(Time to complete = 1.5 hours)

5) Periodic motions for a central force.

By numerical experiments, and trial and error, try to find a period motion that is neither circular nor a straight line for some central force besides $F = -kr$ or $F = -\frac{GmM}{r^2}$.

* See attached Matlab code which was used to check for periodic motions arising from different central forces *

Different central forces that I tried:

$F = \frac{-k}{r}$

$F = -kr^2$

$F = -kr^3$

$F = -ke^r$

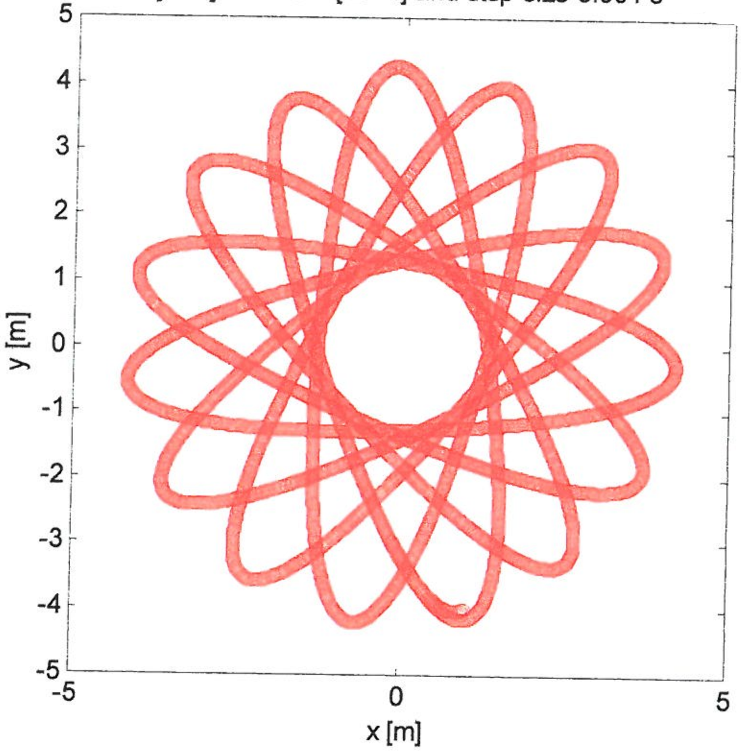
$F = -k\ln(r)$

Cool

Plots of the various trajectories are attached.

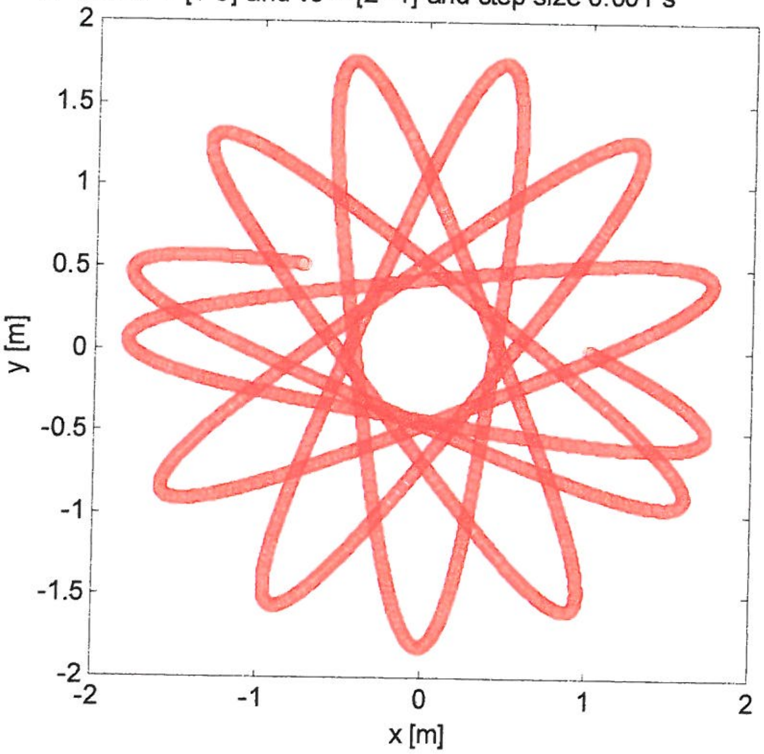
```
%By
%MAE 5735
%HW 2, Problem 5

%This program attempts to find periodic motions for a central force by
%numerical experiments. The motion can be neither circular nor a straight
%line, and must have a central force other than F = -kr or F = -GMm/r^2.

clc
clear all
close all

%Givens
m = 1; %Mass in kg
k = 1; %Spring constant in N/m

%Initial vectors of interest
r = [1 0]'; %Initial position vector
v = [2 -1]'; %Initial velocity vector

%Implementation of Euler's method
step = 0.001; %Step size in seconds used in the Euler method
time_span = 75; %Total time in seconds used to calculate trajectory
for j = 1:(time_span/step)
    F = (-k/(norm(r))^2)*r; %Central force
    a = F./m; %Acceleration vector
    r = r+(step)*v; %Updating the position vector
    v = v+(step)*a; %Updating the velocity vector
    plot(r(1),r(2),'ro') %Plotting the trajectory
    hold on
end
axis square
xlabel('x [m]')
ylabel('y [m]')
title('F = -k/r with r0 = [1 0] and v0 = [2 -1] and step size 0.001 s
    ')
```

$F = -kr^2$ with r0 = [1 -4] and v0 = [-2 -1] and step size 0.001 s



NICE

$F = -kr^3$ with r0 = [1 0] and v0 = [2 -1] and step size 0.001 s



Numerical error?

in general! But they all can.

None are chaotic. *

5) (continued)

As can be seen from the various plots, none of the central forces that I tried produced periodic motions. In addition, in my failed searches the motions did not necessarily have regular patterns and some were chaotic looking. The trajectory for $F = -k\ln(r)$, for example, is especially ~~chaotic~~ looking. The changes in the trajectory do not seem ordered in any way. This is in contrast to the trajectory for $F = -ke^{r}$, which seems to produce more regular elliptical spirals. I am not worried that I was unable to find a central force other than $F = -kr$ or $F = -\frac{GmM}{r^2}$ which produces a periodic motion. This is because of $\boxed{\text{Bertrand's}}$ theorem, which states that the only two types of central forces which produce stable, closed orbits are $F = -kr$ and $F = -\frac{GmM}{r^2}$.

You didn't learn that from me!

NOT RELEVANT?

* quasiperiodic ≠ chaotic. Note $|\vec{v}|$ is always periodic for all of your solns.