3.2 2^{nd} Order ODEs

MATH 293 FALL 1995 PRELIM 2 # 2 293FA95P2Q2.tex

3.2.1 Solve $y" + 2y' + 3y = 0, \quad y(0) = 10, \quad y'(0) = -10$

MATH 294 SPRING 1995 PRELIM 2 # 1 294SP95P2Q1b.tex

3.2.2 Consider the differential equation

$$y" + y' + 2y = cosx$$

i) Find the solution to the homogeneous equation
ii) Find a particular solution
iii) Write down the general solution
iv) Find the solution that satisfies the initial condition $y(0) = 1, \quad y'(0) = 2$
v) Write down the general form of a particular solution to

$$y" + 2y' - 3y = 4 + 7sinx + x^2 + e^x$$

Do not solve for the (undetermined) coefficients.

MATH 294 FALL 1986 FINAL # 8 294FA86FQ8.tex

3.2.3 Solve for a particular solution; $y" + \frac{1}{4}y = f(x)$, where f(x) is the "square wave",

$$f(x) = \begin{cases} -1 & \text{if } (2n-1)\pi \leq x \leq 2n\pi, n \text{ any integer,} \\ 1 & \text{if } 2n\pi \leq x \leq (2n+1)\pi, n \text{ any integer} \end{cases}$$

MATH 294 SPRING 1994 FINAL # 4 294SP94FQ4.tex

3.2.4 Consider the two-dimensional force field

$$\mathbf{F}(x, y) = \frac{2x}{x^2 + y^2}\mathbf{i} + \frac{2y}{x^2 + y^2}\mathbf{j}.$$

a) Show $\mathbf{F}$ to be conservative in the quadrant $x > 0$, $y > 0$ and find a potential function $f(x, y)$ for $\mathbf{F}$.
b) Find the work done by $\mathbf{F}$ along the path $\mathbf{R}(t) = t\mathbf{i} + t^2\mathbf{j}$, from $t = 1$ to $t = 2$.

MATH 293 FALL 1992 FINAL # 1 293FA92FQ1b.tex

3.2.5 Find the solution of the initial value problem

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = 0 \quad with \quad y(0) = 1, \quad \frac{dy}{dx}(0) = 3$$

MATH 294 SPRING 1996 PRELIM 2 # 2 294SP96P2Q2.tex

3.2.6 **a)** Solve $\frac{d^4y}{dx^4} - 81y = 0$

b) The expression $y(x) = D_1e^{-x}sinx + D_2e^{-x}cosx$ (where D_1 and D_2 are constants) satisfies

$$y" + 2y' + 2y = 0$$

Find the solution to the non-homogeneous equation

$$y" + 2y' + 2y = 2 + 10cos2x$$

that satisfies $y(0) = 0, \quad y'(0) = -2$

MATH 294 FALL 1993 FINAL # 2 294FA93FQ2.tex

3.2.7 **a)** Solve $y"(t) + 2y'(t) + 2y(t) = 0, \quad y(0) = 1, \quad y'(0) = 1$

b) Put the equation in (a) into the form $x' = Ax$, where $x_1 = y,\ x_2 = y'$. Put the initial conditions in vector form too. Sketch several trajectories in the phase plane.

c) For the forced equation $y" + 3y' + 2y = coswt$, find a value for w and a particular solution such that the friction term $3y'$ matches the force $coswt$, while the acceleration $y"$ and spring force term $2y$ cancel each other.

MATH 293 FALL 1996 FINAL # 1 293FA96FQ1bc.tex

3.2.8 Simple ODEs. For each of the problems below find the most general solution to the given problem. If no solution exists answer 'no solutions'. You may use any method *except* computer commands.

a) $\frac{d^2x}{dt^2} = -7x \qquad with \qquad x(0) = 3$

b) $\frac{d^2x}{dt^2} = -7 \qquad with \qquad x(0) = 3\ and\ x(3) = 0$

MATH 293 FALL 1994 FINAL # 1 293FA94FQ1a.tex

3.2.9 Find the solution (in real form) of the initial value problem:

$$y" - 6y' + 10y = 0, \qquad y(0) = 7,\ y'(0) = 1$$

MATH 293 FALL 1995 PRELIM 1 # 4 293FA95P1Q4.tex

3.2.10 **a)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0$$

b) Check your solution.

MATH 294 SPRING 1995 FINAL # 2 294SP95FQ2e.tex

3.2.11 Let

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 2 & 6 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

a) Transform this system of equations to a second order equation in x plus a first order equation in x and y that may be solved successively to obtain the same solutions as the original system.

MATH 293 SPRING 1995 FINAL # 2 293SP95FQ2c.tex

3.2.12 Find a solution of the differential equation

$$t\frac{d^2p}{dt^2}(t) + 2p(t) = t^3$$

Do you think that you have found the general solution?

MATH 294 FALL 1992 PRELIM 1 # 4 294FA92P1Q4.tex

3.2.13 **a)** For what value(s) of "L" does the BVP (boundary value problem),

$$y" + 16y = 0, y(0) = 0, y(L) = 0, L > 0;$$

have a *nontrivial* solution?

b) Write down the (nontrivial) solution corresponding to the value(s) of "L" found in part(a).

c) For what values of "L" does the BVP have a *unique* solution?

MATH 293 SPRING 1997 PRELIM 2 # 1 293SP97P2Q1.tex

3.2.14 Solve the initial value problem: $y" + y = e^{-t},\ y(0) = 1,\ y'(0) = 0$

MATH 293 FALL 1996 PRELIM 2 # 1 293FA96P2Q1.tex

3.2.15 **a)** Find $x(2)$ given that $\dfrac{d^2}{dt^2}x = -4x,\ x(0) = 0,\ and\ x(0) = 4$

b) Look at the differential equation $\dfrac{d^2}{dt^2}x = -4x - cos(ct)$ (you don't need to solve it). For what values of c would the solution get infinitely big as t goes to infinity? (explain why if you do not give a solution to the differential equation).

MATH 293 SPRING 1997 PRELIM 2 # 5 293SP97P2Q5.tex

3.2.16 Consider the equation $y" + 4y' + \dfrac{41}{4}y = 0$

a) Find constants a, b, c and d so that $Pe^{at}cos(bt) + Qe^{ct}sin(dt)$ is a solution, regardless of the constants P and Q.

b) What additional information would suffice to determine P and Q?

MATH 294 FALL 1994 PRELIM 2 # 3 294FA94P2Q3a.tex

3.2.17 Solve $y" + 2y' + 3y = 0,\ y(0) = -1,\ y'(0) = -1$

MATH 294 FALL 1994 PRELIM 3 # 1 294FA94P3Q1a.tex

3.2.18 Given $y(t) = e^{-t}sint$, find an ordinary differential equation ($y" + by' + cy = 0$) solved by y.

MATH 293 SPRING 1994 PRELIM 1 # 6 293SP94P1Q6.tex

3.2.19 Consider the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

a) What is the order of (1)?
b) what type of equation is it?
c) Find the characteristic equation of (1)
d) Find the roots of the characteristic equation of (1)
e) Find the general solution of (1)
f) If $y(0) = 1$, and $y'(0) = 0$, find the solution of the initial value problem.

MATH 293 SPRING 1995 FINAL # 1 293SP95FQ1.tex

3.2.20 Consider the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0$$

a) Find the general solution $x(t)$ of this equation, in terms of real-valued functions of t.
b) Find the solution satisfying $x(0) = 1$, $\frac{dx}{dt}(0) = 0$.

MATH 294 FALL 1995 FINAL # 2 294FA95FQ2.tex

3.2.21 Find the general solutions for these two 2nd order ODE's, and find and sketch the solutions for the initial conditions $y(0) = y'(0) = 0$. (Here $a > 0$ is a constant).
a) $y" + a^2y = sinax$
b) $y" - a^2y = sinax$

MATH 294 SPRING 1994 FINAL # 5 294SP94FQ5.tex

3.2.22 For $t \geq 0$ solve the initial value problem

$$y"(t) + y(t) = f(t), \qquad f(t) = \begin{cases} 1 & for\ 0 \leq t \leq 2\pi, \\ 0 & for\ t > 2\pi, \end{cases}$$

with $y(0) = y'(0) = 0$. The solution and its derivative should be continuous for all $t \geq 0$. Hint: Solve each piece separately.

MATH 294 SPRING 1990 FINAL # 7 294SP90FQ7.tex

3.2.23 Find a particular solution of

$$y" + 5y = F(t),$$

where $F(t) \equiv \begin{cases} 1, & -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \\ 0, & \pi \leq t \leq -\frac{\pi}{2},\ \frac{\pi}{2} \leq t \leq \pi \end{cases}$, $and\ F(t + 2\pi) \equiv F(t)$ (i.e., F is 2π-periodic)

MATH 293 SPRING 1993 PRELIM 1 # 1 293SP93P1Q1.tex

3.2.24 Solve the initial value problem

$$\frac{d^2s}{dt^2} + 5\frac{ds}{dt} + 4s = 0$$

if $s = 1$ and $\frac{ds}{dt} = 2$ when $t = 0$

MATH 294 SPRING 1997 PRELIM 1 # 1 294SP97P1Q1.tex

3.2.25 Consider the equation

$$x" + 4x = 2cost + 3sin3t$$

a) Find the general solution of the above equation

b) Find the unique solution of the above equation for the initial conditions $x(0) = 0,\ x'(0) = 0$

MATH 294 SPRING 1993 PRELIM 1 # 2 294SP93P1Q2.tex

3.2.26

i) The O.D.E $u" + 2au' + 9u = 0$ where $a =$ const. has $u(x) = e^{3x}$ as a solution. Determine the value of $a$and the general solution.

ii) Find the general solution of $y" - 4y' + 4y = e^{2x}$

MATH 294 FALL 1993 MAKE UP PRELIM 2 # 3 294FA93MUP2Q3.tex

3.2.27 Find a second solution to

$$x^2y" + 3xy' + y = 0, \qquad x > 0,$$

MATH 294 FALL 1993 MAKE UP PRELIM 2 # 4 294FA93MUP2Q4.tex

3.2.28 Find a general solution to

$$y" + 2y' + y = 2e^{-x}$$

MATH 294 SPRING 1993 PRELIM 1 # 3 294SP93P1Q3.tex

3.2.29 The ODE

$$2x^2y"(x) + 3xy'(x) - y(x) = 0$$

has solutions of the form $y(x) = x^d(d =$const.)

iii) Find the general solution.

iv) Solve the initial-value problem with $y(1) = 3,\ y'(1) = 0$

MATH 293 SPRING 1998 PRELIM 2 # 1 293SP98P2Q1.tex

3.2.30 Solve the following initial value problem:$x" - 2x' + x = 0$ with $x(0) = 2$ and $x'(0) = 3$

MATH 294 SPRING 1997 FINAL # 1 294SP97FQ1.tex

3.2.31 **a**) Solve $y" + 4y = 0, \qquad 0 \leq x \leq 1$ with boundary conditions $y(0) = y(1) = 0$

b) Solve $x" + 100x = 20cos5t + 30sin2t$ with initial conditions $x(0) = 25, \quad x'(0) = 0$

MATH 294 FALL 1993 PRELIM 2 # 3 294FA93P2Q3.tex

3.2.32 Solve $u" + u' + u = 3sint,\ \ u(0) = 1\ \ u'(0) = 0$. where $u' = \dfrac{du}{dt}$

MATH 294 FALL 1993 PRELIM 2 # 4 294FA93P2Q4.tex

3.2.33 Find the general solution to $u" + 9u = cos3t$

MATH 294 FALL 1992 PRELIM 1 # 3 294FA92P1Q3.tex

3.2.34 Consider the equation

$$y" + y' + 3y = cos2x$$

a) Find the homogeneous solution.
b) Find a particular solution.
c) Write down the general solution.
d) Find the solution satisfying $y(0) = 1, y'(0) = -2$.
e) Write down the general form of a particular solution for

$$y" - 2y' + y = x^2e^x + 4sin5x + 1 + x$$

Do not solve for the (undetermined) coefficients.

MATH 293 FALL 1993 PRELIM 1 # 1 293FA93P1Q1.tex

3.2.35 Find the solution of the Initial Value Problem:

$$\frac{d^2s}{dt^2} - 2\frac{ds}{dt} + 5s = 0, \qquad s(0) = 1,\ \frac{ds}{dt}(t = 0) = 0$$

MATH 294 SPRING 1994 PRELIM 2 # 3 294SP94P2Q3.tex

3.2.36 Solve the initial-value problem

$$y" + 9y = 2sinx, \qquad y(0) = 0,\ y'(0) = 0$$

MATH 293 FALL 1994 PRELIM 1 # 5 293FA94P1Q5.tex

3.2.37 **a**) Find the solution of $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0,\ \ y(0) = 1,\ \ \dfrac{dy}{dx}(0) = 0$
b) Find the general solution in real form of the ODE

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

MATH 294 FALL 1996 PRELIM 2 # 3 294FA96P2Q3.tex

3.2.38 **a**) If you know that $y(t) = 4e^{-t}sin(2t)$ is the solution to a homogeneous, second-order linear differential equation with constant coefficients, find the differential equation.
b) What are the initial conditions $y(0)$ and $y'(0)$ that produce the above equation.
c) Write down the general form of the particular solution to the above equation where the right-hand side is no longer zero but rather $2 + 3sint - e^{-t}cos2t$. You do not have to solve for any (undetermined) coefficients. Note that you should not have to solve part (a) to complete this question.

MATH 294 FALL 1994 PRELIM 2 # 2 294FA94P2Q2.tex

3.2.39 Solve the following

a) $y" + 3y' + 2y = 0, \qquad y(0) = -1,\ y'(0) = -1.$

b) $y' = \dfrac{y^2 cosx}{3}$, find general solution.

MATH 294 SPRING 1983 PRELIM 1 # 3 294SP83P1Q3.tex

3.2.40 Consider the system of ordinary differential equations given by

$$\underline{\dot{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underline{\underline{A}}\ \underline{x}$$

MATH 293 SPRING 1995 PRELIM 1 # 5 293SP95P1Q5.tex

3.2.41 **a**) Find the solution of the differential equation

$$y" + 2\delta y' + y = 0, \text{ with } y(0) = 1,\ y'(0) = 0, \text{ with } 0 \leq \delta < \infty \text{ a parameter}$$

b) For what values of δ does the solution oscillate?

MATH 294 SPRING 1996 MAKE UP PRELIM 2 # 2 294SP96MUP2Q2.tex

3.2.42 Consider the equation

$$y" + y' + 3y = cos2x$$

a) Find the solution to the homogeneous equation.

b) Find a particular solution

c) Write down the general solution.

d) Find the solution satisfying $y(0) = 1$ and $y'(0) = -2$

e) Write down the general form of a particular solution for

$$y" + 2y' + y = x^2 e^x + 4sin5x + 1 + x$$

Do not solve for the (Undetermined) coefficients.

MATH 293 FALL 1992 PRELIM 1 # 2 293FA92P1Q2.tex

3.2.43 Find all solutions of the following differential equations and where indicated, find the solution of the initial value problem.

a) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 0$

b) $\dfrac{d^2y}{dt^2} - 4\dfrac{dy}{dt} + 4y = 0$

c) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = 0$

d) $\dfrac{d^2y}{dx^2} + 16y = 0,\ y(0) = 0, \text{ and } \dfrac{dy}{dx}(0) = 1$

MATH 293 FALL 1996 FINAL # 2 293FA96FQ2a.tex

3.2.44 For each of the problems below find the most general solution to the given problem. If no solution exists answer 'no solutions'. You may use any method *except* computer commands.

a) $\dfrac{d^2x}{dt^2} + x = sin(2t)$

MATH 293 SPRING 1996 FINAL # 29 293SP96FQ29.tex

3.2.45 True or False: The differential equation $y" + y' + y = 0$ has no nontrivial real-valued solutions

MATH 294 SPRING 1992 PRELIM 3 # 2 294SP92P3Q2.tex

3.2.46 Find the general solutions of the following differential equations. Express all answers in terms of real valued functions.

a) $\dfrac{d^4y}{dt^4} + 2\dfrac{d^2}{dt^2} + y = 0$

b) $\dfrac{d^2y}{dx^2} - 9y = 6e^{3x}$

MATH 294 FALL 1991 PRELIM 2 # 4 294FA91P2Q4.tex

3.2.47 Determine the general solution of

a) $\dfrac{d^6y}{dx^6} - \dfrac{d^4y}{dx^4} - 6\dfrac{d^2y}{dx^2} = 0, \ \ and$

b) $\dfrac{d^2y}{dt^2} - 4\dfrac{dy}{dt} + 4y = 6e^{2t}$

MATH 294 SPRING 1991 PRELIM 2 # 4 294SP91P2Q4.tex

3.2.48 Determine the general solution of the fifth order equation

$$\frac{d^5y}{dx^5} - 2\frac{d^3y}{dx^3} - 8\frac{dy}{dx} = 0$$

MATH 293 SPRING 1998 PRELIM 2 # 2 293SP98P2Q2.tex

3.2.49 find the real form of the general solution to the following equation

$$\frac{d^4x}{dt^4} + 6\frac{d^2x}{dt^2} + 5x = 0$$

MATH 294 SPRING 1993 PRELIM 2 # 3 294SP93P2Q3.tex

3.2.50 find the steady state solution for

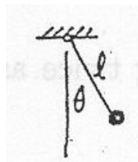
$$x" + 4x' + 13x = \cos wt$$

For what value of w is the amplitude of the response maximized?

MATH 294 SPRING 1996 FINAL # 4 294SP96FQ4.tex

3.2.51 Consider the differential equation

$$\frac{d^2\theta}{dt^2} + (\frac{g}{l})\theta = 0$$



(If $|\theta| << 1$ this governs the small oscillations of a free pendulum of length l in a gravitational field g. However, we allow *theta* to have any value here, and imagine that the pendulum can go around 360^o and more.) Set $x = \theta$ and $y = \frac{dx}{dt}$.

a) Write (1) as a system of first order equations in x and y.

b) Find the eigenvalues of this system and the location of the critical point. Characterize the critical point and make a qualitative sketch of a few trajectories in its neighborhood. Describe the pendulum's oscillation in terms of one of these phase portraits.

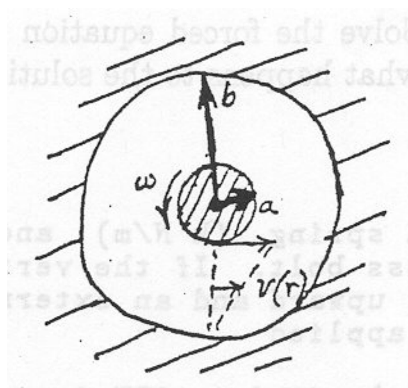
c) Equation (1) is an approximation to the complete equation for a pendulum

$$\frac{d^2\theta}{dt^2} + (\frac{g}{l})\sin\theta = 0. \qquad (2)$$

Where are the critical points of the system corresponding to this equation? Is the motions stable for all of them? (In answering this last equation, it is not necessary to do any analysis or linearization; it is enough to describe the situation in words.)

MATH 294 FALL 1995 PRELIM 2 # 4 294FA95P2Q4.tex

3.2.52 The flow of a fluid between two rotating cylinders is governed by



$$\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \frac{v}{r^2} = 0,$$

where $v(r)$ is the tangential fluid velocity, with boundary conditions $v(a) = wa$, $v(b) = 0$. Try a solution of the form $v = r^\lambda$. Solve for the two values of λ, then let $v = C_1 r^{\lambda_1} + C_2 r^{\lambda_2}$ and use $v(a) = wa$, $v(b) = 0$ to solve for C_1 and C_2.

MATH 294 FALL 1994 FINAL # 6 294FA94FQ6.tex

3.2.53 An object of mass 1 kg is attached to a spring with a spring constant of 2N/n, and is immersed in a viscous medium with damping constant 3 $\frac{Ns}{m}$. At time $t = 0$, the mass is lowered 0.10 m and given an initial velocity of 0.30 m/s in the upward direction. Find the position of the mass as a function of time and sketch this solution. Show that the mass will overshoot the equilibrium position, and then creep back to equilibrium.

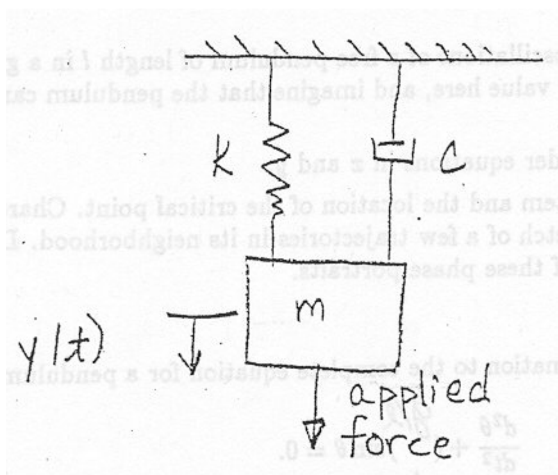
MATH 294 SUMMER 1995 QUIZ 3 # 1 294SU95Q3Q1.tex

3.2.54 A mass-spring-damper system with a periodic applied force is described by the 2nd order differential equation

$$\frac{1}{2}y" + y' + 5y = 8\cos 2t,$$

where y is the displacement of the mass relative to its equilibrium position.

a) What is the spring constant? If you sketch the spring twice as far, by what factor is the force multiplied?



b) Solve for the unforced (homogeneous) equation. State what happens to the motion after a long time.

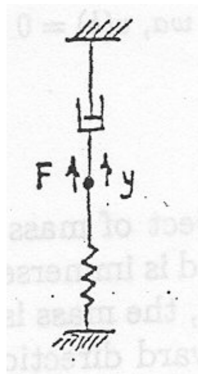
c) Find a particular solution to the forced equation of the form

$$y^p = A\cos bt + C\sin bt.$$

d) Solve the forced equation with $y(0) = 1$, $y'(0) = 0$. Sketch $y(t)$ and state what happens to the solution after a long time.

MATH 294 SPRING 1993 PRELIM 2 # 2 294SP93P2Q2.tex

3.2.55 A spring (K N/m) and damper (τ N/m/sec) are joined by a massless bolt. If the vertical displacement is y upward and an external upward force F is applied



a) find an ODE for the displacement y (i.e., Newton's equation of motion

b) find the general solution when $F(t) = F_o \cos wt$

MATH 294 SPRING 1993 MAKE UP PRELIM 2 # 3 294SP93MUP2Q3.tex

3.2.56 Find the steady state solution for

$$x'' + 4x' + 5x = \sin wt,$$

For what value of w is the amplitude of the response maximized?

MATH 294 SPRING 1993 MAKE UP PRELIM 2 # 1 294SP93MUP2Q1.tex

3.2.57 A damped spring-mass system has unit mass and a natural frequency w. Suppose that it is critically damped.

a) Find the O.D.E. satisfied by the displacement

b) Solve it in general.

c) Find the general solution if the damping coefficient is twice the previous value in (a).

MATH 294 SPRING 1993 FINAL # 2 294SP93FQ2.tex

3.2.58 **a**) A mass-spring-damper system with m = 1, k = 4, is critically damped. Find the most general motion with no initial displacement.

b) The same system is forced by $F(t)$. The steady state motion is $u(t) = 3\sin t - \frac{\pi}{4}$. Find $F(t)$.

MATH 293 SPRING 1997 PRELIM 2 # 4 293SP97P2Q4.tex

3.2.59 Consider the differential equation $z" + tz = 0$ with the initial conditions $z(0) = 0$, $z'(0) = 1$. Do part a) and either part b) or part c).

a) Let $x = z$, $y = z'$, and find the system of first-order equations and initial conditions satisfied by x and y.

b) Euler's method with stepsize h, applied to the system of part a), gives estimates for solution values which start out as $x_o = 0$, $y_o = 1$, $x_1 = h$, $y_1 = 1$, $x_2 = 2h$, $y_2 = 1 - h^3$, etc. Find formulas for x_3 and y_3 which estimate $z(3h)$ and $z'(3h)$.

or

c) Write out a set of Matlab (or C or...) commands which will compute Euler's method for 100 steps of size $h = .02$, and will either print the values found, or graph them.

MATH 294 SPRING 1997 PRELIM 1 # 2 294SP97P1Q2.tex

3.2.60 Consider the eigenvalue problem

$$y" + \lambda y = 0$$

$$y(0) + y'(0) = 0, \quad y(1) = 0$$

It is given that the eigenvalues of this problem are nonnegative. Find all these eigenvalues and the corresponding eigenfunctions. Leave your answers for the eigenvalues in the form "α is a root of this equation". Do not try to find them explicitly.

MATH 294 FALL 1982 FINAL # 4 294FA82FQ4.tex

3.2.61 For what values of λ does the boundary-value problem

$$y" - 2y' + (1 + \lambda)y = 0,$$

$$y(0) = 0, \quad y(1) = 0,$$

possess a nontrivial solution?

MATH 294 SPRING 1998 FINAL # 1 294SP98FQ1.tex

3.2.62 Determine the eigenvalues and the corresponding eigenfunctions for the boundary value problem

$$y" - 2y' + \lambda y = 0 \ on \ 0 < x < \pi;$$

with

$$y(0) = y(\pi) = 0.$$

Hint: Take $\lambda > 1$ and write the roots as complex numbers.

MATH 294 SPRING 1994 PRELIM 2 # 4 294SP94P2Q4.tex

3.2.63 Find the general solution of the following two ODE's. You *must* use **reduction of order**. No other method is allowed:

a) $y" + 10y' + 25y = 0$, $y_1(x) = e^{-5x}$;

b) $x^2y" + 2xy' = 0$, $y_1(x) = 1$.

For what values of x would you expect the solution of (b) to be valid?

MATH 293 SPRING 1998 PRELIM 1 # 6 293SP98P1Q6.tex

3.2.64 For the following equation:

$$y" + \frac{3}{2}y' - y = 0$$

a) Solve the initial value problem with $y(0) = 7$ and $y'(0) = -4$. What is the behavior of the solution as $t \to \infty$?

b) Solve the initial value problem with $y(0) = 1$ and $y'(0) = -2$. What is the behavior of the solution as $t \to \infty$?

MATH 293 FALL 1998 PRELIM 2 # 1 293FA98P2Q1.tex

3.2.65 **a**) Determine the general solution of

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0.$$

b) Determine the solution of this equation that satisfies the initial conditions

$$y(0) = 1 \text{ and } \frac{dy}{dx}(0) = 2.$$