1.2 Solutions of $A\vec{x} = \vec{b}$

MATH 294 SPRING 1983 FINAL # 10 294SP83FQ10.tex

1.2.1 **a**) Find a basis for the vector space of all 2x2 matrices.

b) $\underline{\underline{A}}$ is the matrix given below, $\underline{v}$ is an eigenvector of $\underline{\underline{A}}$. Find any eigenvalue of $\underline{\underline{A}}$.

$$\underline{\underline{A}} = \begin{bmatrix} 3 & 0 & 4 & 2 \\ 8 & 5 & 1 & 3 \\ 4 & 0 & 9 & 8 \\ 2 & 0 & 1 & 6 \end{bmatrix} \text{ with } \underline{v} = [\text{an eigenvector of } \underline{\underline{A}}] = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

c) Find one solution to each system of equations below, if possible. If not possible, explain why not.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} \cdot \underline{x} = \underline{b},\ \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

d) Read carefully. Solve for $\mathbf{x}$ in the equation $\underline{\underline{A}} \cdot \underline{b} = \underline{\mathbf{x}}$ with:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

e) Find the inverse of the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

MATH 294 SPRING 1985 FINAL # 1 294SP85FQ1.tex

1.2.2 For the case $\vec{b} = \vec{0}$, the system

a) Always has at least one solution.
b) May have no solution.
c) Always has more than one solution.
d) Always has an infinite number of solutions.

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1.2.3 For the case $\vec{b} = \vec{0}$, the vector $\vec{x} = \vec{0}$

a) Is always a solution.
b) May or may not be a solution depending on $\underline{\underline{A}}$.
c) Is always the only solution.
d) Is never a solution.

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1.2.4 For the case $\vec{b} \neq \vec{0}$, the vector $\vec{x} = \vec{0}$

a) Is always a solution.

b) May or may not be a solution depending on $\underline{\underline{A}}$.

c) Is always the only solution.

d) Is never a solution.

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1.2.5 For the case $\vec{b} \neq \vec{0}$, you could expect

a) Always a unique solution.

b) Always an infinite number of solution.

c) Always no solution.

d) Any one of the above, (a) or (b) or (c), depending on $\underline{\underline{A}}$ and $\vec{b}$.

MATH 293 FALL 1992 PRELIM 2 # 1 293FA92P2Q1.tex

1.2.6 Find all solution x of the system of equations

$$Ax = b$$

$$\text{where } A = \begin{pmatrix} 1 & 2 & -2 & 0 \\ 3 & 1 & -2 & -1 \\ -1 & 3 & -2 & 1 \end{pmatrix} \; b = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}$$

MATH 293 SPRING 1993 PRELIM 2 # 5 293SP93P2Q5.tex

1.2.7 Suppose $\mathbf{Ax} = \mathbf{b}$ has a particular solution

$$\mathbf{x_b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Suppose $\mathbf{Ax} = \mathbf{0}$ has the general solution.

$$\mathbf{x_h} = c \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}.$$

where c is an arbitrary scalar. Show that

$$\begin{pmatrix} -3 \\ -6 \\ 7 \end{pmatrix}$$

is a solution of $\mathbf{Ax} = \mathbf{b}$.

MATH 293 FALL 1994 FINAL # 2 293FA94FQ2.tex

1.2.8 Find all values of a for which the following linear system

$$x + y - z = 2$$
$$x + 2y + z = 3$$
$$x + y + (a^2 - 5)z = a$$

has:

a) No solution.
b) A unique solution.
c) Infinitely many solutions.

MATH 293 SPRING 1996 PRELIM 2 # 1 293SP96P2Q1.tex

1.2.9 Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 3 & 4 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ -1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$

a) Find *all* $\vec{x}$ for which $C\vec{x} = \vec{b}$, where

$$\vec{b} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}.$$

MATH 293 SPRING 1996 PRELIM 2 # 4 293SP96P2Q4.tex

1.2.10 Let

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 2 & -1 & 2 \\ 1 & 0 & 0 & -1 \end{pmatrix}.$$

Let $\mathbf{x} = (0, \frac{1}{2}, 1, 0)$. We know that $A\mathbf{x} = 0$. True or false:
1. $\mathbf{x}$ is a *trivial* solution to $A\mathbf{x} = 0$.

MATH 293 SPRING 1996 FINAL # 13 293SP96FQ13.tex

1.2.11 The equation $A\mathbf{x} = \mathbf{b}$ has a solution if and only if:

a) $\mathbf{b}$ is in the column space of A^{-1}
b) $\mathbf{b}$ is in the null space of A
c) $\mathbf{b}$ is in the column space of A
d) A augmented by $\mathbf{b}$ is invertible.
e) none of the above.

MATH 293 SPRING 1996 FINAL # 27 293SP96FQ27.tex

1.2.12 True or false: If A is a 3x2 matrix, then $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

MATH 293 SPRING 1996 FINAL # 28 293SP96FQ28.tex

1.2.13 Consider the system

$$2x + ay = 0$$
$$ax + 2y = 0$$

Then there are infinitely many values of the parameter a for which the system has a non trivial solution. True or false.

MATH 293 SPRING 1996 FINAL # 36 293SP96FQ36.tex

1.2.14 If $\mathbf{x_1}$ and $\mathbf{x_2}$ are solutions to $A\mathbf{x} = \mathbf{b}$, then $\frac{1}{4}\mathbf{x_1} + \frac{3}{4}\mathbf{x_2}$ is also a solution to $A\mathbf{x} = \mathbf{b}$. True or false.

MATH 294 SPRING 1997 PRELIM 1 # 8 294SP97P1Q8.tex

1.2.15 Consider the two linear systems

$$\begin{array}{ll} a_{1,1}x_1 + ... + a_{1,9}x_9 = 0 & a_{1,1}x_1 + ...a_{1,9}x_9 = 1 \\ a_{2,1}x_2 + ... + a_{2,9}x_9 = 0 & a_{2,1}x_2 + ...a_{2,9}x_9 = 2 \\ & \text{and} \\ & \cdots\cdots \\ & \cdots\cdots \\ a_{9,1}x_9 + ... + a_{9,9}x_9 = 0 & a_{9,1}x_9 + ...a_{9,9}x_9 = 9 \end{array}$$

Suppose that the homogeneous system on the left has only the trivial solution. Explain why the nonhomogeneous system on the right has a solution.
Be sure to include in your explanation a general *statement or theorem which applies.*

MATH 294 FALL 1997 PRELIM 1 # 2 294FA97P1Q2.tex

1.2.16 Consider three vectors in $\Re^4$:

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 5 \\ -2 \\ a \\ b \end{pmatrix}.$$

a) For what values of a and b does v_3 lie in $span\{v_1, v_2\}$?
b) For what values of a and b is the set $S = \{v_1, v_2, v_3\}$ linearly independent in $\Re^3$?
c) For what values of a and b does

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \\ -1 & 0 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ a \\ b \end{pmatrix}$$

have at least one solution x?

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1.2.17 a) Write the solution set of the system

$$\begin{array}{rcrcrcl} x_1 & - & 3x_2 & - & 2x_3 & = & 0 \\ & & x_2 & - & x_3 & = & 0 \\ -\ 2x_1 & + & 3x_2 & + & 7x_3 & = & 0 \end{array}$$

in parametric form.

b) With

$$A \equiv \begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & -1 \\ -2 & 3 & 7 \end{bmatrix},$$

find all solutions to the system

$$A\overrightarrow{x} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}.$$

c) True or False?

i) The columns of A are linearly independent.

ii) The solution set of $A\overrightarrow{x} = \overrightarrow{b}$ is all vectors of the form $\overrightarrow{w} = \overrightarrow{p} + \overrightarrow{v_h}$ where $\overrightarrow{v_h}$ is any solution of $A\overrightarrow{v_h} = \overrightarrow{0}$ and $A\overrightarrow{p} = \overrightarrow{b}$.

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1.2.18 a)

$$\text{Let } A = \begin{bmatrix} 1 & -7 & 2 & 2 \\ -6 & 5 & 8 & 12 \\ 12 & 0 & -4 & 12 \end{bmatrix}$$

Are the columns of A linearly independent? Why or why not.

b) Determine if the columns of the given matrix form a linearly dependent set. **Hint:** one way to do this is by row operations.

$$A = \begin{bmatrix} 1 & -3 & 0 \\ 3 & -5 & 5 \\ -2 & 6 & -6 \end{bmatrix}$$

c) Let

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}. \text{ Given that } \vec{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

is a solution to $A\vec{x} = \vec{b}$, is this solution unique?

d) For the matrix in (c) is there a solution to $A\vec{x} = \vec{b}$ for all $\vec{b}$ in $\Re^3$? Why or why not?

MATH 294 FALL 1998 PRELIM 2 # 4 294FA98P2Q4.tex

1.2.19* The reduced echelon form of the matrix $A = \begin{bmatrix} 3 & 3 & 2 & 3 \\ -2 & 2 & 0 & 2 \\ 1 & 0 & 1 & -2 \\ 0 & -3 & 2 & -1 \end{bmatrix}$ is

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

a) What is the rank of A.
b) What is the dimenstion of the column space of A?
c) What is the dimension of the null space of A?
d) Find a solution to $A\vec{x} = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$.
e) Find the general solution to $A\vec{x} = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$.
f) What is the row space of A?
g) Would any of your answers above change if you changed A by randomly changing 3 of its entries in the 2nd, third, and fourth columns to different small integers and the corresponding reduced echelon form for B was presented? (yes?, no?, probaly? probaly not?, ?)

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1.2.20 Let $A = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 2 & 5 \\ 2 & 1 & -3 \end{bmatrix}$.

a) Find A^{-1}.
b) Use A^{-1} to solve $A\mathbf{x} = \mathbf{b}$ when $\mathbf{b} = \begin{bmatrix} 5 \\ 2 \\ -5 \end{bmatrix}$.

MATH 293 FALL 1998 PRELIM 2 # 3 293FA98P2Q3.tex

1.2.21 If they exist, use row reduction to find all solutions of the system of equations $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 5 & 1 & 3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}.$$

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1.2.22* Find all solution to the following matrix equation $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 0 \\ 1 & 2 & 2 \end{bmatrix}$$

for each of the following values of $\vec{b}$:

a) $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

b) $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

c) $\vec{b} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$

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1.2.23 Find the general solution of $\mathbf{x}$ to each of the equations below in vector form (partial credit for any form if you forget the meaning of "vector form").
If there is no solution explain why not.

a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{X} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{X} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \mathbf{X} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 4 \\ 0 & 4 \end{bmatrix} \mathbf{X} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \mathbf{X} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$