Typing is great, but not needed. Also, festering on computer typesetting ~~you~~ often leads to not enough, and not good enough sketches. This HW could use a bit more illustration. **Should** use! i.e. FBDs, dimension sketches.

15h

MAE 5735
HW #9

Problem #9

Two masses m1 and m2 are constrained to move frictionlessly on the x axis. Initially they are stationary at positions $x_1(0) = 0$ and $x_2(0) = l_0$. They are connected with a linear spring with constant k and rest length l_0. A force is applied to the second mass. It is a step, or 'Heaviside' function

$$F(t) = F_0 H(t) = \begin{cases} 0 & if\ t < 0 \\ F_0 & if\ t \geq 0 \end{cases}$$

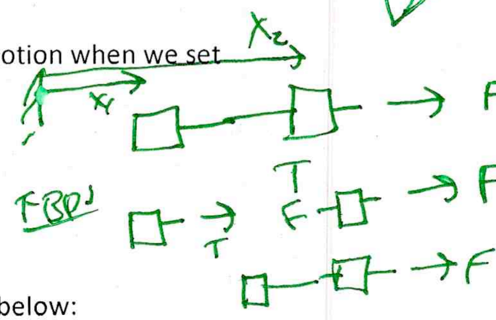
(a) Write code to calculate, plot and (optionally) animate the motions for arbitrary values of the given constrants.

Where are ODEs?

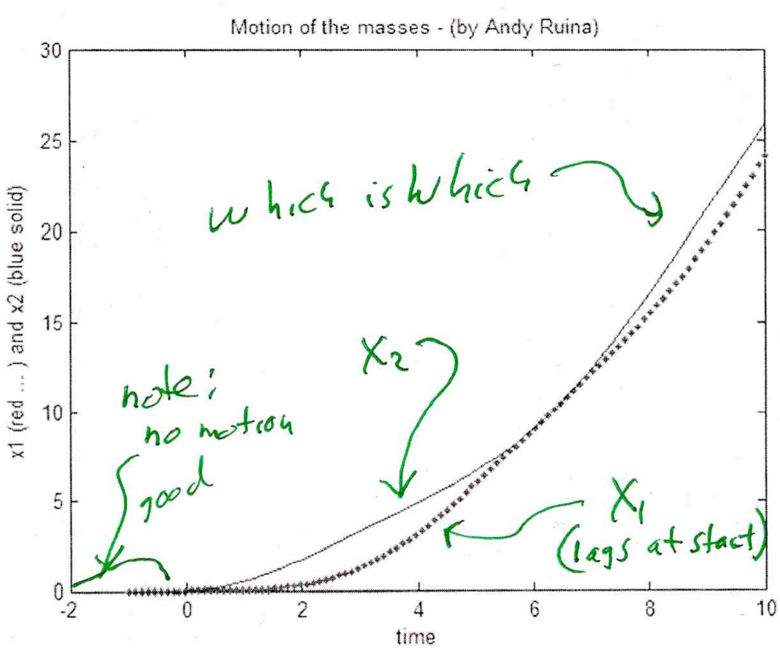
Solution:

Based on the code provided by Prof. Ruina, we can get the motion when we set

$$m_1 = m_2 = 1$$
$$k = 0.5$$
$$l_0 = 3$$
$$F_0 = 1$$



The motion of these two masses can be shown as the figure below:



Don't be afraid to mark up your computer output by hand!

(b) Within numerical precision, should your numerical solution always have the property that $F = (m_1 + m_2)a_G$ where $x_G = (x_1 m_1 + x_2 m_2)/(m_1 + m_2)$?

Solution:

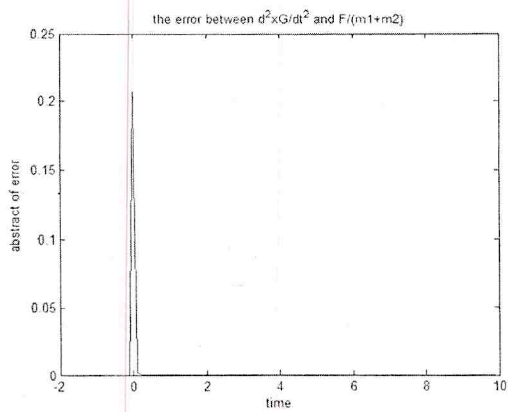
Here, I wrote some code to calculate the error between $\frac{d^2x_G}{dt^2}$ and $\frac{F}{m_1+m_2}$, the code is listed below:

```
function twomasses1()
clear all
clc
%Minimal code.  Not good enough to hand in.
p.m  = [1 1]';  p.k  = .5;
p.L0 = 3;       p.F0 = 1;

tspan = linspace(-1,10,101);
z0    = [0 p.L0 0 0]' ;

[t zarray] = ode23(@rhs,tspan, z0,[],p);

x1 = zarray(:,1);    x2 = zarray(:,2);
x1= x1 - x1(1);      x2 = x2-x2(1);
xG=(x1*p.m(1)+x2*p.m(2))/(p.m(1)+p.m(2));
aG1=diff(xG,2)/(t(2)-t(1))^2;
for i=1:length(aG1)
    tplot(i)=t(i+1);
    if tplot(i)<0
        aG2(i)=0;
    else
        aG2(i)=p.F0/(p.m(1)+p.m(2));
    end
end
error=abs(aG1-aG2');
j=0;
for i=1:length(error)
    if tplot(i)>0.11
        j=j+1;
        tplotnew(j)=tplot(i);
        errornew(j)=error(i);
    end
end
figure(1)
plot(tplot,error)
title('the error between d^2xG/dt^2 and F/(m1+m2)')
xlabel('time')
ylabel('abstract of error')
figure(2)
plot(tplotnew,errornew)
title('the error between d^2xG/dt^2 and F/(m1+m2)after t=0.11')
xlabel('time')
ylabel('abstract of error')
end


function zdot =rhs(t,z,p)
x=z(1:2); v=z(3:4);

F=0;
```

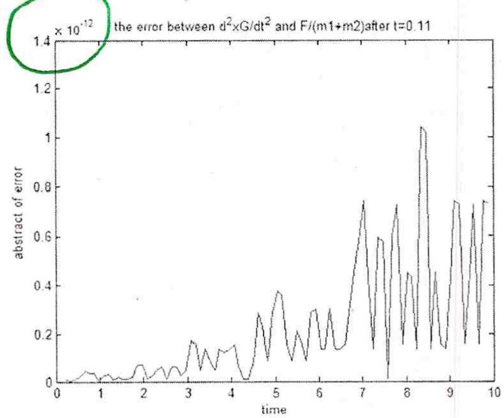
```
if t>=0, F=p.F0; else F=0;   end

T = p.k*(x(2)-x(1) - p.L0);

  xdot  = v;
  vdot  = [ T   (-T+F) ]'./ p.m;

zdot  = [xdot;vdot];
end
```

Is this what you expected? why? why not?

the results are:



10^-12



As shown above, because of the interpolation error, the error around $t = 0$ is very large, but after $t = 0.11$, as we can see from the right picture, the error is very small and acceptable. Another conclution is that the error will increase with time.

(c) Use your numeric to demonstrate that if k is large the motion of each mass is, for time scales large compared to the oscillations, close to the center of mass motion.

Solution:

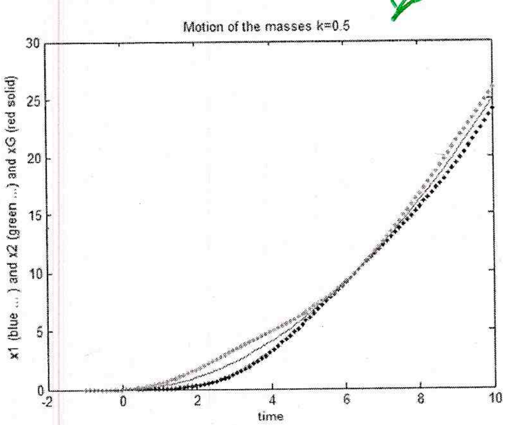
Here, we plot the motion of x_1, x_2, x_G, and change the value of k, and see what is going on. The code for this section is listed as below:

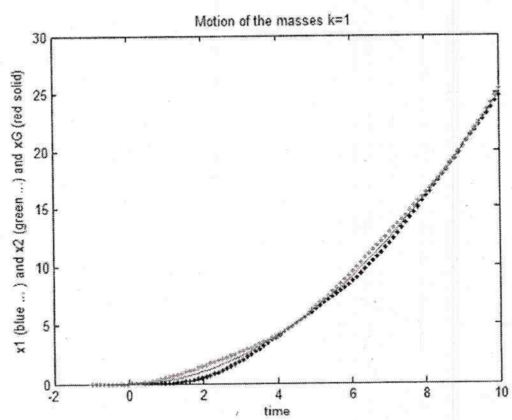
```
function twomasses1()
p.m   = [1 1]';  p.k  = .5;
p.L0  = 3;        p.F0 = 1;

tspan  = linspace(-1,10,101);
z0     = [0 p.L0 0 0]' ;

[t zarray] = ode23(@rhs,tspan, z0,[],p);

x1  = zarray(:,1);    x2  = zarray(:,2);
x1= x1 - x1(1);       x2  = x2-x2(1);
```

```
xG=(p.m(1)*x1+p.m(2)*x2)/(p.m(1)+p.m(2));

plot(t,x1,'b.', t,x2,'g.',t, xG,'r')
  title('Motion of the masses')
  xlabel('time')
  ylabel('x1 (blue ... ) and x2 (green ...) and xG (red solid)')
end


function zdot =rhs(t,z,p)
x=z(1:2); v=z(3:4);

F=0;
if t>=0, F=p.F0; else F=0;  end


T = p.k*(x(2)-x(1) - p.L0);

  xdot = v;
  vdot = [ T  (-T+F) ]'./ p.m;

zdot = [xdot;vdot];
end
```

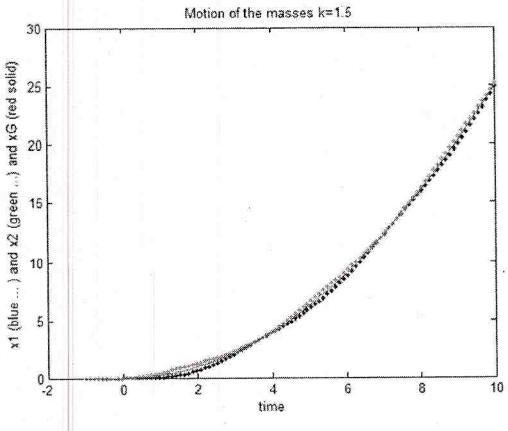
labels of curves . point out on curve what you are trying to show.

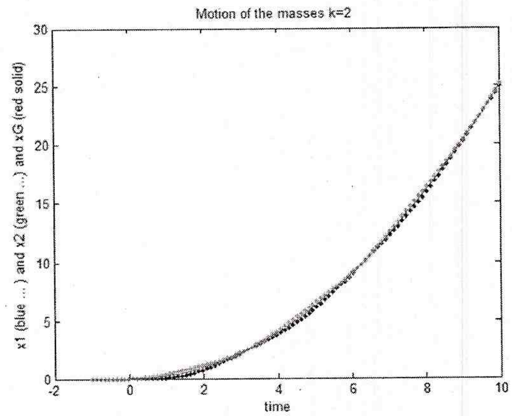
The results are shown below:





point out issues on plots





As we can see, the difference between each mass motion and center mass motion decreases with the increasing of k. For all init. conditions?

(d) Using analytic arguments, perhaps inspired by and buttressed with numerical examples, make the following statement as precise as possible:

For high values of k the system nearly behaves like a single mass.

Solution:

First I define z as:

$$z = x_2 - x_1 - L_0$$

Then I say, the measure of the extent to which the system is 1DOF should be:

$$\text{measure} = \max(|z|)$$

Now, we need to get the value of z.

For the system, we have the equation of motion as below:

$$\ddot{x_1} = \frac{k(x_2 - x_1 - L_0)}{m_1}$$

$$\ddot{x_2} = \frac{F_0 - k(x_2 - x_1 - L_0)}{m_2}$$

So, if we use the second equation minus the first one, we get:

$$(\ddot{x_2} - \ddot{x_1}) = \frac{F_0}{m_2} - k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)(x_2 - x_1 - L_0)$$

So,

$$\ddot{z} + k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)z = \frac{F_0}{m_2}$$

To solve this equation, we get,

$$z = C1cos\sqrt{k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}t + C2sin\sqrt{k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}t + \frac{F_0 m_1}{k(m_1 + m_2)}$$

Using the initial condition, $z(0) = 0, \dot{z}(0) = 0$, we know,

$$C1 = -\frac{F_0 m_1}{k(m_1 + m_2)}$$

$$C2 = 0$$

So,

$$z = -\frac{F_0 m_1}{k(m_1 + m_2)} cos \sqrt{k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)} t + \frac{F_0 m_1}{k(m_1 + m_2)}$$

So,

$$measure = \max(|z|) = \frac{2F_0 m_1}{k(m_1 + m_2)},$$

Now, the statement should be,

$$\lim_{k\to\infty} measure = 0$$

Which means when k tends to be ∞, the distance between two masses tends to be L_0 all the time.

For ICs of ~~the~~ where initial spring stretch is zero.