Your year in school (semester, degree sought):

Your name:

Cornell MAE 4735/5735

Final Exam

Dec 10, 2012

No calculators, books or notes allowed.

5 Problems, 150 minutes, no extra time (Cornell rules)

How to get the highest score?

Please do these things:

- ? If, when working on a problem, you have any *questions* about what you should or should not assume or write, please read these directions again.
- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- $\vec{v}_{\text{vect}}$ Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- □ TIDILY REDUCE and box in your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{n}} \ldots$) and signs ($\pm$) with sketches, equations or words.
- $\rightarrow\!|$ **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- $\approx$ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.
  Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 7: ____ /25

Problem 8: ____ /25

Problem 9: ____ /25

Problem 10: ____ /25

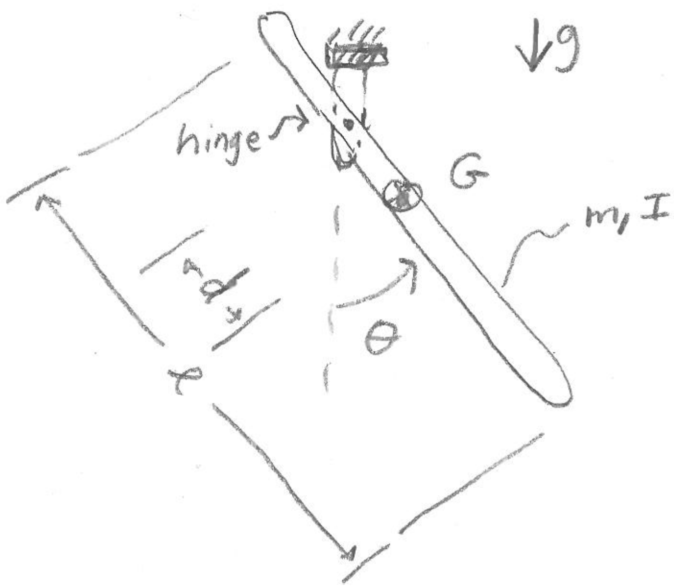
Problem 11: ____ /25

7a) 2D. A stick with length ℓ, mass m and moment of inertia I about its center of mass G is suspended from a hinge on the stick a distance $d < \ell/2$ from G.

(i) Find the equations of motion.

(ii) For given $\theta, \dot{\theta}, m, I, \ell$ and d find the force of the hinge on the pendulum (using any base vectors you like).

(iii) For given ℓ, m, I and gravitational acceleration g for what d is the period of small oscillation minimized. If you can reduce this to finding the root of a polynomial or transcendental equation, that is good enough, you need not find the root. [Hints: Find the equations of motion → Solve them → Find the period of small oscillations → Minimize.]



7b) A block with mass M slides without friction on a flat level surface. The top surface has slope γ. A smaller block with mass m slides without friction on the sloped top of the lower block.

(i) Find two scalar equations from which you could, if you liked, solve for $\ddot{s}$ and $\ddot{x}$ in terms of some or all of $x, \dot{x}, s, \dot{s}, \gamma, m, M$ and g [that is, you need not invert the mass matrix].

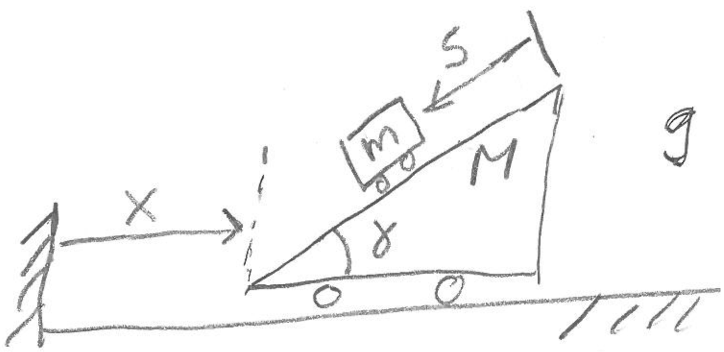
(ii) Some of the statements below are true, some are false, some may be partially true. Say which, and say why (with equations and/or words) clearly enough so that a skeptic would be convinced.

(a) system potential energy is conserved

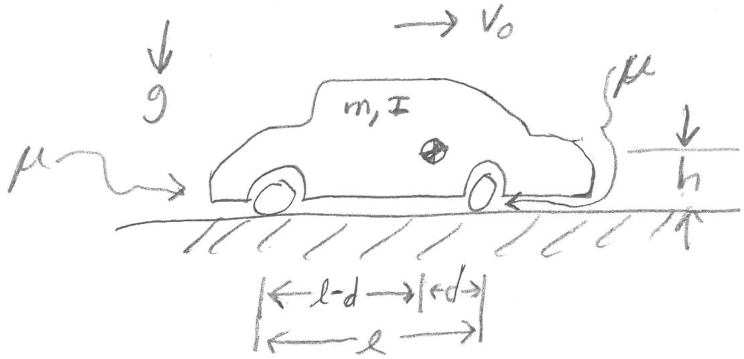
(b) system kinetic energy is conserved

(c) system total energy is conserved

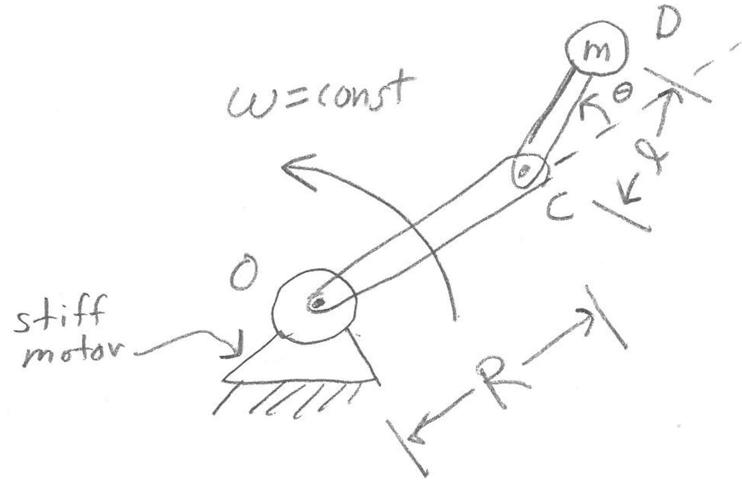
(d) system linear momentum is conserved



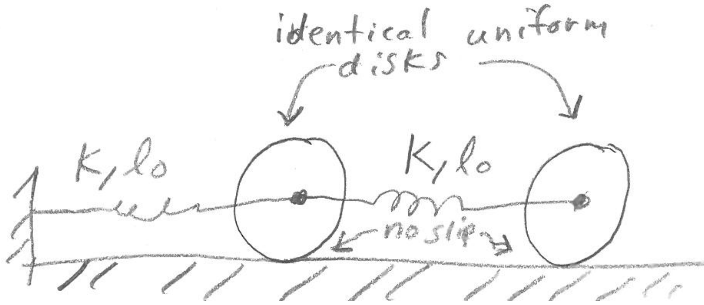
8a) A car with mass m moves at speed v_0 and suddenly slams on the brakes. All four wheels skid with friction coefficient μ (and friction angle ϕ with $\tan\phi = \mu$). Assume the suspension is rigid. In terms of some or all of $d, \ell, h, \mu, \phi, m, I$ and gravity g find how long it takes for the car to come to a stop.



8b) No gravity. One link of a pendulum has radius R and is powered by a stiff motor to rotate at a fixed rate ω. The second link has length ℓ, is massless, and has a point mass m at the end. Find $\ddot{\theta}$ in terms of some or all of $\theta, \dot{\theta}, R, \ell$ and ω.



9) Two uniform round disks with mass m roll without slipping on a flat plane. They are connected to each other and to the left wall with two springs, both with stiffness k and rest length ℓ_0. Find one normal mode and it's corresponding frequency.



10a) This problem concerns the classical 1 DOF spring-mass system with m, k and c.

(a) Make a clear picture of the system.

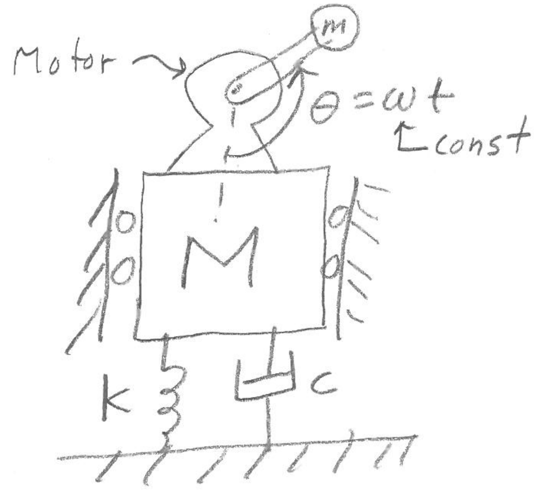
(b) Using any mechanics method you like, find the equations of motion: $m\ddot{x} + c\dot{x} + kx = 0$.

(c) Reduce this to standard form: $\ddot{x} + 2\omega\eta\dot{x} + \omega^2 x = 0$

(d) Define both ω and η with equations. Explain the meaning of both terms with words.

(e) Find an equation for the damped natural frequency ω_d.

10b) A platform and motor with total mass M are supported by a spring (k, ℓ_0) and a dashpot (c). The motor spins at constant rate ω so that $\theta = \omega t$. The motor spins an eccentric mass m which is a distance d from the motor shaft. Find the equations of motion for the mass M consisting of the motor and platform (relative to the static equilibrium position).



11) A system has 7 degrees of freedom paramaterized by the components of the 7-element vector $\vec{\boldsymbol{x}}$. The equations of motion, for small motion, are:

$$M\ddot{\vec{\boldsymbol{x}}} + C\dot{\vec{\boldsymbol{x}}} + K\vec{\boldsymbol{x}} = \vec{\boldsymbol{0}}$$

where the matrices M and K are given and symmetric.

(i) Assume that $C = 0$. Write MATLAB commands to find a constant vector $\vec{\boldsymbol{v}}$ and ω so that

$$\vec{\boldsymbol{x}}(t) = \sin(\omega t)\vec{\boldsymbol{v}}$$

is a solution of the governing equations. Just one normal mode solution is desired.

(ii) Clearly define at least one non-zero damping matrix C so that, with an appropriate change of variables the equations can be re-written as a set of decoupled scalar equations of the form:

$$\ddot{r}_i + 2\omega_i \eta_i \dot{r}_i + \omega_i^2 r_i = 0.$$

As for all problems, justify your result.

(iii) Clearly define the most general damping matrix C for which, with an appropriate change of variables the equations can be re-written as a set of decoupled scalar equations (as written above).