1.3 Vector and Matrix Equations

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1.3.1 Vector \( \vec{p} \) contains \( m \) elements, and vector \( \vec{q} \) contains \( n \) elements. The vector \( \vec{r} = \vec{p} + \vec{q} \)

a) Is always defined.
b) Is only defined when \( m = n \).
c) Is never defined.
d) Is defined for \( m = n \) and \( m \neq n \).

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1.3.2 The following [question] refer to the real 2x2 matrix \( A \).

Solve for the vector \( \vec{u} + \vec{v} \) in \( V_4 \) if

\[
3\vec{u} + 4\vec{v} = (0, 1, 0, 1) \quad \text{and} \quad -2\vec{u} + 7\vec{v} = (-1, 2, -7, 0)
\]

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1.3.3 a) Express the vectors \( \vec{u}, \vec{v} \) in terms of \( \vec{a}, \vec{b} \), given that

\[
3\vec{u} + 2\vec{v} = \vec{a}, \quad \vec{u} - \vec{v} = \vec{b}
\]

b) If \( \vec{a}, \vec{b} \) are linearly independent, find a basis for the span of \( \{\vec{u}, \vec{v}, \vec{a}, \vec{b}\} \)

c) Find \( \vec{u}, \vec{v} \), if \( \vec{a} = (-1, 2, 8), \vec{b} = (-2, -1, 1) \).

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1.3.4 If \( A \) is a \( n \times n \) matrix and if \( x \) and \( y \) are \( (n \times 1) \) vector, that satisfy the equations

\[
Ax = b, \quad Ay = c \quad (1,2)
\]

find the solution \( z \) of the equation

\[
Az = 2b - 3c \quad (3)
\]

and verify that \( z \) is a solution of \( (3) \).

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1.3.5 Fill in the blanks of the following statements.

In what follows \( A \) is an \( m \times n \) matrix

a) The dimension of the row space is \( m \).
The dimension of the null space is \( n \).
The number of columns of \( A \) is \( \infty \).

b) \( Ax = b \) has a solution \( x \) if and only if \( b \) is in the \( \infty \) space of \( A \).

c) If \( Ax = 0 \) and \( Ay = 0 \) and if \( C_1 \) and \( C_2 \) are arbitrary constants then \( A(C_1x + C_2y) = \infty \).
Given that:

\[ Ax = b \]

has a particular solution

\[ x_p = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \]

and

\[ Ax = 0 \]

has the general solution

\[ x_h = s \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \]

where \( s \) and \( t \) are arbitrary parameters, determine which of the following vectors are solutions of \( Ax = b \).

a) \( x = \begin{pmatrix} 0 \\ -3 \\ 0 \\ 3 \end{pmatrix} \)

b) \( x = \begin{pmatrix} 1 \\ -3 \\ -2 \\ 1 \end{pmatrix} \)

One serving of Kellogg's Crackin' Oat Bran supplies 110 calories, 3g of protein, 21g of carbohydrate, and 3g of fat. One serving of Kellogg's Crispix supplies 110 calories, 2g of protein, 25g of carbohydrate, and 0.4g of fat. It is desired to have \( b_1, b_2, b_3 \) and \( b_4 \) calories, and grams of protein, carbohydrate, and fat, respectively.

a) Set up, but do not solve, a matrix equation that would tell you how many servings of each cereal to eat.

b) Find a \( b \) (the column vector with \( b_1, b_2, b_3, b_4 \) as entries) where the matrix equation has a solution and find the solution. (One example with numbers is desired, not the general case.)

c) Is there a solution for all \( b \)? If so explain why, if not find a \( b \) where there is no solution and explain why there is no solution.