2.6 Finite Difference Equations and Markov Chains

MATH 294 FALL 1997 PRELIM II # ?

2.6.1 Consider the linear difference equation

$$y_{k+3} - 2y_{k+2} - y_{k+1} + 2y_k = 0.$$

a) What is the dimension of the solution set of this equation?
b) Find a basis for this subspace of S.
c) Suppose $u = \{u_k\}$ is a solution to this difference equation where $u_0 = 1, u_1 = 0$, and $u_4 = 4$. Find a formula for u_k. (Hint: Use a linear combination of the basis vectors that you found in part (b) above).

MATH 294 SPRING 1998 PRELIM 3 # 2

2.6.2 Consider the difference equation

$$y_{k+2} + 4y_{k+1} + y_k = 0$$

for $k = 1, 2, ..., N-2$
a) Find its general solution.
b) Find the particular solution that satisfies the boundary conditions $y_1 = 5000$ and $y_N = 0$.
(The answer involves N.)

MATH 294 FALL 1998 PRELIM 3 # 4

2.6.3 The three "spaces" on the simple board game shown are labeled "C", "I", and "D" for coin, tetrahedron, and dice. On one turn a player advances clockwise a random number of spaces as determined by shaking and dropping the object on their present space (From the C position a player moves 1 or 2 spaces with equal probabilities, from the T space a player moves 1-4 spaces with equal probabilities, and from the D space a player moves 1-6 spaces with equal probabilities.).
In very long game what function of the moves end up on the D space on average? [Hint: Use exact arithmetic rather than truncated decimal representations.]