,2) and back along C_2 from (1,2) to (0,0). Check this answer using Green's Theorem.

MATH 294 FALL 1991 FINAL # 6 294FA91FQ6.tex

4.2.26 Find the work done in moving a particle from (2,0,0) to $(0,2,3\frac{\pi}{2})$ along a right circular helix

$$\mathbf{R}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + 3t\mathbf{k}$$

if the force field $\mathbf{F}$ is given by

$$\mathbf{F} = (4xy - 3x^2z^2)\mathbf{i} + 2x^2\mathbf{j} - 2x^3z\mathbf{k}$$

MATH 294 SPRING 1992 PRELIM 3 # 1 294SP92P3Q1.tex

4.2.27 Calculate the work done by the force field

$$\vec{F} = x^3\hat{i} + (\sin y - x)\hat{j}$$

along the following paths

a) $C_1 : \vec{R}_1(t) = \sin t\hat{i} + t\hat{j}, \ 0 \le t \le \pi$.

b) $C_2 : \vec{R}_2(t) = t\hat{j}, \ 0 \le t \le \pi$.

MATH 294 SPRING 1992 FINAL # 6 294SP92FQ6.tex

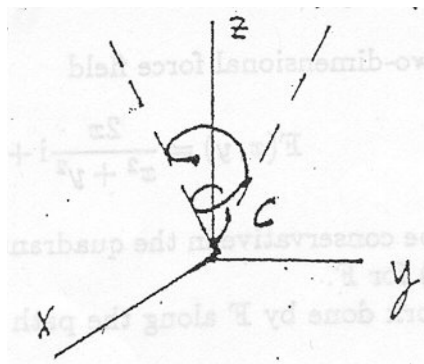
4.2.28 Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{R}$ where $\mathbf{F}(x,y) = (x+y^2)\hat{i} + (2xy+1)\hat{j}$ and C is the curve given by $\mathbf{R}(t) = \sin(t^2\pi)\hat{i} + t^3\hat{j}, \ 0 \le t \le 1$.

MATH 294 FALL 1992 FINAL # 2 294FA92FQ2.tex

4.2.29 Consider the curve $C : \mathbf{r}(t) = t\cos t\mathbf{i} + t\sin t\mathbf{j} + t\mathbf{k},\ 0 \le t \le 4\pi$, which corresponds to the conical spiral shown below.

a) Set up, but do not evaluate, the integral yielding the arc-length of C.

b) Compute $\int_C (y+z)dx + (z+x)dy + (x+y)dz$.



MATH 294 SPRING 1993 FINAL # 8 294SP93FQ8.tex

4.2.30 Evaluate $\int_C yz^2dx + xz^2dy + 2xyzdz$, where C is <u>any</u> path from the origin to the point (1,1,2).

MATH 294 FALL 1993 PRELIM 1 # 6 294FA93P1Q6.tex

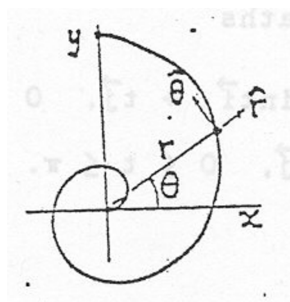
4.2.31 Evaluate $\int_C 2xyzdx + x^2zdy + x^2ydz$, where C is any path from (2,3,-1) to the origin.

MATH 294 FALL 1993 FINAL # 3 294FA93FQ3.tex

4.2.32 For $\mathbf{F} = 3\hat{\imath} - y\hat{\jmath}$, evaluate ∇ x F.

For the same vector $\mathbf{F}$, evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the spiral curve $r = 2\theta$ that runs from $\theta = 0$ to $\theta = 5\frac{\pi}{2}$.

Note that in cartesian coordinates $d\mathbf{r} = dx\hat{\imath} + dy\hat{\jmath}$. If you wish to use polar coordinates, $d\mathbf{r} = dr\hat{r} + rd\theta\hat{\theta}$.



MATH 294 SPRING 1994 FINAL # 2 294SP94FQ2.tex

4.2.33 Consider the force field

$$\mathbf{F}(x, y, z) = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{\sqrt{x^2 + y^2 + z^2}}$$

a) Show $\mathbf{F}$ to be conservative in the region $x > 0,\ y > 0,\ z > 0$.

b) Find a potential function $f(x, y, z)$ for $\mathbf{F}$.

MATH 294 SPRING 1994 FINAL # 4 294SP94FQ4.tex

4.2.34 Consider the two-dimensional force field

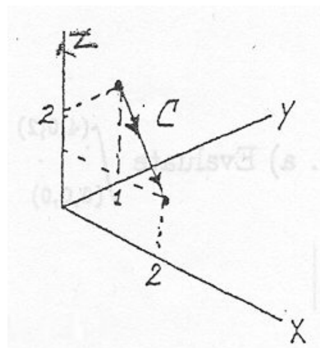
$$\mathbf{F}(x, y) = \frac{2x}{x^2 + y^2}\mathbf{i} + \frac{2y}{x^2 + y^2}\mathbf{j}.$$

a) Show $\mathbf{F}$ to be conservative in the quadrant $x > 0,\ y > 0$ and find a potential function $f(x, y)$ for $\mathbf{F}$.

b) Find the work done by $\mathbf{F}$ along the path $\mathbf{R}(t) = t\mathbf{i} + t^2\mathbf{j}$, from $t = 1$ to $t = 2$.

MATH 294 FALL 1994 PRELIM 1 # 1 294FA94P1Q1.tex

4.2.35 C is the line segment from (0,1,2) to (2,0,1).



a) which of the following is a parametrization of C?

i) $x = 2t,\ y = 1 - t,\ z = 2 - t,\ 0 \leq t \leq 1.$

ii) $x = 2 - 2t,\ y = -2t,\ z = 1 - 2t,\ 0 \leq t \leq \frac{1}{2}.$

iii) $x = 2\cos t,\ y = \sin t,\ z = 1 + \sin t,\ 0 \leq t \leq \frac{\pi}{2}.$

b) evaluate $\int_C 3z\hat{j} \cdot d\vec{r}.$

MATH 294 FALL 1994 FINAL # 1 294FA94FQ1a.tex

4.2.36 Evaluate $\int_C \cos y dx - x \sin y dy + dz$ where C is some curve from the origin to (2, $\frac{\pi}{2}$, 5),

MATH 294 SPRING 1995 PRELIM 1 # 3 294SP95P1Q3.tex

4.2.37 **a**) Find a potential function for $\vec{F} = (2xyz + \sin x)\hat{i} + x^2 z\hat{j} + x^2 y\hat{k}.$

b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is any curve from (π, 0, 0) to (1,1, π).

MATH 294 SPRING 1995 FINAL # 1 294SP95FQ1.tex

4.2.38 Find

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F}(x, y) = y\hat{i} + x\hat{j}$ and C is the curve given by $\vec{r}(t) = e^{\sin(t)}\hat{i} + t\hat{j},\ 0 \leq t \leq \pi.$

MATH 294 FALL 1995 PRELIM 2 # 1 294FA95P2Q1b.tex

4.2.39 Evaluate $\int_C y^2 z^2 dx + 2xyz^2 dy + 2xy^2 z dz$, where C is a path from the origin to the point (5,2,-1).

MATH 294 FALL 1995 PRELIM 1 # 2 294FA95P1Q2.tex

4.2.40 **a**) Evaluate $\oint_{C_1} 2dx + xdy$ where C_1 is the unit circle counterclockwise.

b) Evaluate $\oint_{C_2} 2dx + xdy$ where C_2 is the part of C_1 where $y \geq 0$.

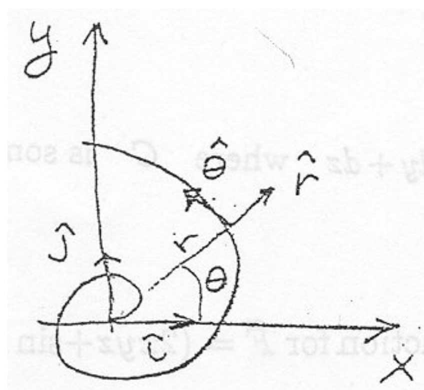
MATH 294 SPRING 1996 PRELIM 1 # 1 294SP96P1Q1a.tex

4.2.41 Evaluate $\int_{(0,0,0)}^{(4,0,2)} 2xz^3dx + 3x^2z^2dz$ on any path.

MATH 294 FALL 1996 PRELIM 1 # 1 294FA96P1Q1.tex

4.2.42 For $\mathbf{F} = 4\hat{\imath} - y\hat{\jmath}$. evaluate ∇ x $\mathbf{F}$.

For the same vector $\mathbf{F}$, evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the spiral curve $r = 2\theta$ that runs from $\theta = 0$ to $theta = \frac{5\pi}{2}$.



MATH 294 FALL 1996 PRELIM 1 # 2 294FA96P1Q2.tex

4.2.43 A three-dimensional curve C is parametrically represented by

$$\mathbf{r}(t) = t\cos t\hat{\imath} + t\sin t\hat{\jmath} + t\hat{k},\ 0 \leq t \leq 4\pi$$

Describe the curve and sketch it, clearly indicating the start and end points. Set up, but do not evaluate, an integral over t that gives the length of C.

MATH 294 FALL 1996 PRELIM 1 # 6 294FA96P1Q6.tex

4.2.44 The following figures show vector fields derived from real systems:

a) the electric field **E** emanating from a point charge near a conducting sphere (also shown are constant lines of potential $\nabla f = \mathbf{E}$, and

b) the velocity field **V** surrounding Jupiter's Great Red Spot.

For these vector fields, state whether the following quantities are $> 0, < 0, = 0$, or indeterminate from what's given.

Provide mathematical reasons for your choices.

For part a

$$\int_A^B \mathbf{E} \cdot d\mathbf{r}$$

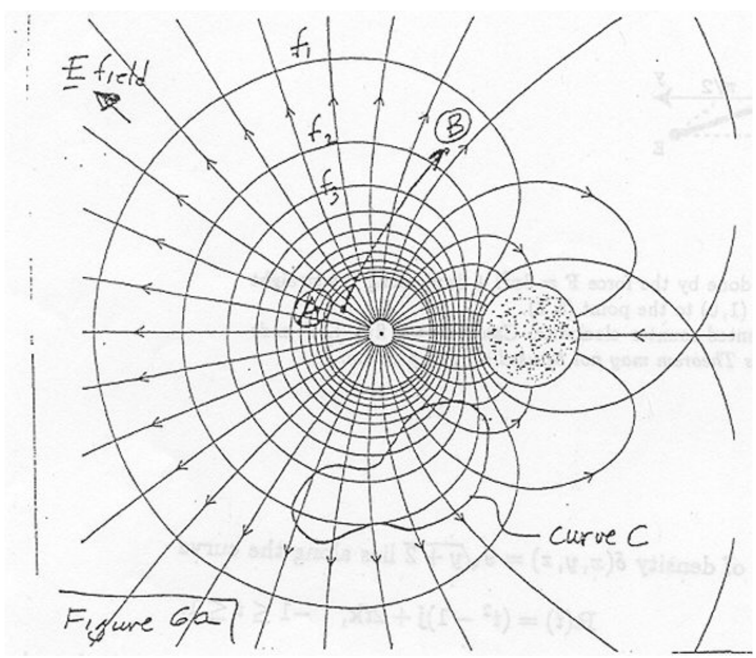
$$\oint \mathbf{E} \cdot d\mathbf{r}$$

For part b with the area A of interest being some ellipsoidal boundary of the Red Spot

$$\int\int_A (\nabla \text{ x } \mathbf{V}) \cdot \hat{n} dA$$

$$\int\int_A \nabla \cdot \mathbf{V} dA;$$

$\hat{n}$ is out of the paper



MATH 293 FALL 1996 PRELIM 3 # 3 293FA96P3Q3.tex

4.2.45 Let C be the curve parametrized by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ with $-2 \le t \le 2$. Let $\mathbf{F} = \frac{1}{5}\mathbf{k}$

a) Sketch the curve C.

b) From your sketch explain why $\int_C \mathbf{F} \cdot d\mathbf{r}$ is a positive or negative.

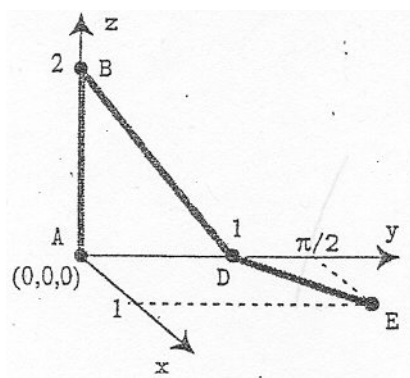
c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

MATH 293 FALL 1996 FINAL # 3 293FA96FQ3.tex

4.2.46 **Integrals.** Any method allowed except MATLAB.

a) Evaluate $\int\int_S \mathbf{F} \cdot \mathbf{n} d\sigma$ with $\mathbf{F} = \mathbf{r}$ and S the sphere of radius 7 centered at the origin. [$\mathbf{n}$ is the outer normal to the surface, $\mathbf{r} \equiv x\hat{\imath} + y\hat{\jmath} + z\hat{k}$].

b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ with $\mathbf{F} = \sin(y)e^z\hat{\imath} + x\cos(y)e^z\hat{\jmath} + x\sin(y)e^z\hat{k}$ and the path C is made up of the sequence of three straight line A to B, B to D, and D to E.



MATH 293 FALL 1997 PRELIM 3 # 5 293FA97P3Q5.tex

4.2.47 **a)** Find the work $\int_C \mathbf{F} \cdot d\mathbf{r}$ done by the force $\mathbf{F} = 6x^2\hat{\imath} + 6xy\hat{\jmath}$ along the straight line segment C from the point (1,0) to the point (5,8).

b) Now C is the unit circle oriented counter-clockwise. Calculate the flux $\int_C \mathbf{F} \cdot \mathbf{n} ds$ if $\mathbf{F} = y^2\hat{\imath} + xy\hat{\jmath}$. *Note: Green's Theorem may not be used on this problem*

MATH 294 SPRING 1994 FINAL # 1 294SP94FQ1.tex

4.2.48 A wire of density $\delta(x, y, z) = 9\sqrt{y+2}$ lies along the curve

$$\mathbf{R}(t) = (t^2 - 1)\mathbf{j} + 2t\mathbf{k}, \ -1 \le t \le 1.$$

Find (a) its total mass and (b) its center of mass. Then sketch the wire and center of mass on a suitable coordinate plane.

MATH 293 FALL 1998 PRELIM 3 # 4 293FA98P3Q4.tex

4.2.49 Calculate the work done by the vector field

$$\mathbf{F} = xz\mathbf{i} + y\mathbf{j} + x^2\mathbf{k}$$

along the line segment from (0,-1,0) to (1,1,3).

MATH 293 FALL 1998 PRELIM 3 # 5 293FA98P3Q5.tex

4.2.50 Find the circulation and the flux of the vector field

$$\mathbf{F} = 2x\mathbf{i} - 3y\mathbf{j}$$

in the $x-y$ plane around and across $x^2+y^2 = 4$ traversed once in a counter-clockwise direction. Do this by direct calculation, not by Green's theorem.

MATH 294 FALL 1998 FINAL # 1 293FA98FQ1.tex

4.2.51 If ∇ x $\mathbf{F} = 0$, the vector field $\mathbf{F}$ is conservative.

a) Show that $\mathbf{F} = (sec^2 x + \ln y)\mathbf{i} + (\frac{x}{y} + ze^y)\mathbf{j} + e^y\mathbf{k}$ is conservative.

b) Calculate the value of the integral

$$\int_{r_1}^{r_2} \mathbf{F} \cdot d\mathbf{r}$$

along any path from $\mathbf{r}_1 = \frac{\pi}{4}\mathbf{i} + \mathbf{j}$ to $\mathbf{r}_2 = \mathbf{j} + \mathbf{k}$.