2.2 Intro to Bases

MATH 294 FALL 1981 PRELIM 1 # 3

2.2.1 **a**) Show that the set of vectors

$$\{1+t, 1-t, 1-t^2\}$$

is a basis for the vector space of all polynomials

$$\vec{p} = a_0 + a_1 t + a_2 t^2$$

of degree less than three.

b) Express the vector

$$2 + 3t + 4t^2$$

in terms of the above basis.

MATH 294 SPRING 1982 PRELIM 1 # 2

2.2.2 Let V be the space of all solutions of

$$\vec{x} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \vec{x}.$$

Consider the vectors

$$\vec{x}_1(t) = \begin{pmatrix} e^{-t} \\ 0 \\ -e^{-t} \end{pmatrix}, \vec{x}_2(t) = \begin{pmatrix} e^{t} \\ 0 \\ e^{t} \end{pmatrix}.$$

a) Do $\vec{x}_1(t)$, $\vec{x}_2(t)$ belong to V?

b) Are $\vec{x}_1(t)$, $\vec{x}_2(t)$ linearly independent? Give reasons for your answer.

c) Do the vectors $\vec{x}_1(t)$, $\vec{x}_2(t)$ form a basis for V? Give reasons for your answer.

MATH 294 SPRING 1983 FINAL # 10

2.2.3 **a)** Find a basis for the vector space of all 2×2 matrices.

b) A is the matrix given below, $\vec{v}$ is an eigenvector of A. Find any eigenvalue of A.

$$A = \begin{bmatrix} 3 & 0 & 4 & 2 \\ 8 & 5 & 1 & 3 \\ 4 & 0 & 9 & 8 \\ 2 & 0 & 1 & 6 \end{bmatrix} \text{ with } \vec{v} = [\text{an eigenvector of A}] = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}$$

c) Find one solution to each system of equations below, if possible. If not possible, explain why not. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} \cdot \vec{x} = \vec{b}\ ,\ \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

d) Read carefully. Solve for $\vec{x}$ in the equation $A \cdot \vec{b} = \vec{x}$ with: $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

e) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

MATH 294 SPRING 1984 FINAL # 2

2.2.4 Determine whether the given vectors form a basis for S, and find the dimension of the subspace. S is the set of all vectors of the form $(a, b, 2a, 2b)$ in $\Re^4$. The given set is $\{(1,0,2,0),(0,1,0,3),(1,-1,2,-3)\}$.

MATH 294 FALL 1986 FINAL # 1

2.2.5 The vectors (1,0,2,-1,3),(0,1,-1,2,4),(-1,1,-2,1,-3),(0,1,1,-2,-4), and (1,4,2,-1,3) span a subspace S of $\Re^5$.

a) What is the dimension of S?

b) Find a basis for S.

MATH 294 FALL 1986 FINAL # 2

2.2.6 **a)** Solve the linear system $A\vec{x} = \vec{b}$, where $A = \begin{bmatrix} 1 & 0 & -2 & 4 \\ 2 & 1 & -4 & 6 \\ -1 & 2 & 5 & -3 \\ 3 & 3 & -5 & 4 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 4 \\ 9 \\ 9 \\ 15 \end{bmatrix}$.

b) Solve the linear system $A\vec{x} = \vec{0}$, where $A = \begin{bmatrix} -3 & -1 & 0 & 1 & -2 \\ 1 & 2 & -1 & 0 & 3 \\ 2 & 1 & 1 & -2 & 1 \\ 1 & 5 & 2 & -5 & 4 \end{bmatrix}$ Express your answer in vector form, and give a basis for the space of solutions.

MATH 294 FALL 1987 PRELIM 3 # 6

2.2.7 Find an orthonormal basis for the subspace of $\Re^3$ consisting of all 3-vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ such that $x + y + z = 0$.

MATH 294 FALL 1989 PRELIM 3 # 3

2.2.8 Let W be the following subspace of $\Re^3$,

$$W = Comb\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -3 \end{bmatrix} \right)$$

a) Show that $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, is a basis for W

For b) and c) below, let T be the following linear transformation $T : W \to \Re^3$.

$$T\left(\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

for those $\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$ in $\Re^3$ which belong to W. [You are allowed to use a) even if you did not solve it.]

b) What is the dimension of Range(T)? (Complete reasoning, please.)

c) What is the dimension of Ker(T)? (Complete reasoning, please.)

MATH 293 SPRING 1990 PRELIM 1 # 3

2.2.9 Find the dimension and a basis for the following spaces

a) The space spanned by $\{(1, 0, -2, 1), (0, 3, 1, -1), (2, 3, -3, 1), (3, 0, -6, -1)\}$

b) The set of all polynomials $p(t)$ in P^3 satisfying the two conditions

i) $\frac{d^3}{dt^3}p(t) = 0$ for all t

ii) $p(t) + \frac{d}{dt}p(t) = 0$ at $t = 0$

c) The subspace of the space of functions of t spanned by $\left\{e^{at}, e^{bt}\right\}$ if $a \neq b$.

d) The space spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ in W, given that $\{\vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is a basis for W.

MATH 293 SPRING 1990 PRELIM 1 # 4

2.2.10 **a**) Show that $B = \left\{t^2 - 1, t^2 + 1, t\right\}$ is a basis for P^2

b) Express the vectors in $\left\{1, t, t^2\right\}$ in terms of those in B and find the components of $p(t) = (1 + t)^2$ with respect to B.

c) Find the components of the vector $\vec{x} = (1, 2, 3)$ with respect to the basis $\{(1,0,0),(1,1,0),(1,1,1)\}$.

MATH 293 FALL 1990 PRELIM 2 # 1

2.2.11 **a)** Express the vectors $\vec{u}, \vec{v}$ in terms of $\vec{a}, \vec{b}$, given that

$$3\vec{u} + 2\vec{v} = \vec{a}, \vec{u} - \vec{v} = \vec{b}$$

b) If $\vec{a}, \vec{b}$ are linearly independent, find a basis for the span of $\{ \vec{u}, \vec{v}, \vec{a}, \vec{b} \}$

c) Find $\vec{u}, \vec{v}$, if $\vec{a} = (-1, 2, 8), \vec{b} = (-2, -1, 1)$

MATH 293 FALL 1991 PRELIM 3 # 1

2.2.12 Consider the matrix

$$A = \begin{pmatrix} 2 & -1 & 1 & 3 \\ -1 & 2 & -2 & -2 \\ 2 & 5 & -4 & 1 \\ 1 & 4 & -4 & 0 \end{pmatrix}$$

a) Find a basis for the row space of A.

b) Find a basis for the column space of A.

MATH 293 SPRING 1992 PRELIM 3 # 6

2.2.13 Given $A = \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 3 & 5 \\ 2 & -1 & 3 & 4 \end{pmatrix}$.

a) Find a basis for the null space of A.

b) Find the rank of A.

MATH 293 SUMMER 1992 PRELIM 7/21 # 3

2.2.14 Given a matrix $A = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 2 & 1 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & -2 & 0 & -3 \end{pmatrix}$.

a) Find a basis for the row space W_1 of A.

b) Find a basis for the range W_2 of A.

c) Find the rank of A.

d) Are the two space W_1 and W_2 the same subspace of V_4? Explain your answer carefully in order to get credit for this part.

MATH 293 SPRING 1992 FINAL # 2

2.2.15 **a)** Find a basis for V_4 that contains at least two of the following vectors:

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}$$

b) A is a 3×3 matrix. If $A \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 7 \end{pmatrix}$ and $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$ is a basis for the nullspace of A, then find the general solution $\vec{x}$ of the equation $A\vec{x} = \begin{pmatrix} 0 \\ 4 \\ 7 \end{pmatrix}$.

Find, also, the determinant of A.

MATH 293 SUMMER 1992 PRELIM 7/21 # 4

2.2.16 Given four vectors in V_4

$$\vec{v}_1 = \begin{pmatrix} 2 \\ 4 \\ -2 \\ -4 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ -2 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} 4 \\ 4 \\ 0 \\ -6 \end{pmatrix}, \vec{v}_4 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

a) Find the space W spanned by the vectors $(\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)$

b) Find a basis for W.

c) Find a basis for V_4 that contains as many of the vectors $\vec{v}_1$,$\vec{v}_2$,$\vec{v}_3$ and $\vec{v}_4$ as possible.

MATH 293 FALL 1992 PRELIM 3 # 2

2.2.17 Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2 & 2 \\ 2 & 0 & -6 & -2 \\ -1 & 1 & 5 & 3 \end{pmatrix}$$

a) Find a basis for the column space of A from among the set of column vectors.

b) Find a basis for the row space of A.

c) Find a basis for the null space of A.

d) What is the rank of A and the dimension of the null space (the nullity)?

MATH 293 FALL 1992 PRELIM 3 # 3

2.2.18 Let $C(-\pi,\pi)$ be the vector space of continuous functions on the interval $-\pi \leq x \leq \pi$. Which of the following subsets S of $C(-\pi,\pi)$ are subspaces? If it is not a subspace say why. If it is, then say why and find a basis.
Note: You must show that the basis you choose consists of linearly independent vectors. In what follows a_0, a_1 and a_2 are arbitrary scalars unless otherwise stated.

a) S is the set of functions of the form $f(x) = 1 + a_1\sin(x) + a_2\cos(x)$

b) S is the set of functions of the form $f(x) = 1 + a_1\sin(x) + a_2\cos(x)$, subject to the condition $\int_{-\pi}^{\pi} f(x)\,\mathrm{d}x = 2\pi$

c) S is the set of functions of the form $f(x) = 1 + a_1\sin(x) + a_2\cos(x)$, subject to the condition $\int_{-\pi}^{\pi} f(x)\,\mathrm{d}x = 0$

MATH 293 FALL 1992 FINAL # 3

2.2.19 **a**) Let A be an $n \times n$ nonsingular matrix. Prove that $\det(A^{-1}) = \frac{1}{\det(A)}$. Hint: You may use the fact that if A and B are $n \times n$ matrices $\det(AB) = \det(A)\det(B)$.

b) An $n \times n$ matrix A has a nontrivial null space. Find $\det(A)$ and explain your answer.

c) Given two vectors $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ in V_3. Find a vector (or vectors) $\vec{w}_1$,$\vec{w}_2$,... in V_3 such that the set $\{\ \vec{v}_1,\vec{v}_2,\vec{w}_1,...\ \}$ is a basis for V_3.

d) Let S be the set of all vectors of the form $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k}$ where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are the usual mutually perpendicular unit vectors. Let W be the set of all vectors that are perpendicular to the vector $\vec{v} = \vec{i} + \vec{j} + \vec{k}$. Is W a vector subspace of V_3? Explain your answer.

MATH 293 SPRING 1993 PRELIM 3 # 2

2.2.20 Given the matrix $B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 3 & 3 & 4 & 5 \end{pmatrix}$

a) Find a basis for the row space of B

b) Find a basis for the null space of B

MATH 293 SPRING 1993 PRELIM 3 # 14

2.2.21 Consider the following vectors in $\Re^4$

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix},\ \vec{v}_2 = \begin{pmatrix} 2 \\ -3 \\ -8 \\ 2 \end{pmatrix},\vec{v}_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 0 \end{pmatrix},\vec{v}_4 = \begin{pmatrix} 3 \\ 1 \\ -1 \\ 3 \end{pmatrix}$$

Let W be the subspace of $\Re^4$ spanned by the vectors $\vec{v}_1$,$\vec{v}_2$,$\vec{v}_3$ and $\vec{v}_4$.
Find a basis for W which is contained in (is a subset of) the set $\{\ \vec{v}_1,\vec{v}_2,\vec{v}_3,\ \vec{v}_4\ .\}$

MATH 293 SPRING 1993 PRELIM 3 # 5

2.2.22 **a**) Consider the vector space V whose elements are 3×3 matrices.

i) Find a basis for the subspace W_1 of V which consists of all upper-triangular 3×3 matrices.

ii) Find a basis for the subspace W_1 of V which consists of all upper-triangular 3×3 matrices with zero trace.
The trace of a matrix is the sum of its diagonal elements.

b) Consider the polynomial space P^3 of polynomials with degree ≤ 3 on $0 \leq t \leq 1$. Find a basis for the subspace W of P^3 which consists of polynomials of degree ≤ 3 with the constraint

$$\left[\frac{d^2p}{dt^2} + \frac{dp}{dt}\right]_{t=0} = 0.$$

MATH 293 FALL 1994 PRELIM 3 # 1

2.2.23 Let A be the matrix $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 2 & -1 & 2 \\ 1 & 0 & 0 & -1 \end{bmatrix}$

a) Find a basis for the Null Space of A. What is the nullity of A?
b) Find a basis for the Row Space of A. What is its dimension?
c) Find a basis for the Column Space of A. What is its dimension?
d) What is the rank of A?

MATH 293 FALL 1994 FINAL # 4

2.2.24 **a**) Find a basis for the space spanned by: {(1,0,1),(1,1,0),(-1,-4,-3)}.
b) Show that the functions $e^{2x}\cos(x)$ and $e^{2x}\sin(x)$ are linearly independent.

MATH 293 SPRING 1995 PRELIM 3 # 3

2.2.25 Let P_3 be the space of polynomials $p(t)$ of degree ≤ 3. Consider the subspace $S \subset P_3$ of polynomials that satisfy

$$p(0) + \left.\frac{dp}{dt}\right|_{t=0} = 0$$

a) Show that S is a subspace of P_3.
b) Find a basis for S.
c) What is the dimension of S?

MATH 293 SPRING 1995 PRELIM 3 # 5

2.2.26 **a**) Find a basis for the plane $P \subset \Re^3$ of equation

$$x + 2y + 3z = 0$$

b) Find an orthonormal basis for P.

MATH 293 FALL 1995 PRELIM 3 # 5

2.2.27 Let P_3 be the space of polynomials $p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ of degree ≤ 3. Consider the subset S of polynomials that satisfy

$$p''(0) = 4p(0) = 0$$

Here $p''(0)$ means, as usual, $\left.\frac{d^2 p}{dt^2}\right|_{t=0}$.

a) Show that S is a subspace of P_3. Give reasons.
b) Find a basis for S.
c) What is the dimension of S? Give reasons for your answer.
Hint: What constraint, if any, does the given formula impose on the constants a_0, a_1, a_2, and a_3 of a general $p(t)$?

MATH 293 FALL 1995 FINAL # 2

2.2.28 Consider the subspace W of $\Re^4$ which is defined as

$$W = span\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

a) Find a basis for W.
b) What is the dimension of W?
c) It is claimed that W is a "plane" in $\Re^4$. Do you agree? Give reasons for your answer.
d) It is claimed that the "plane" W can be described as the intersection of two 3-D regions $S-1$ and S_2 in $\Re^4$. The equations of $S-1$ and S_2 are:

$$\begin{array}{rc} S_1: & x - u = 0 \\ S_2: & ax + by + cz + du = 0 \end{array}$$

where $\begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix}$ is a generic point in $\Re^4$ and a, b, c, d are real constants.
Find one possible set of values for the constants a, b, c, and d.

MATH 293 SPRING 1996 PRELIM 3 # 1

2.2.29 The set W of vectors in $\Re^3$ of the form (a, b, c), where $a + b + c = 0$, is a subspace of $\Re^3$.

a) Verify that the sum of any two vectors in W is again in W.

b) The set of vectors

$$S = (1, -1, 0), (1, 1, -2), (-1, 1, 0), (1, 2, -3)$$

is in W. Show that S is linearly dependent.

c) Find a subset of S which is a basis for W.

d) If the condition $a + b + c = 0$ above is replaced with $a + b + c = 1$, is W still a subspace? Why/ why not?

MATH 293 SPRING 1996 PRELIM 3 # 2

2.2.30 Which of the following subsets are bases for $\Re^2$? Show any algebra involved or state a theorem to justify your answer.

$$S_1 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, S_2 = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}, S_3 = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -6 \end{bmatrix} \right\}.$$

MATH 293 SPRING 1996 FINAL # 22

2.2.31 Let

$$W = Span \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix} \right\}.$$

Then an orthonormal basis for W is

a) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}, \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{bmatrix} \right\}$

e) none of the above

MATH 294 FALL 1997 PRELIM 2 # 2

2.2.32 Consider the vector space P_2 of all polynomials of degree ≤ 2. Consider two bases of P_2:

$S : \{1, t, t^2\}$, the standard basis, and

$H : \{1, 2t, -2 + 4t^2\}$, the Hermite basis.

a) Find the matrices $P_{S \leftarrow H}$ and $P_{H \leftarrow S}$.

b) Consider $p_1(t) = 1 + 2t + 3t^2$ in P_2, and $p_2(t) = \frac{d}{dt} p_1(t)$. Find

$$[p_1(t)]_S, [p_2(t)]_S, [p_1(t)]_H, [p_2(t)]_H,$$

i.e. the coordinates of p_1 and p_2 in the bases S and H.

MATH 294 FALL 1997 PRELIM 2 # 3

2.2.33 Let W be the subspace of $\Re^4$ defined as

$$W = span \left(\begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -6 \\ 4 \end{pmatrix} \right).$$

a) Find a basis for W. What is the dimension of W?

b) It is claimed that W can be described as the intersection of two linear spaces S_1 and S_2 in $\Re^4$. The equations of S_1 and S_2 are

$$S_1 : x - y = 0,$$

and

$$S_2 : ax + by + cz + dw = 0,$$

where a, b, c, d are real constants that must be determined. Find one possible set of values of a, b, c and d.

MATH 294 FALL 1997 PRELIM 2 # 6

2.2.34 Let V be the vector space of 2×2 matrices.

a) Find a basis for V.

b) Determine whether the following subsets of V are subspaces. If so, find a basis. If not, explain why not.

i) $\{ A \text{ in } V \big| \det A = 0\}$

ii) $\{ A \text{ in } V | A \begin{pmatrix} 0 \\ 1 \end{pmatrix} = A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \}$.

c) Determine whether the following are linear transformations. Give a short justification for your answers.

i) $T : V \to V$, where $T(A) = A^T$,

ii) $T : V \to \Re^1$, where $T(A) = \det A$,

MATH 294 FALL 1998 FINAL # 4

2.2.35 Here we consider the vector spaces P_1, P_2, and P_3 (the spaces of polynomials of degree 1,2 and 3).

a) Which of the following transformations are linear? (Justify your answer.)

i) $T : P_1 \to P_3, T(p) \equiv t^2 p(t) + p(0)$

ii) $T : P_1 \to P_1, T(p) \equiv p(t) + t$

b) Consider the linear transformation $T : P_2 \to P_2$ defined by $T(a_0 + a_1 t + a_2 t^2) \equiv (-a_1 + a_2) + (-a_0 + a_1)t + (a_2)t^2$. With respect to the standard basis of P_2, $B = \{1, t, t^2\}$, is $A = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Note that an eigenvalue/eigenvector pair of A is $\lambda = 1, \vec{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Find an eigenvalue/eigenvector (or eigenfunction) pair of T. That is, find λ and $g(t)$ in P_2 such that $T(g(t)) = \lambda g(t)$.

c) Is the set of vectors in $P_2 \{3+t, -2+t, 1+t^2\}$ a basis of P_2? (Justify your answer.)

MATH 293 SPRING ? FINAL # C

2.2.36 Give a definition for addition and for scalar multiplication which will turn the set of all pairs $(\vec{u}, \vec{v})$ of vectors, for $\vec{u}, \vec{v}$ in V_2, into a vector space V.

a) What is the zero vector of V?

b) What is the dimension of V?

c) What is a basis for V?

MATH 294 FALL 1987 PRELIM 3 # 2 MAKE-UP

2.2.37 On parts (a) - (g), answer true or false.

a) $span\,(\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4) = \Re^3$, where $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

b) The four vectors in (a) are independent.

c) Referring to a again, all vectors $\vec{v} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in $span\,(\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)$ satisfy a linear equation $ax_1 + bx_2 + cx_3 = 0$ for scalars a,b,c not all 0.

d) The rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$ is 3.

e) In Re^n n distinct vectors are independent.

f) $n + 1$ distinct vectors always span $\Re^n$, for $n > 1$.

g) If the vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ span $\Re^n$, then they are a basis for $\Re^n$.

MATH 293 UNKNOWN PRACTICE # 4a

2.2.38 **a**) Find a basis for the row space of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 3 & 6 & 1 & 12 \\ 9 & 18 & 1 & 36 \end{bmatrix}$$

UNKNOWN UNKNOWN UNKNOWN # ?

2.2.39 If A is an $m \times n$ matrix show that $B = A^T A$ and $C = AA^T$ are both square. What are their sizes? Show that $B = B^T, C = C^T$

MATH 294 FALL ? FINAL # 1 MAKE-UP

2.2.40 Consider the homogeneous system of equations $B\vec{x} = \vec{0}$, where

$$B = \begin{bmatrix} 0 & 1 & 0 & -3 & 1 \\ 2 & -1 & 0 & 3 & 0 \\ 2 & -3 & 0 & 0 & 4 \end{bmatrix}, \ \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \text{ and } \vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

a) Find a basis for the subspace $W \subset \Re^5$, where W = set of all solutions of $B\vec{x} = \vec{0}$

b) Is B 1-1 (as a transformation of $\Re^5 \to \Re^3$)? Why?

c) Is B:$\Re^5 \to \Re^3$ onto why?

d) Is the set of all solutions of $B\vec{x} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$ a subspace of $\Re^5$? Why?