(30 min) 24. Use double pendulum solution (with assumptions) to see if normal modes exist.

$$\ddot{\theta}_2 = \frac{-\frac{mgl}{2} \cdot \sin\theta_2}{\frac{ml^2}{4} + I_2} \quad \text{where } I_2 = 0 \text{ and } \sin\theta_2 = \theta_2$$

$$= \frac{-\frac{\cancel{m}g\cancel{l}}{2} \cdot \theta_2}{\frac{\cancel{m}l^{\cancel{2}}}{\cancel{4}2}} = \frac{2g\theta_2}{l}$$

$$\frac{l}{2}(3mg\sin\theta_1 - \sin\theta_2) = \left(\frac{5ml^2}{4} + I_1 + \frac{ml^2}{2}\cos(\theta_2-\theta_1)\right)\ddot{\theta}_1 + \left(I_2 + m\frac{l^2}{4} + m\frac{l^2}{2}\cos(\theta_2-\theta_1)\right)\left[\frac{-mgl\sin\theta_2}{ml^2/2 + 2I_2}\right]$$

$$\frac{l}{2}(3mg\theta_1 - \theta_2) = \left(\frac{5ml^2}{4} + \frac{ml^2}{2}\right)\ddot{\theta}_1 + \left(m\frac{l^2}{4} + m\frac{l^2}{2}\right)\left[\frac{-mgl\theta_2}{\cancel{m}l^2/2}\right]$$

$$\frac{\cancel{l}}{2}(3mg\theta_1 - \theta_2) = \frac{7}{4}ml^2\ddot{\theta}_1 + \frac{3}{4}m\cancel{l^2} \cdot \frac{2gl\theta_2}{\cancel{l}} = \frac{7}{4}ml^2\ddot{\theta}_1 + \frac{3}{2}mgl\theta_2$$

$$3mg\theta_1 - \theta_2 = \frac{7}{2}ml\ddot{\theta}_1 + 3mg\theta_2$$

$$\ddot{\theta}_1 = \frac{3mg\theta_1 - \theta_2 - 3mg\theta_2}{7/2\, ml} = \frac{6mg\theta_1 - 2\theta_2(1-3mg)}{7ml}$$

Putting into form of $M\ddot{\vec{\theta}} + K\vec{\theta} = \vec{0}$

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\ddot{\vec{\theta}} + \begin{bmatrix} \frac{6g}{7l} & \frac{-2(1-3mg)}{7ml} \\ 0 & \frac{2g}{l} \end{bmatrix}\vec{\theta} = \vec{0}$$

See attached Matlab code for plotted normal modes

Contents

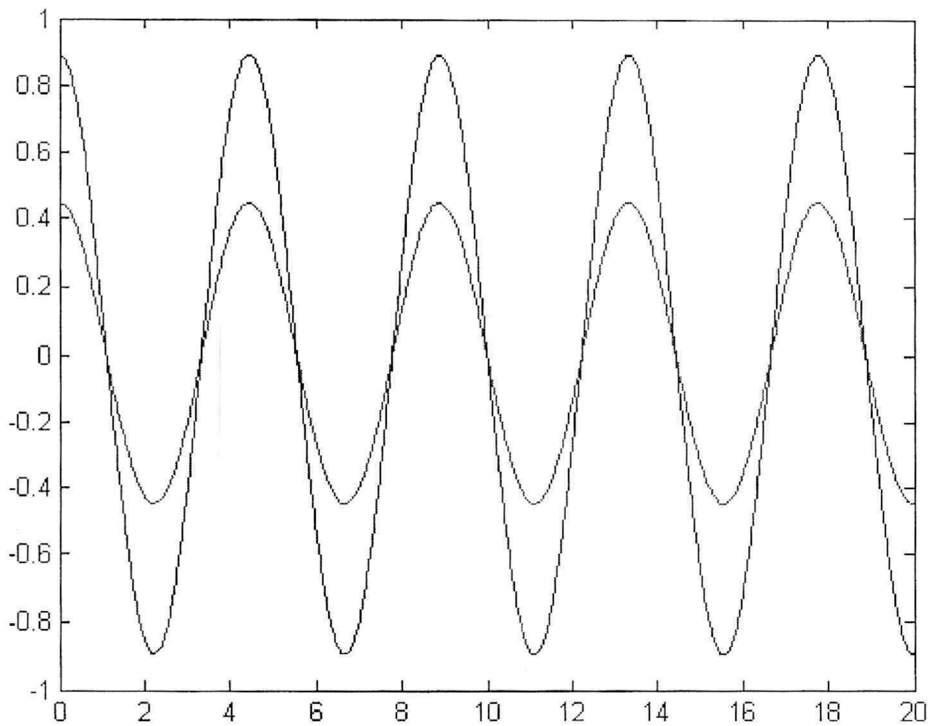

```
function HW24()
```

Problem Statement

Problem Setup

```
clear all
close all
clc

% Define parameters
m = 1;
g = 1;
l = 1;

M = [m     0;...
     0     m];
K = [(6*g)/(7*l)      -2*(1-3*m*g)/(7*m*l);...
     0                2*g/l];

[p, lambda] = eig(M^-1*K)

eVect1 = p(:,1);
lambda1 = lambda(1);

eVect2 = p(:,2);
lambda2 = lambda(2);

x0 = [eVect2(1)      eVect2(2)]';
v0 = [0    0]';
zzero = [x0;v0];

n= 1000;

tspan = linspace(0, 20, n);

%Method I: ODE soln
[t zmatrix] = ode45(@method, tspan, zzero, [], M, K);

x1 = zmatrix(:,1);
x2 = zmatrix(:,2);

plot (t,x1,'r',t,x2,'b');
```

```
p =

    1.0000    0.4472
         0    0.8944


lambda =

    0.8571         0
         0    2.0000
```




```
end

function zdot = method(t, z, M, K)
x = z(1:2);
v = z(3:4);

xdot = v;
vdot = -M^-1 * K*x;


zdot = [xdot; vdot];

end
```

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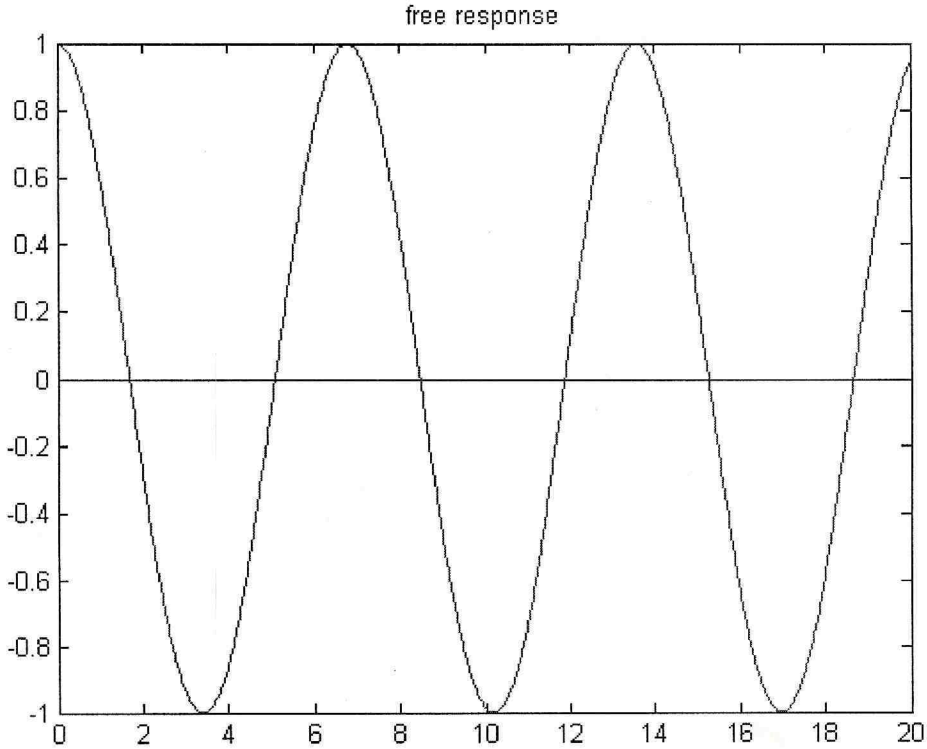


```
end

function zdot = method(t, z, M, K)
x = z(1:2);
v = z(3:4);

xdot = v;
vdot = -M^-1 * K*x;


zdot = [xdot; vdot];

end
```

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Check w/ full non-lin. eqs?

(see if normal mode is an approx soln.).