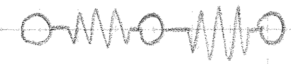
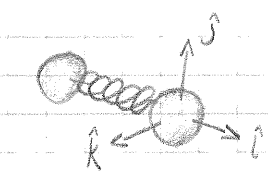
31. For each of the following systems, find as many normal modes, and their frequencies, as you can <u>without</u> doing matrix calculations.

a) 3 equal masses in a line connected by two springs. No points are grounded.



1. Translation of all 3 masses; frequency $= 0$
2. First and last mass oscillating out of phase; frequency $= \sqrt{k/m}$
3. First and last mass oscillating in phase, both out of phase with middle mass; frequency $= \sqrt{\frac{3k}{m}}$

b) Two unequal masses, m_1 and m_2, are at points $\vec{r}_1$ and $\vec{r}_2$ in 3D space and are connected by one spring k. No points are grounded.

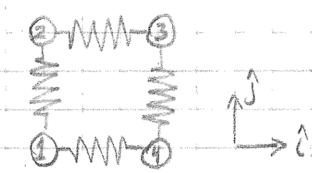


1-2. Both masses in rotation about $\hat{j}$ and $\hat{k}$; frequency $= 0$

3-5. Both masses in translation along $\hat{i}$, $\hat{j}$, and $\hat{k}$; frequency $= 0$

6. Masses oscillating along $\hat{i}$; frequency $= \sqrt{\frac{(m_1+m_2)k}{m_1 m_2}}$

c) 4 point masses are arranged in a square; the 4 edges are equal massless springs



1. All 4 masses in rigid rotation; frequency $= 0$

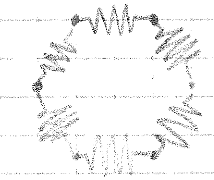
2-3. All 4 masses in rigid translation along $\hat{i}$ and $\hat{j}$; frequency $= 0$

4. Masses 1 and 2 oscillating out of phase from masses 3 and 4; frequency $= \sqrt{\frac{2k}{m}}$
5. Masses 2 and 3 oscillating out of phase from masses 1 and 4; frequency $= \sqrt{\frac{2k}{m}}$
6. Masses 2 and 3 orbiting their barycenter and masses 1 and 4 orbiting their barycenter; frequency $= 0$
7. Masses 1 and 2 orbiting their barycenter and masses 3 and 4 orbiting their barycenter; frequency $= 0$
8. "Breathing", All 4 masses moving radially in-phase; frequency $= \sqrt{\frac{k}{2m}}$

31. (continued)

d) A regular hexagon has point masses at the vertices and springs on the edges. No points are grounded. Find one mode of vibration that does not have zero frequency.

1) "Breathing", all masses moving radially in-phase; frequency $= \sqrt{\frac{3K}{2M}}$



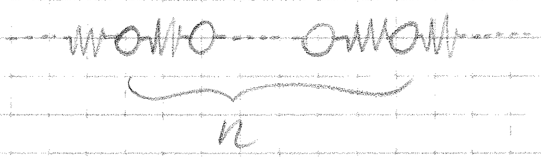
e) An infinite line of point masses m is connected by equal springs k. One normal mode and frequency is:

$\vec{V} = [\ldots 1 \ \ -1 \ \ 1 \ \ -1 \ldots]$ $\quad \omega = 2\sqrt{k/m}$

Find one or more other modes and frequencies.

One simple mode is the rigid motion of all masses (i.e. $\vec{V} = [\ldots 1 \ \ 1 \ \ 1 \ldots]$) with frequency $= 0$, but this is a boring answer.

Consider a subset of n masses:



We can dictate that the first and last masses in this subset remain stationary by simply assuming that the n-mass subsets that precede and follow this one will have the same motion as this one, but with opposite magnitude. These preceding and following subsets will also have stationary endpoints if we continue the process inductively. Thus, the problem becomes that of finding normal modes of a system of n masses between two rigid walls. In fact the infinite set of normal modes for this infinite system is just the space of all such normal modes for $n \in [0, \infty)$ (see reverse)

31. e) (continued)

Effectively this means that the normal modes will have amplitudes determined by sine waves over the space of masses, with wavelengths of $\frac{\pi}{n}$. I suspect that this is, in fact, this is the basis of energy quantization in quantum mechanics, as there is a minimum wavelength determined by the spacing of the masses.